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Data-Driven Model-Free Adaptive Control with Prescribed Performance: a rigorous Sliding-Mode based approach

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Abstract:
In this paper, the data-driven control approach known as Model-Free Adaptive Control technique is applied to the control of a class of general discrete-time Single-Input Single-Output nonlinear systems, making use of model obtained adopting a dynamic linearization technique based on pseudo-partial derivatives. The present study is inspired by the very recent paper [Liu and Yang, 2019], where a data-driven adaptive sliding mode controller has been proposed able to account also for prescribed performance constraints. In particular, a rigorous stability analysis is here proposed, achieved modifying the forms of the sliding surface and of the control law but still retaining the main setup presented in the source paper. The careful analysis of the closed loop system here provided is shown to lead to the definition of suitable constraints on the gain of the sliding-mode based control term. A comparative study, by simulation, is also provided, performed using a test taken from the literature. Results show a remarkable improvement of control accuracy.

Keywords: Data-driven Control, Model-Free Adaptive Control, Prescribed performance Control, Sliding Mode Control.

1. INTRODUCTION

Design techniques belonging to modern control theory are usually derived on the basis of the fundamental assumption that the mathematical model or nominal model of the controlled plant is a priori known, at least with some degree of accuracy. Nonetheless, when facing the complexity of real world processes, the designer has first to solve the tough problem of building a suitable model, descriptive enough to capture the plant behavior but sufficiently simple to allow the application of model-based control techniques. Due to the complexity of nowadays industrial plants, building an accurate model is a difficult and expensive task, often leading to high order and strongly nonlinear models which produce, in turn, serious difficulties in analysis and control design. Also, realization and maintenance costs of complicated controllers are often unacceptable. On the other hand, the development of information science and technology in recent years have induced significant changes in industrial sites so that large amount of process data, generated by plants and industrial processes, are available to control designers. Therefore, data-driven control methods [Yin et al., 2014] have been derived in order to use the available process data to develop efficient control and optimization methods for industrial processes when accurate process models are unavailable. Many different data-driven control approaches are available, all sharing the fact that control design depends only on Input/Output (I/O) data of the controlled plant and on the information about the process operations implicitly contained therein [Hou and Xu, 2009] [Hou and Jin, 2011]. Even the well known Ziegler-Nichols tuning procedure for PID controllers could be potentially ascribed to the category of data-driven approaches [Hou and Jin, 2011] [Wang et al., 2016b]. Among the various available techniques, the approach known as Model-Free Adaptive Control (MFAC) has been recently proposed for a class of general discrete-time nonlinear systems. First discussed in [Hou, 1994], extended in [Hou and Jin, 2011] and finally thoroughly formalized in [Hou and Jin, 2014], MFAC makes use of an equivalent dynamic linearization model obtained adopting a dynamic linearization technique based on pseudo-partial derivatives (PPD). In addition to a number of interesting features, discussed in [Hou and Jin, 2011] and ranging from low cost and easy applicability, successful implementation in practical applications, and absence of training phases, it is worth to notice that BIBO stability and closed loop convergence of the tracking error has been theoretically proved under mild assumptions [Hou and Jin, 2011]. In this framework, the proposal has been very recently presented [Liu and Yang, 2019] to couple the so called Prescribed Performance Control (PPC) [Bechlioulis and Rovinthakis, 2008] [Bechlioulis and Rovinthakis, 2009] with MFAC, in order to embed a constraint on the tracking error within the MFAC mechanism. In the original formulation of PPC, transient and steady-state features of the tracking error are constrained according to given performance bounds constructing a suitable error transformation so that the unconstrained transformed error (instead of the constrained tracking error) is easily stabilized. Although most of the available results about PPC refer to model-based approaches [Zhang and Yang, 2017], [Zhai et al., 2017], [Wang et al., 2016a], [Wang and Yang, 2017], [Nguyen et al., 2018], the paper [Liu and Yang, 2019] presents, perhaps for the first time, a data-driven adaptive sliding mode controller for nonlinear discrete-time systems with prescribed performance constraints. This very interesting approach, however, suffers from limitations deriving from an unsatisfactory theoretical proof of convergence. In particular, the presented stability analysis is performed using, for the process, the *estimated* Partial Form Dynamic Linearization (PFDL) model of the plant and not the *real* model, this leading to a poorly meaningful theoretical development. Therefore, inspired by the work [Liu and Yang, 2019] and according to the previous considerations, the main contributions of the present paper are the following:

- still retaining the main setup presented in [Liu and Yang, 2019], the forms of the sliding surface and of the control law have been modified in order to provide a rigorous proof ensuring the boundedness of the control law;
- a rigorous stability of the closed loop system is provided, leading to the definition of suitable constraints on the gain of the sliding-mode based control term;
- a comparative analysis has been performed using a test taken from the literature, showing a remarkable improvement of control accuracy (ranging from 30% to 35%) with respect to both the approaches described in [Liu and Yang, 2019] and [Hou and Jin, 2011].

The paper is organized as follows. Section 2 presents some preliminary issues about the considered model and the control problem. The main technical results are reported in Section 3, where closed loop stability is proved and ensured for suitable choices of the control and estimation parameters. A simulation study is finally reported in Section 4, showing a noticeable performances improvement, in terms of tracking accuracy, of the proposed control law. The paper ends with a few comments reported in Section 5.

2. PROBLEM STATEMENT

2.1 Model

Consider the following discrete-time SISO nonlinear system [Hou and Jin, 2011]:

$$y(k+1) = f(y(k)\dots y(k-n_y), u(k), u(k-1), \dots, u(k-n_u)) \tag{1}$$

where $u(k)$ and $y(k)$ are the system input and output at time k, n_y and n_u are unknown orders, and $f(\dots)$ is an unknown nonlinear function. The Partial Form Dynamic Linearization (PFDL) of the plant (1) is based on the following Assumptions.

Assumption 2.1. The partial derivatives of $f(\dots)$ with respect to the control input $u(k)$, $u(k-1)$,...,$u(k-d_u)$ are continuous, d_u being a positive discrete constant known as control input length constant of linearization for the discrete-time nonlinear system.

Define: $\Delta y(k+1) = y(k+1) - y(k)$, $\Delta u(k-i) = u(k-i) - u(k-i-1)$, $i = 0, 1, \dots, d_u - 1$, $\mathbf{\Delta U}(k) = [\,\Delta u(k), \Delta u(k-1), \dots, \Delta u(k-d_u+1)\,]$, with $u(k) = 0$ for $k \le 0$.

Assumption 2.2. The plant $f(\dots)$ is generalized Lipschitz, i.e. $|\Delta y(k+1)| \le b||\mathbf{\Delta U}(k)||$, $b \in \mathbb{R}_+$, and $||\mathbf{\Delta U}(k)|| \ne 0$ $\forall k > 0$

The Pseudo Partial Derivative (PPD) based model of the plant (1) relies on the following Theorem [Hou and Jin, 2011].

Theorem 2.1. For the nonlinear system (1) satisfying Assumptions 2.1 and 2.2, there exists a parameter vector $\mathbf{\Phi}(k)$, called the PPD vector, such that the plant can be transformed into the following equivalent PFDL description

$$\Delta y(k+1) = \mathbf{\Phi}(k)^T \mathbf{\Delta U}(k) \tag{2}$$

where $\mathbf{\Phi}(k) = [\,\phi_1(k), \phi_2(k), \dots, \phi_{du}(k)\,]$ and $||\mathbf{\Phi}(k)|| \le b$.

where b is a positive constant. Following [Hou and Jin, 2011], the estimate $\hat{\mathbf{\Phi}}(k)$ of the unknown PPD vector $\mathbf{\Phi}(k)$ can be derived using the modified projection algorithm starting from the following cost function:

$$\begin{aligned} J(\hat{\mathbf{\Phi}}(k)) = |y(k) - y(k-1) - \hat{\mathbf{\Phi}}(k)^T \mathbf{\Delta U}(k-1)|^2 \\ + \mu||\hat{\mathbf{\Phi}}(k) - \hat{\mathbf{\Phi}}(k-1)||^2; \quad \mu > 0 \end{aligned} \tag{3}$$

obtaining:

$$\begin{aligned} \hat{\mathbf{\Phi}}(k) &= \hat{\mathbf{\Phi}}(k-1) + \\ &+ \frac{\eta \mathbf{\Delta U}(k-1)(\Delta y(k) - \hat{\mathbf{\Phi}}(k-1)^T \mathbf{\Delta U}(k-1))}{\mu + ||\mathbf{\Delta U}(k-1)||^2}; \end{aligned} \tag{4}$$

$$\begin{aligned} &\hat{\mathbf{\Phi}}(k) = \hat{\mathbf{\Phi}}(1), \\ &\quad if\ ||\hat{\mathbf{\Phi}}(k)|| \le \epsilon\ or\ sign(\hat{\phi}_1(k)) \ne sign(\hat{\phi}_1(1)) \end{aligned} \tag{5}$$

with $\eta \in (0, 2)$ and ϵ is a positive design constant.

Remark 1. Due to (5), it can be assumed, without loss of generality, that $\hat{\phi}_1(k) > 0$.

2.2 Control Problem

The control problem addressed in this paper is the tracking of a constant reference output variable y^*. In addition, following [Liu and Yang, 2019], a Prescribed Performance Control (PPC) requirement is considered. In particular, a positive, decreasing, discrete-time sequence $\rho(k)$ is defined as follows:

$$\rho(k+1) = (1 - \theta_1)\rho(k) + \theta_1 \rho_\infty, \quad \theta_1 \in (0, 1) \tag{6}$$

with $\rho(0) > \rho_\infty > 0$. It is required that

$$-\rho(k) < e(k) < \rho(k) \tag{7}$$

where $e(k) = y^* - y(k)$ is the tracking error. According to [Liu and Yang, 2019], the following transformed error is introduced:

$$\tau(k) = \frac{1}{2} \ln \left(\frac{\rho(k) + e(k)}{\rho(k) - e(k)} \right) \tag{8}$$

thus transforming the initial problem containing the constraint on the tracking error into an unconstrained problem. Using (8), the following sliding surface is defined:

$$s(k+1) = s(k) + \tau(k+1) \tag{9}$$

differently from [Liu and Yang, 2019].

Remark 2. In this paper, a sliding mode based control law will be provided solving the PPC problem. The reason for proposing the sliding surface (9) is that a rigorous stability analysis will be provided guaranteeing both the achievement of the tracking performances and the boundedness of the closed loop variables. Such stability analysis will be performed, differently from [Liu and Yang, 2019], studying the behavior of the "true" plant (2) (not the estimated one $\Delta y(k+1) = \hat{\mathbf{\Phi}}(k)^T \mathbf{\Delta U}(k)$) fed by the proposed sliding mode control law.

3. CONTROL DESIGN

Before proceeding to the design of the sliding mode based control input, the following Assumption [Hou and Jin, 2011] is required in order to prove stability and convergence of the overall control scheme.

Assumption 3.1. The first element of the PPD vector satisfies $\phi_1(k) > \delta \ \forall k$, where δ is an arbitrary small positive constant.

Further, it should be recalled that, since $\mathbf{\Phi}(k)$ and $\hat{\mathbf{\Phi}}(k)$ are bounded in view of Theorem 2.1, there exists $\lambda_{min} > 0$ such that for $\lambda > \lambda_{min}$ it holds [Hou and Jin, 2011]

$$\left| \frac{\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \right| < M_1 < \frac{0.5}{b}; \quad M_2 \leq \left| \frac{\phi_1(k)\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \right| \leq 0.5 \tag{10}$$

where M_1 and M_2 are positive constant, and there exists $\rho_1 \in (0,1)$ such that

$$\left| 1 - \rho_1 \frac{\phi_1(k)\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \right| < 1 \tag{11}$$

Consider the following control input

$$\Delta u(k) = u_M(k) + u_S(k) \tag{12}$$

where

$$u_S(k) = \frac{\rho_1 |\hat{\phi}_1(k)|}{\lambda + \hat{\phi}_1(k)^2} \Gamma_s(k) sign(s(k)) \tag{13}$$

with $\Gamma_s(k) > 0$ to be defined in the following, and

$$u_M(k) = \frac{\rho_1 \hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \left(e(k) - \sum_{i=2}^{du} \rho_i \hat{\phi}_i(k) \Delta u(k-i+1) + \frac{\rho(k+1)}{1+\xi(k)} \right) \tag{14}$$

where $\xi(k) = \exp(-\alpha\tau(k))$, $\alpha \in (0,1)$ and with $\rho_i \in (0,1)$ properly chosen positive constants. From the definition of the tracking error, one has:

$$e(k+1) = e(k) - \sum_{i=1}^{du} \phi_i(k) \Delta u(k-i+1) \tag{15}$$

Defining $\Delta s(k+1) = s(k+1) - s(k)$, it follows:

$$\Delta s(k+1) = \tau(k+1) = \frac{1}{2} \ln \left(\frac{\rho(k+1) + e(k+1)}{\rho(k+1) - e(k+1)} \right) \tag{16}$$

and after some manipulations one gets:

$$\Delta s(k+1) = \frac{1}{2} \ln \left(\frac{\alpha(k) + M_a(k) - \phi_1(k) u_S(k)}{\beta(k) - M_a(k) + \phi_1(k) u_S(k)} \right) \tag{17}$$

where

$$\alpha(k) \triangleq \rho(k+1) \left(1 - \frac{\rho_1 \phi_1(k)\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \frac{1}{1+\xi(k)} \right) \tag{18}$$

$$\beta(k) \triangleq \rho(k+1) \left(1 + \frac{\rho_1 \phi_1(k)\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \frac{1}{1+\xi(k)} \right) \tag{19}$$

$$M_a(k) \triangleq e(k) \left(1 - \frac{\rho_1 \phi_1(k)\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \right) + \sum_{i=2}^{du} \left(\frac{\rho_i \phi_1(k)\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \hat{\phi}_i(k) - \phi_i(k) \right) \Delta u(k-i+1) \tag{20}$$

Lemma 3. Consider the plant (2) satisfying Assumption 3.1, controlled by the control input (12). Then there exists $\bar{M}_a > 0$ such that $|M_a(k)| \leq \bar{M}_a \ \forall k$.

The proof is omitted for brevity.

Defining:

$$\gamma(k) \triangleq \frac{\rho_1 \phi_1(k)\hat{\phi}_1(k)}{\lambda + \hat{\phi}_1(k)^2} \tag{21}$$

in view of Remark 1, Assumption 3.1 and (10), it holds:

$$0 < \gamma(k) < 0.5 \tag{22}$$

and introducing the following definitions:

$$\nu_b(k) \triangleq \frac{1+\xi(k)}{1+\xi(k)-\gamma(k)} \tag{23}$$

$$\nu_c(k) \triangleq \frac{1+\xi(k)}{1+\xi(k)+\gamma(k)} \tag{24}$$

$$\delta(k) \triangleq \frac{1+\xi(k)-\gamma(k)}{1+\xi(k)+\gamma(k)} \tag{25}$$

due to (22), $\nu_b(k)$, $\nu_c(k)$ and $\delta(k)$ are bounded as follows:

$$1 \leq \nu_b(k) \leq 2 \tag{26}$$

$$\frac{2}{3} < \nu_c(k) \leq 1 \tag{27}$$

$$1/3 < \delta(k) < 1 \tag{28}$$

Moreover, comparing (18), (19) and (23), one has:

$$\alpha(k) = \frac{\rho(k+1)}{\nu_b(k)} \tag{29}$$

$$\beta(k) = \frac{\rho(k+1)}{\nu_c(k)} \tag{30}$$

As a consequence, $\Delta s(k+1)$ can be written as:

$$\Delta s(k+1) = \frac{1}{2} \ln \left(\frac{\rho(k+1) + (M_a(k) - \phi_1(k)u_S(k))\nu_b(k)}{\rho(k+1) - (M_a(k) - \phi_1(k)u_S(k))\nu_c(k)} \right) \delta(k) \tag{31}$$

Theorem 3.1. Consider the plant (2) satisfying Assumption 3.1, controlled by the control input (12). Under the condition:

$$\begin{cases} \bar{M}_a < \min \left\{ \dfrac{\rho_\infty}{\left(6 + \frac{1}{M_2}\right)}, \dfrac{\left(2 - \dfrac{2}{3M_2}\right)\rho_\infty}{\left(2 + \frac{1}{M_2}\right)} \right\} \\ \dfrac{1}{3} < M_2 \leq \dfrac{1}{2} \end{cases} \tag{32}$$

for $\lambda > \lambda_{min}$ in (10) with suitable values of $\lambda_{min} > 0$, the gain $\Gamma_s(k)$ can be properly designed such that the sliding variable $s(k)$ is bounded. As a consequence, $\tau(k)$ is bounded and the condition (7) is satisfied.

Proof The proof consists of two steps. First it will be proved that $s(k)\Delta s(k+1) < 0$, next it will be shown that a region exists bounding the sliding variable.
Step 1, $s(k) > 0$.
Define:

$$\eta(k) \triangleq M_a(k) - \frac{\rho_1 \phi_1(k)|\hat{\phi}_1(k)|}{\lambda + \hat{\phi}_1(k)^2}\Gamma_s(k)$$

The condition $\Delta s(k+1) < 0$ requires that

$$\begin{cases} \eta(k) < 0 \\ \rho(k+1) + \eta(k)\nu_b(k) > 0 \end{cases} \tag{33}$$

Moreover, due to (33), it follows:

$$\rho(k+1) + \eta(k)\nu_b(k) < \rho(k+1) - \eta(k)\nu_c(k) \tag{34}$$

Taking the worst case, and defining $\theta(k) \triangleq \dfrac{\rho_1 \phi_1(k)|\hat{\phi}_1(k)|}{\lambda + \hat{\phi}_1(k)^2}$, conditions (33) correspond to:

$$\bar{M}_a < \theta(k)\Gamma_s(k) < \frac{\rho(k+1)}{\nu_b(k)} - \bar{M}_a \tag{35}$$

and the following strongest condition will be considered instead:

$$\bar{M}_a < \theta(k)\Gamma_s(k) < \frac{\rho(k+1)}{\nu_b(k)} - 3\bar{M}_a \tag{36}$$

Recalling that, according to (10), $M_2 \leq \dfrac{\theta(k)}{\rho_1} \leq 0.5$, condition (36) is fulfilled if:

$$\begin{cases} \Gamma_s(k) > \dfrac{\bar{M}_a}{\rho_1 M_2} \\ \Gamma_s(k) < \dfrac{\rho(k+1) - 6\bar{M}_a}{\rho_1} \end{cases} \tag{37}$$

which requires:

$$\left(6 + \frac{1}{M_2}\right)\bar{M}_a < \rho(k+1) \tag{38}$$

or, as a stronger condition:

$$\left(6 + \frac{1}{M_2}\right)\bar{M}_a < \rho_\infty \tag{39}$$

Step 1, $s(k) < 0$.
Define:

$$\bar{\eta}(k) \triangleq M_a(k) + \theta(k)\Gamma_s(k) \tag{40}$$

The condition $\Delta s(k+1) > 0$ requires that:

$$\begin{cases} \bar{\eta}(k) > 0 \\ \rho(k+1) - \bar{\eta}(k)\nu_c(k) > 0 \\ \rho(k+1) + \bar{\eta}(k)\nu_b(k) > 3(\rho(k+1) - \bar{\eta}(k)\nu_c(k)) \end{cases} \tag{41}$$

Considering the worst case and definition (40), inequalities (41) corresponds to:

$$\frac{2}{3}\rho(k+1) < \theta(k)\Gamma_s(k) + M_a(k) < \rho(k+1) \tag{42}$$

The following strongest condition will be considered instead:

$$\frac{2}{3}\rho(k+1) < \theta(k)\Gamma_s(k) + M_a(k) < a_1\rho(k+1) \tag{43}$$

with $\dfrac{2}{3} < a_1 < 1$.

Taking into account Lemma 3 and recalling that, according to (10), $M_2 \leq \dfrac{\theta(k)}{\rho_1} \leq 0.5$, from (43) the following conditions on $\Gamma_s(k)$ can be derived:

$$\begin{cases} \Gamma_s(k) < \dfrac{2\left[a_1\rho(k+1) - \bar{M}_a\right]}{\rho_1} \\ \Gamma_s(k) > \dfrac{\frac{2}{3}\rho(k+1) + \bar{M}_a}{M_2\rho_1} \end{cases} \tag{44}$$

which requires:

$$\left(2 + \frac{1}{M_2}\right)\bar{M}_a < \left(2a_1 - \frac{2}{3M_2}\right)\rho(k+1) \tag{45}$$

or, as a stronger condition:

$$\left(2 + \frac{1}{M_2}\right)\bar{M}_a < \left(2a_1 - \frac{2}{3M_2}\right)\rho_\infty \tag{46}$$

In order to have a feasible solution interval for M_2, recalling (10), and considering also (39), one has the final inequalities system on M_2:

$$\begin{cases} \left(6 + \frac{1}{M_2}\right)\bar{M}_a < \rho_\infty \\ \left(2 + \frac{1}{M_2}\right)\bar{M}_a < \left(2a_1 - \dfrac{2}{3M_2}\right)\rho_\infty \\ \dfrac{1}{3} < M_2 \leq \dfrac{1}{2} \end{cases} \tag{47}$$

i.e.:

$$\begin{cases} \bar{M}_a < \min\left\{\dfrac{\rho_\infty}{\left(6 + \frac{1}{M_2}\right)}, \dfrac{\left(2a_1 - \dfrac{2}{3M_2}\right)\rho_\infty}{\left(2 + \frac{1}{M_2}\right)}\right\} \\ \dfrac{1}{3} < M_2 \leq \dfrac{1}{2} \end{cases} \tag{48}$$

which corresponds to (32).

Step 2. Consider the expression (31). Due to conditions derived in *Step 1* when $s(k) > 0$, it holds $\Delta s(k+1) < 0$, i.e.

$$\Delta s(k+1) = \frac{1}{2}\ln\left[\frac{N_1(k)}{D_1(k)}\right] = \frac{1}{2}\cdot$$

$$\ln\left\{\frac{\left[\rho(k+1) + M_a(k)\nu_b(k) - \theta(k)\Gamma_s(k)\nu_b(k))\right]\delta(k)}{\rho(k+1) - M_a(k)\nu_c(k) + \theta(k)\Gamma_s(k)\nu_c(k)}\right\} < 0$$

$$\Rightarrow \frac{N_1(k)}{D_1(k)} < 1 \tag{49}$$

Moreover, using (36) and taking the worst case, one has:

$$\begin{aligned} N_1(k) &\geq \\ &\left(\rho(k+1) - 2\bar{M}_a - \left(\frac{\rho(k+1)}{\nu_b(k)} - 3\bar{M}_a\right)\nu_b(k)\right)\delta(k) \\ &\geq \frac{\bar{M}_a}{3} \triangleq N_1^{\min} \end{aligned} \tag{50}$$

$$\begin{aligned} D_1(k) &\leq \\ &\left(\rho(k+1) + \bar{M}_a + \left(\frac{\rho(k+1)}{\nu_b(k)} - 3\bar{M}_a\right)\nu_c(k)\right) \\ &\leq 2\rho(k+1) - 2\bar{M}_a \leq 2\rho(k+1) \leq 2\rho(0) \triangleq D_1^{\max} \end{aligned} \tag{51}$$

Taking into account (49), (50) and (51), it follows:

$$\frac{\bar{M}_a}{6\rho(0)} = \frac{N_1^{\min}}{D_1^{\max}} < \frac{N_1(k)}{D_1(k)} < 1 \tag{52}$$

Consider again the expression (31). Due to conditions derived in *Step 1* when $s(k) < 0$, it holds $\Delta s(k+1) > 0$, i.e.

$$\Delta s(k+1) = \frac{1}{2}\ln\left(\frac{N_2(k)}{D_2(k)}\right) =$$
$$\frac{1}{2}\ln\left\{\frac{[\rho(k+1)+M_a(k)\nu_b(k)+\theta(k)\Gamma_s(k)\nu_b(k)]\,\delta(k)}{\rho(k+1)-M_a(k)\nu_c(k)-\theta(k)\Gamma_s(k)\nu_c(k)}\right\} > 0$$
$$\Rightarrow \frac{N_2(k)}{D_2(k)} > 1 \tag{53}$$

Using (43) and taking the worst case, one has:

$$N_2(k) \le \left\{\rho(k+1) + 2\bar{M}_a + 2\left[\rho(k+1) - \bar{M}_a\right]\right\}$$
$$\le 3\rho(k+1) \le 3\rho(0) \triangleq N_2^{\max} \tag{54}$$
$$D_2(k) \ge \rho(k+1) - a_1\rho(k+1) \ge (1-a_1)\rho(k+1)$$
$$\ge (1-a_1)\rho_\infty \triangleq D_2^{\min} \tag{55}$$

Taking into account (53), (54) and (55), it follows:

$$1 < \frac{N_2(k)}{D_2(k)} < \frac{N_2^{\max}}{D_2^{\min}} = \frac{3\rho(0)}{(1-a_1)\rho_\infty} \tag{56}$$

Collecting the results obtained in both cases $s(k) > 0$ and $s(k) < 0$ one gets:

$$\bar{\alpha} \triangleq \frac{1}{2}\ln\left(\frac{\bar{M}_a}{6\rho(0)}\right) < \Delta s(k+1) \tag{57}$$
$$< \frac{1}{2}\ln\left(\frac{3\rho(0)}{(1-a_1)\rho_\infty}\right) \triangleq \bar{\beta}$$

where $\bar{\alpha} < 0$ and $\bar{\beta} > 0$. Define

$$\Lambda = \left\{\bar{\alpha} \le s(k) \le \bar{\beta}\right\} \tag{58}$$

It is straightforward to see that, once this region is entered by the sliding variable $s(k)$ at the time instant k, then $s(k+1) \in \Lambda(k+1)$. In fact, if $s(k) > 0$ and $s(k) \in \Lambda$, it means that $0 < s(k) \le \bar{\beta}$. Since $\bar{\alpha} < \Delta s(k+1) < 0$, one has $\bar{\alpha} < s(k+1) < s(k) \le \bar{\beta}$. Analogously, if $s(k) < 0$ and $s(k) \in \Lambda$, it means that $\bar{\alpha} \le s(k) < 0$. Since $\bar{\beta} \ge \Delta s(k+1) > 0$ one has $\bar{\beta} > s(k+1) > s(k) \ge \bar{\alpha}$. □

4. SIMULATION TESTS

The proposed controller has been tested by intensive simulations using the models proposed in [Liu and Yang, 2019]. In particular, the following steam-water heat exchanger has been considered:

$$x(k) = 1.5u(k) - 1.5u(k)^2 + 0.5u(k)^3$$
$$y(k+1) = 0.6y(k) - 0.1y(k-1) + 1.2x(k) - 0.1x(k-1) \tag{59}$$

Following [Liu and Yang, 2019], the following parameters have been used for simulation tests: $d_u = 4$, $\alpha = 0.1$, $\vartheta = 0.08$, $\rho_\infty = 0.01$, $\rho(0) = 3$, $\mu = 0.6$, $\lambda = 15$, $\eta = 1$, $\hat{\mathbf{\Phi}}(0) = [0.8\ 0\ 0\ 0]^T$, $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 1$. According to [Liu and Yang, 2019], the desired trajectory is constant $y_d = 2.5$. The control law has been designed choosing $M_2 = 1/18$ and $\bar{M}_a = \rho(k+1)/24$. This value in fact satisfies (38) and (45) for the feasible value, in the worst case, of $a_1 = 0.57$. In particular, in the case $s(k) > 0$ the term $\Gamma_s(k)$ has been set as $\Gamma_s(k) = \rho(k+1)/2 - 3\bar{M}_a$, which fulfills (37) for any admissible ρ_1. In the case $s(k) < 0$, the controller has been designed as $\Gamma_s(k) = \rho(k+1)$, which satisfies both constraints (44) for the chosen value of $\bar{M}_a$ and for the selected value of ρ_1.

Some of the performed tests have been reported in Figs.1-2. Fig.1 shows the tracking error and proves the fulfillments of the requirements about the prescribed performance of the tracking accuracy. The upper panel of Fig.2 depicts the sliding surface proving its boundedness are theoretically proved, while the boundedness of the variable $\tau(k)$ is proved in the lower panel of the same figure. A comparison is reported with the performances obtained with the control algorithms described in [Hou and Jin, 2011] and [Liu and Yang, 2019]. In particular, both algorithms have been set with $\lambda = 10$, and with $\Gamma_s = 0.01$ for the algorithm [Liu and Yang, 2019], values which provided best result.

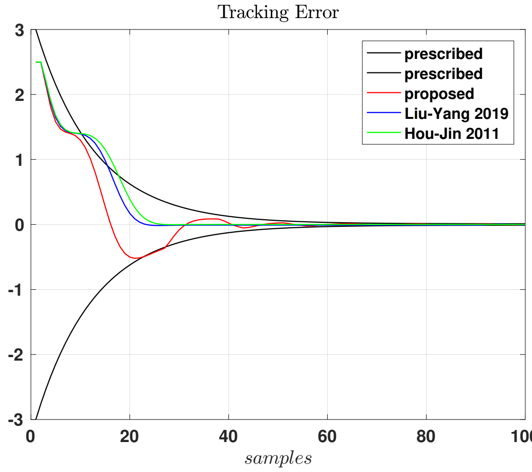

Fig. 1. Tracking Error.

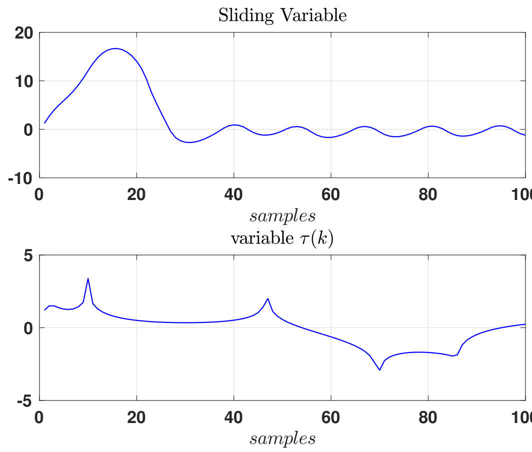

Fig. 2. Sliding Variable $s(k)$.

The comparison shows an improved performance of the proposed controller in terms of fulfillment of the prescribed performance requirement. In order to confirm this result, the IAE criterion, i.e. the integral of the absolute value of the tracking error, has been computed for the considered controllers over a time interval of 1000 samples. The results are reported in Tab. 1, showing that an improvement ranging from 30% to 35% is obtained using the proposed control algorithm.

The response reported in Fig. 1 shows an overshoot, which is nonetheless compliant with the prescribed behavior defined by the chosen sequence $\rho(k)$. It could be of interest to notice that the response can be shaped by properly setting the parameters ρ_i. Just as an example, setting

Table 1. Performance comparison: IAE of the tracking error (1000 samples).

Proposed Approach	Liu-Yang 2019	Hou-Jin 2011
29.72	42.73	45.70

$\rho = 0.9; \rho_2 = \rho_3 = \rho_4 = 1$ the obtained behavior is that reported in Fig. 3 where a reduction of the amplitude of the overshoot (from 0.5 to 0.4) is achieved still retaining good tracking performances. Of course, the overall response is slower, and the corresponding IAE increases to 31.91. This suggests that the desired tradeoff between tracking accuracy and promptness of response can be achieved by suitably tuning the design parameters.

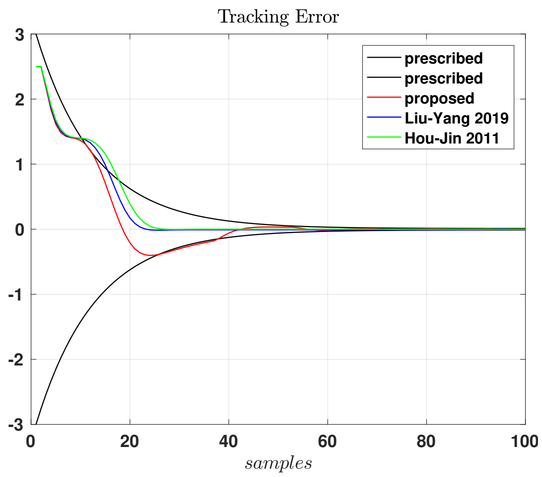

Fig. 3. Tracking error with different settings.

5. CONCLUSIONS

The present study is inspired by the very recent paper [Liu and Yang, 2019], where a data-driven MFAC sliding mode controller has been proposed able to account also for prescribed performance constraints. Due to the presence of an unsatisfactory theoretical proof of convergence in the original paper, a rigorous stability analysis is here proposed, achieved modifying the forms of the sliding surface and of the control law but still retaining the main setup presented in the source paper. In addition, a number of constraints on the gain of the sliding-mode based control term have been shown to arise from a careful analysis of the closed loop system. The presented comparative analysis, performed using a test taken from the literature, has been shown to provide very satisfactory results, showing an improvement of control accuracy ranging from 30% to 35% with respect to the available literature.

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