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A two-step approach to interval estimation for continuous-time switched linear systems

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Abstract: This paper proposes a two-step interval estimation method for continuous-time switched linear systems subject to unknown but bounded disturbance and measurement noise. We first use an L_∞ norm-based approach to attenuate the effect of uncertainties in observer design. Then, based on the obtained observer, interval estimation can be achieved via analyzing the bounds of estimation error. The proposed method is intuitive and independent of cooperativity constraint, which is main restriction of interval observer theory. The performance of the proposed method is demonstrated through a numerical simulation.

Keywords: Interval estimation, observer design, switched linear system, L_∞ norm

1. INTRODUCTION

As an effective way to deal with uncertainties (e.g. disturbance and measurement noise), interval observer has received increasing attention in recent years, see Efimov and Raïssi (2016); Tang et al. (2019b); Efimov et al. (2016); Chebotarev et al. (2015); Mazenc and Dinh (2014) for example. Under a general assumption that the uncertainties are unknown but bounded, interval observer can obtain an envelope that includes all possible trajectories of states. Some applications of interval observer can be found in the literature, e.g. Gouzé et al. (2000); Heeks et al. (2002); Moisan et al. (2009), just name a few.

The concept of interval observer is first proposed by Gouzé et al. (2000). The basic idea is to design two sub-observers such that the error dynamic systems are stable and cooperative, please see Efimov and Raïssi (2016) for more details. Note that classical observers only require that the error dynamic systems are stable. Compared with the design conditions of classical observers, extra cooperative constraint makes the design of interval observer more difficult. To cope with this limitation, coordinate transformation is presented to acquire relaxed design conditions, see e.g. Raïssi et al. (2012); Mazenc and Bernard (2011). However, as pointed out in Chambon et al. (2016), the observer gain matrix and the coordinate transformation matrix cannot be simultaneously synthesized to satisfy the disturbance attenuation and the cooperativity properties. Moreover, the interval will be enlarged during the process of inverse coordinate transformation. To overcome this limitation, Wang et al. (2018) proposes a novel interval observer structure, which can provide more design degrees of freedom. This method is further extended to discrete-time Takagi-Sugeno fuzzy systems in Li et al. (2019a) and continuous linear parameter-varying systems in Li et al. (2019b). However, the method in Wang et al. (2018) still suffers from the cooperativity constraint, which may lead to some conservatism.

On the other hand, as an important class of hybrid systems, switched systems attract much attention in control society, see, e.g. Fei et al. (2017, 2018); Shi et al. (2018b,a). Interval observer design for switched systems can be found in a few literatures, see e.g. Marouani et al. (2018); Guo and Zhu (2017); Ethabet et al. (2017, 2018); He and Xie (2015); Ifqir et al. (2018); He and Xie (2016). Marouani et al. (2018) and Guo and Zhu (2017) consider the case of discrete-time switched linear systems based on a time-varying coordinate transformation. Ethabet et al. (2017) and Ethabet et al. (2018) consider the case of continuous-time case based on a switching coordinate transformation. He and Xie (2015) studies the nonlinear switched systems under dwell-time constraints and further extends the design method to control field by He and Xie (2016). Ifqir et al. (2018) applies the interval observer design method to the estimation of vehicle dynamics. However, all these methods are considered based on cooperativity constraint or coordinate transformation.

To overcome the aformentioned drawbacks, we propose a two-step method to design interval observers for continuous-time switched linear systems. This idea is motivated by the fact that interval estimation can be achieved by integrating observer design and error analysis, see Tang et al. (2019a) and Tang et al. (2019b) for more details. The main contributions of this paper are two folds. First,

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a two-step method is presented to circumvent the design rectriction caused by cooperativity constraint and provide an alternative solution to interval observer design. Second, we implement this method to addresss the interval estimation problem for switched linear systems, and further apply an L_∞ norm-based approach to enhance the estimation performance.

The remainder of this paper is structured as follows: In Section 2, we present the problem statement and some preliminary results. In Section 3, main results on the computation of state estimation and interval estimation for switched linear systems are presented. In Section 4, numerical examples are provided to illustrate the proposed methods. In Section 5, some conclusions are drawn.

2. PROBLEM FORMULATION

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n–dimensional Euclidean space and the set of $n \times m$ real matrices, respectively. $\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$. 0 and I denote the zero and identity matrices with compatible dimensions, respectively. The absolute value operator $|\cdot|$ and the symbols $\geq, >, \leq$ and $<$ should be understood elementwise. For a matrix A, A^T stands for its transposition and $\mathbf{He}(A)$ is used to denote $\mathbf{He}(A) := A + A^T$. $P \succ 0$ and $P \prec 0$ indicate that matrix P is positive definite and negative definite, respectively. An asterisk $\star$ is used to represent a term induced by symmetry. $\xi_s(i) = (\underbrace{0, \cdots, 0, \overbrace{1}^{i\text{th}}, 0, \cdots, 0}_{s \text{ components}})^T \in \mathbb{R}^s, s \geq 1$. e is the exponential constant.

For a measurable and locally essentially bounded signal $u : \mathbb{R}_+ \to \mathbb{R}^p$, its L_∞ norm is defined as the supremum over all time, i.e.

$$||u||_\infty = \sup\{||u(t)||, t \in \mathbb{R}_+\},$$

where $||\cdot||$ denotes the Euclidean norm.

Consider the following switched linear system

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + w(t) \\ y(t) = C_{\sigma(t)}x(t) + v(t) \end{cases}, t \in \mathbb{R}_+, \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the input vector, $y(t) \in \mathbb{R}^{n_y}$ is the output vector, $w(t) \in \mathbb{R}^{n_x}$ denotes the process disturbance and $v(t) \in \mathbb{R}^{n_y}$ denotes the measurement noise. $\sigma(t)$ is a known piecewise constant function which denotes the switching signal. $\{(A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}) : \sigma(t) \in \mathscr{Q}\}$ are a family of matrices parameterized by an index set $\mathcal{Q} = \{1, 2, \cdots, s\}$, where s is the number of linear subsystems. Let $q = \sigma(t)$ be the index of the active subsystem, $A_q \in \mathbb{R}^{n_x\times n_x}$, $B_q \in \mathbb{R}^{n_x\times n_u}$ and $C_q \in \mathbb{R}^{n_y\times n_x}$ are known constant matrices.

We consider the following assumptions.

Assumption 1. $||w(t)|| \leq ||w||_\infty \leq \overline{w}$ and $||v(t)|| \leq ||v||_\infty \leq \overline{v}$, where $\overline{w}$ and $\overline{v}$ are known constants.

Assumption 2. The pair $(A_q, C_q), q \in \mathcal{Q}$ is observable.

Design objective. This manuscript aims to generate two consecutive signals $\overline{x}(t)$ and $\underline{x}(t)$ such that the condition $\underline{x}(t) \leq x(t) \leq \overline{x}(t)$, $t \geq 0$ always holds. One step further, we hope that the interval $\overline{x}(t) - \underline{x}(t)$ is tight enough so that the obtained state estimation is accurate.

3. MAIN RESULTS

In this section, we propose a two-step interval estimation method for system (1). First, a robust observer is designed to obtain state estimation. Second, peak-to-peak analysis is used to analyze the bounds of error and to get the interval estimation.

3.1 State estimation

For system (1), we consider the following observer

$$\dot{\hat{x}}(t) = A_q\hat{x}(t) + B_qu(t) + L_q(y(t) - C_q\hat{x}(t)), \tag{2}$$

where $\hat{x}(t) \in \mathbb{R}^{n_x}$ is the state estimation vector and $L_q \in \mathbb{R}^{n_x\times n_y}, q \in \mathcal{Q}$, is the observer gain matrix to be designed. Define the estimation error as

$$e(t) = x(t) - \hat{x}(t),$$

it follows that

$$x(t) = \hat{x}(t) + e(t). \tag{3}$$

Then, if we can estimate the bounds $e_b(t)$ of $e(t)$ such that

$$|e(t)| \leq e_b(t), t \in \mathbb{R}_+,$$

then from (3), the interval estimation of state can be obtained by

$$\begin{aligned} \overline{x}(t) &= \hat{x}(t) + e_b(t), \\ \underline{x}(t) &= \hat{x}(t) - e_b(t). \end{aligned} \tag{4}$$

From (1) and (2), the dynamic error system is governed by

$$\dot{e}(t) = (A_q - L_qC_q)e(t) + w(t) - L_qv(t), \tag{5}$$

To attenuate the effect of disturbance and measurement noise, in this paper, we apply an L_∞ norm-based approach, see in Han et al. (2018), which results in the following theorem.

Theorem 1. Given a scalar $\eta > 0$, if there exist scalars $\gamma > 0, \mu > 0$ and matrices $P = P^T \succ 0 \in \mathbb{R}^{n_x\times n_x}$ and $W_q \in \mathbb{R}^{n_x\times n_y}$ for $\forall q \in \mathcal{Q}$ such that

$$\begin{bmatrix} \mathbf{He}(PA_q - W_qC_q) + \eta P & \star & \star \\ P^T & -\mu I & \star \\ -W_q^T & 0 & -\mu I \end{bmatrix} \prec 0, \tag{6}$$

$$\begin{bmatrix} \eta P & 0 & I \\ 0 & (\gamma - \mu)I & 0 \\ I & 0 & \gamma I \end{bmatrix} \succ 0, \tag{7}$$

then observer (2) is a robust observer for system (1) and satisfies the following performance

$$||e(t)||^2 \leq \gamma\eta \mathrm{e}^{-\eta t}V(0) + \gamma^2||d||^2_\infty, \tag{8}$$

where

$$V(0) = e^T(0)Pe(0),\ ||d||_\infty = \sqrt{||w||^2_\infty + ||v||^2_\infty}.$$

An optimal solution can be found by solving

$$\min_{\substack{s.t.\ (6)-(7)}} \gamma, \tag{9}$$

and the gain matrix L_q is obtained from

$$L_q = P^{-1}W_q.$$

Proof. Consider the following common quadratic Lyapunov function

$$V(t) = e^T(t)Pe(t).$$

By pre-multiplying and post-multiplying inequality (6) with

$$\left[e^T(t)\ d^T(t)\right]$$

and its transpose, we have

$$\dot{V}(t) + \eta V(t) \le \mu d^T(t)d(t) \tag{10}$$

where $d(t) = \left[w^T(t)\ v^T(t)\right]^T$.

Note that if the uncertainty $d(t)$ is zero, we have

$$\dot{V}(t) \le -\eta V(t) \le 0.$$

Thus, the error system (5) is asymptotically stable.

By iterating, inequality (10) follows that

$$\begin{aligned} V(t) &\le \mathrm{e}^{-\eta t}V(0) + \mu\int_0^t \mathrm{e}^{-\eta(t-\tau)}d^T(\tau)d(\tau)d\tau \\ &\le \mathrm{e}^{-\eta t}V(0) + \frac{\mu}{\eta}(1-\mathrm{e}^{-\eta t})d^T(t)d(t) \\ &\le \mathrm{e}^{-\eta t}V(0) + \frac{\mu}{\eta}(1-\mathrm{e}^{-\eta t})||d||_\infty^2 \\ &\le \mathrm{e}^{-\eta t}V(0) + \frac{\mu}{\eta}||d||_\infty^2. \end{aligned} \tag{11}$$

Additionally, using Schur complement Boyd et al. (1994), inequality (7) is equivalent to

$$\begin{bmatrix} \eta P - \frac{1}{\gamma}I & 0 \\ 0 & (\gamma-\mu)I \end{bmatrix} \succ 0 \tag{12}$$

By pre- and post-multiplying inequality (12) with

$$\left[e^T(t)\ d^T(t)\right]$$

c and its transpose, we have

$$\begin{aligned} |||e(t)||^2 &\le \gamma(\eta V(t) + (\gamma-\mu)d^T(t)d(t)) \\ &\le \gamma\eta V(t) + \gamma(\gamma-\mu)||d||_\infty^2 \end{aligned} \tag{13}$$

Combining (11) and (13), we have (8). □

Note that inequality (8) indicates that for $\forall t \in \mathbb{R}_+$, the amplitude of $e(t)$ is bounded by $\sqrt{\gamma\eta\mathrm{e}^{-\eta t}V(0) + \gamma^2||d||_\infty^2}$. Thus, by calculating the design parameter γ, the interval estimation of state can be achieved through (4). However, the obtained estimation results may be too conservative since for all entries of $e(t)$ and $d(t)$, a common disturbance attenuation coefficient γ is applied. Facing this reality, we attempt to use different coefficients to characterize the disturbance attenuation level of different entries of $d(t)$ to different entries of $e(t)$.

3.2 Interval estimation

After getting observer gain matrices $L_q, q \in \mathcal{Q}$ by solving the optimization problem (9), state estimation $\hat{x}(t)$ can be synthesized through (2). To obtain a tight envelope of state, we rewrite the dynamic error system in (5) as follows.

$$\dot{e}(t) = \tilde{A}_q e(t) + \sum_{i=1}^{n_d} \tilde{B}_q \xi_{n_d}(i) d_i(t) \tag{14}$$

where $d_i(t)$ is the ith entry of $d(t)$ and

$$\tilde{A}_q = A_q - L_q C_q,\ \tilde{B}_q = \left[I\ -L_q\right], n_d = n_x + n_y$$

Note that the jth entry of error $e(t)$ can be expressed as

$$e_j(t) = \xi_{n_x}^T(j)e(t), \tag{15}$$

then for the jth entry $e_j(t), j \in \{1, 2, \cdots, n_x\}$ of error, the following state-space system can be obtained:

$$\begin{cases} \dot{e}(t) = \tilde{A}_q e(t) + \sum_{i=1}^{n_d} \tilde{B}_q \xi_{n_d}(i) d_i(t), \\ e_j(t) = \xi_{n_x}^T(j) e(t), \end{cases} \tag{16}$$

Remark 1. The reason for deriving error subsystem (16) from system (5) is simple. In this way, it is convenient to analyze the effect of the ith entry $d_i(t)$ of disturbance on the jth entry $e_j(t)$ of error.

For calculating the envelopes of $e_j(t)$ in error system (16), we propose the following theorem.

Theorem 2. For jth entry error $e_j(t)$, $j = \{1, 2, \cdots, n_x\}$, given a scalar $\lambda > 0$, if there exist scalars $\gamma_{ij} > 0, \mu_{ij} > 0$ and matrices $P_j = P_j^T \succ 0 \in \mathbb{R}^{n_x \times n_x}$ for $\forall q \in \mathcal{Q}, i \in \{1, 2, \cdots, n_d\}$ such that

$$\begin{bmatrix} \mathbf{He}(P_j\tilde{A}_q) + \lambda P_j & \star & \star & \cdots & \star \\ (P_j\tilde{B}_q\xi_{n_d}(1))^T & -\mu_{1j}I & \star & \cdots & \star \\ (P_j\tilde{B}_q\xi_{n_d}(2))^T & 0 & -\mu_{2j}I & \cdots & \star \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (P_j\tilde{B}_q\xi_{n_d}(n_d))^T & 0 & 0 & \cdots & -\mu_{(n_d)j}I \end{bmatrix} \prec 0, \tag{17}$$

$$\begin{bmatrix} \lambda P_j & \star & \cdots & \star & \star \\ 0 & (\gamma_{1j}-\mu_{1j})I & \cdots & \star & \star \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & (\gamma_{n_dj}-\mu_{n_dj})I & 0 \\ \xi_{n_x}^T(j) & 0 & \cdots & 0 & \gamma_{0j}I \end{bmatrix} \succ 0. \tag{18}$$

Then, the jth entry error $e_j(t)$ in (16) satisfies

$$||e_j(t)||^2 \le \gamma_{0j}(\lambda\mathrm{e}^{-\lambda t}V_j(0) + \sum_{i=1}^{n_d}\gamma_{ij}||d_i||_\infty^2), \tag{19}$$

where $||d_i||_\infty$ is the upper bound of the ith entry of $d(t)$ and $V_j(0) = e^T(0)P_je(0)$.

A satisfactory envelope of $e_j(t)$ can be obtained by solving

$$\min_{\substack{s.t. \\ (17)-(18)}} \gamma_{0j} + \gamma_{1j} + \cdots + \gamma_{n_dj}. \tag{20}$$

Proof. Define a Lyapunov function for each component e_j

$$V_j(t) = e^T(t)P_je(t).$$

By pre-multiplying and post-multiplying (17) with

$$\left[e^T(t)\ d_1^T(t)\ \cdots\ d_{n_d}^T(t)\right]$$

and its transpose, we have

$$\dot{V}_j(t) + \lambda V_j(t) \le \sum_{i=1}^{n_d}\mu_{ij}d_i^T(t)d_i(t) \tag{21}$$

By iterating, inequality (21) follows that

$$
\begin{aligned}
V_j(t) &\le \mathrm{e}^{-\lambda t}V_j(0) + \sum_{i=1}^{n_d}\mu_{ij}\int_0^t \mathrm{e}^{-\lambda(t-\tau)}d_i^T(\tau)d_i(\tau)d\tau \\
&\le \mathrm{e}^{-\lambda t}V_j(0) + \sum_{i=1}^{n_d}\frac{\mu_{ij}}{\lambda}(1-\mathrm{e}^{-\lambda t})d_i^T(t)d_i(t) \\
&\le \mathrm{e}^{-\lambda t}V_j(0) + \sum_{i=1}^{n_d}\frac{\mu_{ij}}{\lambda}(1-\mathrm{e}^{-\lambda t})||d_i||_\infty^2 \\
&\le \mathrm{e}^{-\lambda t}V_j(0) + \sum_{i=1}^{n_d}\frac{\mu_{ij}}{\lambda}||d_i||_\infty^2. \qquad (22)
\end{aligned}
$$

Additionally, using Schur complement Boyd et al. (1994), inequality (18) is equivalent to

$$
\begin{bmatrix}
\Gamma & \star & \cdots & \star \\
0 & (\gamma_{1j}-\mu_{1j})I & \cdots & \star \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & (\gamma_{n_dj}-\mu_{n_dj})I
\end{bmatrix} \succ 0, \qquad (23)
$$

where $\Gamma = \lambda P_j - \frac{1}{\gamma_{0j}}\xi_{n_x}(j)\xi_{n_x}^T(j)$.

By pre- and post-multiplying inequality (23) with

$$\begin{bmatrix} e^T(t) & d_1^T(t) & \cdots & d_{n_d}^T(t)\end{bmatrix}$$

and its transpose, we have

$$
\begin{aligned}
||e_j(t)||^2 &= ||\xi_{n_x}^T(j)e(t)||^2 \\
&\le \gamma_{0j}(\lambda V_j(t) + \sum_{i=1}^{n_d}(\gamma_{ij}-\mu_{ij})d_i^T(t)d_i(t)) \\
&\le \gamma_{0j}(\lambda V_j(t) + \sum_{i=1}^{n_d}(\gamma_{ij}-\mu_{ij})||d_i||_\infty^2) \qquad (24)
\end{aligned}
$$

Substituting (22) into (24), we have (19). □

Remark 2. From (19), we know that when $t \to \infty$, the effect of the ith entry $d_i(t)$ of disturbance on the jth entry $e_j(t)$ of error is characterized as the multiplication of γ_{0j} and γ_{ij}. Intuitively, to obtain a tight envelop of $e_j(t)$, the multiplication of γ_{0j} and γ_{ij} should be as small as possible. Obviously, the minimization of $\gamma_{0j}\gamma_{ij}$ subject to linear matrix inequalities (17)-(18) is not a convex optimization problem. For sake of solvability, we choose to solve (20) to obtain a suboptimal solution.

Remark 3. From (19), it seems that design parameter μ_{ij}, $\forall i \in \{1,2,\cdots,n_d\}, j\in\{1,2,\cdots,n_x\}$ possesses none effect on the upper bound of $e_j(t)$. In fact, μ_{ij} is an intermediate variable. It plays the role of a bridge between (21) and (24).

By repeating to solve (20) for $j = \{1,2,\cdots,n_x\}$, we can obtain the upper bound of each entry of estimation error $e(t)$. If we define the jth entry of the upper bound of $e(t)$ as $e_{bj}(t), j = \{1,2,\cdots,n_x\}$, then

$$||e_{bj}||^2(t) = \gamma_{0j}(\lambda \mathrm{e}^{-\lambda t}e^T(0)P_je(0) + \sum_{i=1}^{n_d}\gamma_{ij}||d_i||_\infty^2).$$

Note that P_j is positive and symmetric, thus, we can always find a matrix T such that

$$
\begin{cases}
T^{-1}P_jT = diag(p_{j1},\cdots,p_{jn_x}) \\
TT^{-1} = I
\end{cases} \qquad (25)
$$

where $p_{ji}, i = 1,\cdots n_x$ denote the eigenvalues of matrix P_j.

Using (25), we have

$$
\begin{aligned}
e^T(0)P_je(0) &= e^T(0)TT^{-1}P_jTT^{-1}e(0) \\
&= e^T(0)Tdiag(p_{j1},\cdots,p_{jn_x})T^{-1}e(0) \\
&\le e^T(0)Tdiag(p_{mj},\cdots,p_{mj})T^{-1}e(0) \\
&= p_{mj}||e(0)||^2 \\
&\le p_{mj}e_b^T(0)e_b(0)
\end{aligned}
$$

where p_{mj} is the maximum eigenvalue of P_j and $e_b(0)$ denotes the bound of $e(0)$ satisfying $||e(0)|| \le e_b(0)$. One step further, the envelopes of states can be obtained by

$$||e_{bj}||^2(t) \le \gamma_{0j}(\lambda \mathrm{e}^{-\lambda t}p_{mj}e_b^T(0)e_b(0) + \sum_{i=1}^{n_d}\gamma_{ij}||d_i||_\infty^2).$$

For clarity, we summarize the presented interval estimation method as Algorithm 1.

Algorithm 1 A two-step interval estimation method

Input: System matrices $A_q, B_q, C_q, q \in \mathcal{Q}$ and switching signal $\sigma(t)$.

Output: Envelopes of states $\overline{x}(t)$ and $\underline{x}(t)$.

1: **initialization**: $j = 1$, $\eta > 0, \lambda > 0$ and the bound $e_b(0)$ of $e(0)$.
2: Solve (9) to obtain gain matrix $L_q, q \in \mathcal{Q}$.
3: Generate state point estimation $\hat{x}(t)$ using observer (2).
4: **while** $j \le n_x$ **do**
5: Solve (20) to obtain design parameters $\gamma_{0j}, \gamma_{1j}, \cdots, \gamma_{n_dj}$ and P_j.
6: Calculate the maximum eigenvalue p_{mj} of P_j.
7: Calculate the jth entry $e_{bj}(t)$ of the bound of $e_j(t)$

$$e_{bj}(t) \le \sqrt{\gamma_{0j}(\lambda \mathrm{e}^{-\lambda t}p_{mj}e_b^T(0)e_b(0) + \sum_{i=1}^{n_d}\gamma_{ij}||d_i||_\infty^2)}.$$

8: $j = j + 1$.
9: **end while**
10: Construct the bound of error by concatenating the entries $e_{bj}(t), j = \{1,2,\cdots,n_x\}$

$$e_b(t) = \begin{bmatrix} e_{b1}(t) & e_{b2}(t) & \cdots & e_{bn_x}(t)\end{bmatrix}^T.$$

11: **return**: Generate $\overline{x}(t)$ and $\underline{x}(t)$ based on (4).

Remark 4. Note that (19) can only be used in theoretical analysis, for practical implementation, the upper bound $e_b(0)$ of error $e(0)$ should be used because $e(0)$ may not be available.

4. SIMULATIONS

In this section, a benchmark from Ethabet et al. (2017) is adopted to demonstrate the effectiveness of the proposed method. The system is described as (1) with

$$
A_1 = \begin{bmatrix} -1.5 & 0.262 \\ 0 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} -0.5 & 2 \\ 0 & -1 \end{bmatrix},
$$

$$
A_3 = \begin{bmatrix} -0.6 & 1.5 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},
$$

$$
C_1 = [1 \ 0], C_2 = [1 \ 1], C_3 = [1 \ 1.5],
$$

$$
\begin{bmatrix} -0.03 \\ -0.03 \end{bmatrix} \le w(t) \le \begin{bmatrix} 0.03 \\ 0.03 \end{bmatrix}, -0.3 \le v(t) \le 0.3.
$$

Then we have $||d_1(t)||_\infty = 0.03$, $||d_2(t)||_\infty = 0.03$ and $||d_3(t)||_\infty = 0.3$. Note that for this system, we cannot

Table 1. Design parameters obtained by solving Theorem 2.

Parameters	Values	Parameters	Values
γ_{01}	1.6495	γ_{02}	1.4715
γ_{11}	0.5144	γ_{12}	0.2855
γ_{21}	0.3645	γ_{22}	0.9097
γ_{31}	0.7745	γ_{32}	0.2807
μ_{11}	0.5141	μ_{12}	0.2853
μ_{21}	0.3642	μ_{22}	0.9094
μ_{31}	0.7742	μ_{32}	0.2804
p_{m1}	0.4091	p_{m2}	0.4544

find a matrix P such that $P(A_q - L_q C_q)P^{-1}$ is Metzler, thus the method in Efimov and Raïssi (2016); Mazenc and Dinh (2014); Raïssi et al. (2012) fails to be applied. Ethabet et al. (2017) and Ethabet et al. (2018) overcome this deficiency using the switching matrices $P_q, q \in \mathcal{Q}$, which are also based on the coordinate of transformation and need to satisfy the cooperativity constraint. However, as pointed out in Chambon et al. (2016), the method of coordinate of transformation will enlarge the estimated intervals and thus lead to inevitable conservatism. To avoid such conservatism, in this paper, we estimate the envelopes of system states using the proposed two-step method, which is independent of coordinate transformation.

Following Algorithm 1, we first solve (9) in Theorem 1. Choose $\eta = 1$, we have $\gamma = 1.1045$ and

$$P = \begin{bmatrix} 0.9247 & -0.0623 \\ -0.0623 & 1.1072 \end{bmatrix}, L_1 = \begin{bmatrix} 1.1983 \\ 0.0674 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 1.2659 \\ 1.0683 \end{bmatrix}, L_3 = \begin{bmatrix} 1.2995 \\ 1.5687 \end{bmatrix}.$$

Next, setting $\lambda = 1.5$ and solving (20) in Theorem 2, we have the design parameters in Table 1 and

$$P_1 = \begin{bmatrix} 0.4070 & -0.0221 \\ -0.0221 & 0.1793 \end{bmatrix}, P_2 = \begin{bmatrix} 0.2009 & -0.0121 \\ -0.0121 & 0.4539 \end{bmatrix}.$$

Thus, the maximum eigenvalues of each entry should be

$$p_{m1} = 0.4091, p_{m2} = 0.4544.$$

In the simulation, the switching between the three subsystems is governed by the signal depicted in Figure 1. The input signal $u(k)$ is set as a constant value 0.5, the initial state is $x(0) = [0\ 0]^T$, the initial state estimation is $\hat{x}(0) = [1\ 1]^T$, then the error of estimation is $e(0) = [1\ 1]^T$. For simplicity, in this simulation, we set that $e_b(0) = [1\ 1]^T$.

The simulation results acquired by the presented method and that by the method in Ethabet et al. (2017) are depicted in Figure 2 and Figure 3. In the simulation, the solid black lines are the components of system state $x(t)$, the dash-dotted red lines represent the interval estimations obtained by the method in Ethabet et al. (2017), the dashed green lines denote the interval estimations obtained by the proposed Algorithm 1 and the solid blue lines depict the center of the intervals obtained by Algorithm 1. From Figure 2 and Figure 3, it can be seen that under the same simulation conditions, the interval estimation of the presented method is more accurate than that in Ethabet et al. (2017). The reason is that the proposed method is independent of coordinate transformation and the effect of uncertainties are attenuated using an L_∞ norm-based approach. The results of simulation exhibit the effectiveness and superiority of the proposed two-step method.

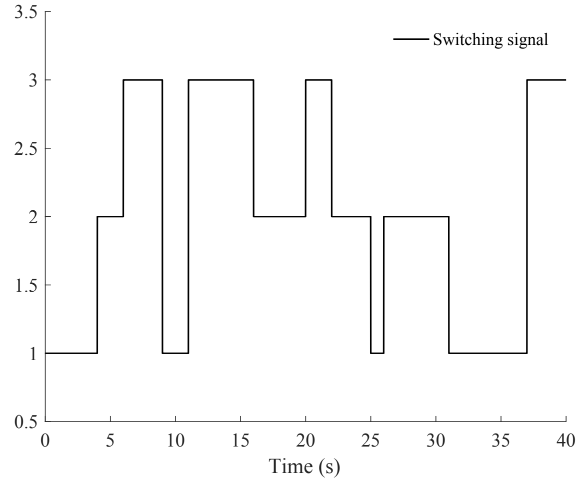

Fig. 1. Switching signal

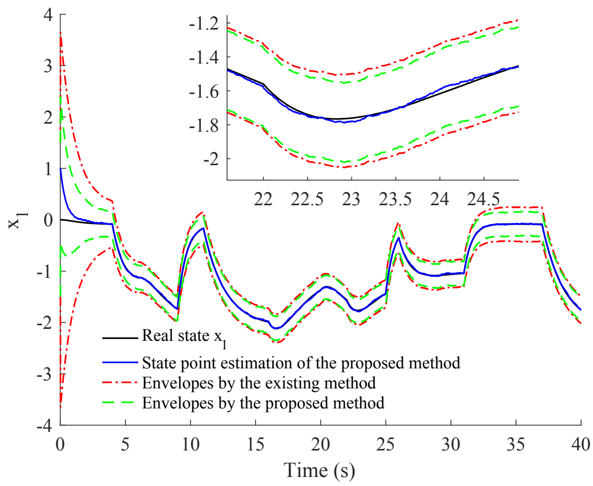

Fig. 2. State $x_1(t)$ by different methods

5. CONCLUSIONS

This manuscript studies the interval estimation for switched linear systems. The main contribution of this work consists in the derivation of a two-step interval estimation method. We use an L_∞ norm-based approach to vanish the effect of disturbance and obtain the state point estimation, followed by analyzing the estimation error dynamic systems to capture the bounds of each entry of error signals. Finally, the state interval estimation is synthesized by combining the state point estimation and the error entries bounds. Consequently, the cooperativity constraint in the interval observer theory is perfectly circumvented. Simulation results illustrate the viability and validity of the proposed method. In this work, the stability analysis is achieved based on a common Lyapunov function, which may result in some conservatism. In the future, more advanced approaches (e.g. approaches based on average dwell time, see in Fei et al. (2017)) may be exploited to reduce such conservatism.

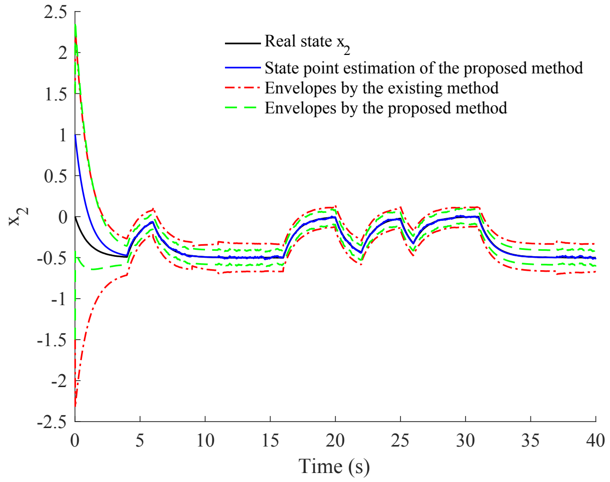

Fig. 3. State $x_2(t)$ by different methods

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