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An attempt to self-organized polygon formation control of swarm robots under cyclic topologies

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Abstract: In this paper, we consider the problem of controlling a swarm of mobile robots to formulate a prescribed formation shape under cyclic topologies. By mimicking the shape transformation of elastic strings, we design a distributed control strategy using local sensing quantities. To implement this control algorithm, only a small number of robots need to be regulated to achieve the desired formation shape. It is shown that the desired polygon formation can be globally asymptotically stabilized under the proposed control strategy. Furthermore, in order to adapt to the situations where external influence is not exerted on the "right" robots, we propose a new self-organized control strategy that is capable of transferring the external influence through interacting with their nearest neighboring robots. Simulations are carried out to demonstrate the effectiveness of the proposed control strategies.

Keywords: Formation control, swarm robots, cooperative control

1. INTRODUCTION

In recent years, cooperative control of multi-robot systems has attracted considerable attention due to its broad applications. In many applications, an essential task is controlling a group of robots to form the desired geometric shape, which is common in surveillance and reconnaissance (Saska et al. (2014)), cooperative transportation (Chen and Kai (2018)), and mapping and monitoring (Carrillo et al. (2015)).

The distributed control method relies on local information, which is efficient when a large number of robots are involved. In behavior-based control method, a set of predefined basic behaviors are utilized to form the collective behavior of the swarm robots (Xu et al. (2014)). However, it is difficult to analyze the convergence of the system mathematically. In Wiech et al. (2018), the robots are connected to their nearest neighbors via virtual spring dampers to achieve self-organizing swarm collective behavior. Besides, motived by the collective behavior of animals in nature, researchers investigate the rules to form a specific shape. In Turing reaction-diffusion model, robots exchange two morphogen-like signals through a set of reaction-diffusion differential equations. In Ikemoto et al. (2005), robots decide their own movements according to the signal value, and finally form a specific polygon formation. Gene regulatory network (GRN) technique is used in Meng et al. (2013), where each agent has two genes, and the expression of genes is controlled by a function encoding the desired shape information. Genes can produce proteins to control the movement of the agent. However, most previous works require all robots knowing the information of the desired shape.

In Lin et al. (2014), a formation control method using the complex Laplacian matrix is proposed, in which the formation size can be controlled. This method allows two leaders knowing the desired distance between them and the rest are driven to appropriate positions via inner couplings imposed by the complex Laplacian matrix. Stress-matrix-based formation control has gained increasing attention due to its superiority in changing formation shapes. In Zhao (2018), to achieve the affine transformation (defined as a linear transformation plus a translation) of the formation, it requires three robots to be informed of the desired formation information. Yang et al. (2019) realizes the formation scaling control in the circumstance that only one robot knows the desired scaling size. These methods can realize formation stabilization/transformation under the constraints that only a few robots know the desired formation information, but the target formations are restricted to only affine transformations of the nominal configuration. In addition, when the number of robots is large, the calculation of the stress matrix will become extremely complicated.

The goal of this paper is to design a distributed control strategy for swarm robots to form an arbitrary polygon formation with few external inputs. In our strategy, it is

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assumed that the sensing topology of the robots is cyclic, which implies that each robot i has only two neighbors $i-1$ and $i+1$(Fathian et al. (2019)). In this paper, we first design a distributed control strategy under the condition that the external influence is applied to appropriate robots. These robots are selected in accordance with the desired formation shape. In this strategy, all robots only need to measure the relative positions with respect to their nearest neighbors in local coordinate systems. Then, to remove the tedious calculation on the "chosen robots", a self-organized control strategy is designed by developing an adaptive influence-transition mechanism, within which the external influence can be applied to arbitrary robots.

2. PRELIMINARIES

2.1 Notations

Let $\mathbf{0}_n$ be the n-dimensional column vector with all zeros. The cardinality of a set X is denoted by $|X|$. We use $||x||$ and x^T to represent the 2-norm and transpose of a vector x, respectively. The symbol mod denotes the modulo operator. Let $\text{sgn}:\mathbb{R}\to\mathbb{R}$ be the signum function. We use $\mathbb{N}_n$ to denote the set of nonnegative integers that are less than n, written as $\{0,1,\cdots,n-1\}$.

2.2 Graph theory

In this paper, an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ is used to represent the interactions among the networked n robots, where $\mathcal{V}=\{0,1,\ldots,n-1\}$ is the node set, $\mathcal{E}\in\mathcal{V}\times\mathcal{V}$ is the edge set, and the weighted adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{n\times n}$ is defined by $a_{ij}=0$ if $(j,i)\notin\mathcal{E}$, and $a_{ij}=1$, otherwise. $(j,i)\in\mathcal{E}$ indicates that robot i and j can sense each other, namely, the relative position between them can be acquired. The set of robots that are adjacent to robot i is denoted by the neighbor set $\mathcal{N}_i=\{j|(i,j)\in\mathcal{E}\}$. A topology is said to be *cyclic* if neighbors of robot $i\in\mathcal{V}$ are robots $i-1$ and $i+1$(mod n)(Fathian et al. (2019)).

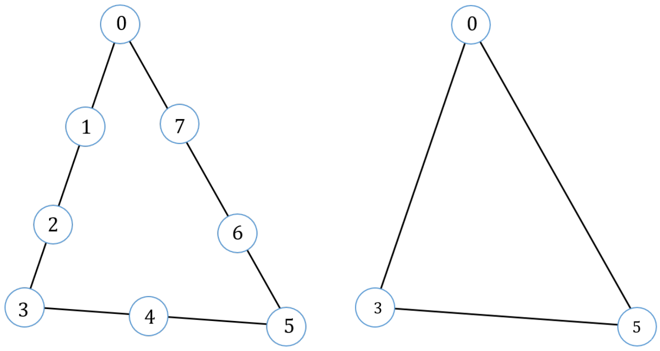
A *configuration* $q\in\mathbb{R}^{n\times 2}$ is a finite collection of n labeled robots $q_i\in\mathbb{R}^2$, denoted by $q=[q_0,q_1,\ldots,q_{n-1}]^T$. A *framework* $(\mathcal{G},q)$ can be obtained by assigning an undirected graph $\mathcal{G}$ to a set of agents with a feasible configuration q (Yang et al. (2019)).

2.3 Polygon

In a framework $(\mathcal{G},q)$, a robot is called a *vertex robot* if it's not collinear with its neighbors. We use $\overline{\mathcal{V}}=\{\overline{v}_0,\overline{v}_1,..,\overline{v}_{\overline{n}-1}\}\subseteq\mathcal{V}$ to represent the set of vertex robots with $\overline{n}$ being the number of vertex robots. Note that $\overline{v}_i$ is the index associated with some vertex robot and the set $\overline{\mathcal{V}}$ may change as robots move. The vector $b=[b_0,b_1,\cdots,b_{\overline{n}-1}]^T$ is used to represent the number of edges between vertex robots in the graph $\mathcal{G}$ and b_i is defined as $b_i=(\overline{v}_{i+1}-\overline{v}_i)\bmod n$. Define $\overline{q}=[\overline{q}_0,\overline{q}_1,\cdots,\overline{q}_{\overline{n}-1}]^T$ with $\overline{q}_i=q_{\overline{v}_i}-q_{\overline{v}_{(i+1)}}$. From definition of b, we can easily get that

$$\sum_{i\in\mathbb{N}_{\overline{n}}} b_i=n. \tag{1}$$

In this paper, we use $(b,\overline{q})$ to denote the polygon formation. Define an implicit function $\mathcal{F}:\mathbb{R}^{n\times 2}\to\mathbb{R}^{\bar{n}}\times\mathbb{R}^{\bar{n}\times 2}$ to represent the polygon formation abstracted from the configuration.



(a) A configuration with a cyclic topology and the vertex robot set $\overline{\mathcal{V}}=\{0,3,5\}$.

(b) The polygon formation $(b,\overline{q})$ obtained from the configuration with $b_0=3$, $b_1=2$, and $b_2=3$.

Fig. 1. An example of forming $(b,\overline{q})$

2.4 Problem Statement

We focus on a swarm of n robots with single-integrator dynamics, i.e.

$$\dot{q}_i=u_i, \tag{2}$$

where $x_i\in\mathbb{R}^2$ is the position of robot i and $u_i\in\mathbb{R}^2$ is the control input. We aim to solve the following problem. Given a desired polygon formation $(b^*,\overline{q}^*)$ subject to the following constraints

- Each robot i has only two neighbors $i-1$ and $i+1$,
- Robots can only sense the relative position with their neighbors,
- Only a small number of robots can receive the external information of the desired formation.

The objective is to design distributed control laws for each robot i, such that

$$\mathcal{F}(\lim_{t->\infty} q)=(b^*,\overline{q}^*). \tag{3}$$

3. SWARM CONTROL STRATEGY

In this section, we first introduce a distributed control strategy that enables robots over cyclic topologies to form an arbitrary polygon formation. Note that the external control inputs are injected to only a few robots during the whole evolution. Then, we will analyze the stability and convergence properties of the closed-loop system.

3.1 Controller design

Let $u_{ij}\in\mathbb{R}^2$ denote the influence over robot i exerted by robot j, which is modeled as

$$u_{ij}=k(q_j-q_i)+\frac{(i-j)\epsilon}{2}g(q_j-q_i), \tag{4}$$

where k and ϵ are positive constants, and the function $g:\mathbb{R}^2\to\mathbb{R}^2$ is defined by

$$g(x)\triangleq\frac{x}{||x||}. \tag{5}$$

The first term of (4) represents the inner coupling generated by the virtual spring. The second one can be viewed as an offset, whose direction always starts from robot j to i, $j > i$, and magnitude is determined by ϵ. Then the control input for each robot i is designed as

$$u_i = \sum_{j\in\mathcal{N}_i} u_{ij} + u_i^e, \tag{6}$$

where $u_i^e \in \mathbb{R}^2$ denotes the external control input that robot i receives. $u_i^e = \mathbf{0}$ if robot i is independent of any external signal. In this case, robot i will stop at somewhere along the line in parallel with its neighbors. The robots who receive external control inputs are served as the vertex robots, and thus the geometric shape of the overall formation is determined. However, to implement such a control strategy, one problem needs to be addressed: the explicit expression of the external input.

Consider the desired polygon formation $(b^*, \overline{q}^*)$ with $|b^*| = m$. Let $S = \{s_0, s_1, \cdots, s_{m-1}\}$ satisfying

$$(s_{i+1} - s_i) \mod n = b_i^*, i \in \mathbb{N}_m \tag{7}$$

be the set of robots that need to be regulated by the external control input. The purpose of (7) is to ensure that the number of edges between vertex robots is consistent with b^* when the system is stablilized. Let $f_i^e \in \mathbb{R}^2, i \in \mathbb{N}_m$ represents the external control inputs, given by

$$f_i^e = f_{i+}^e + f_{i-}^e, \tag{8}$$

where

$$\begin{aligned} f_{i+}^e &= \frac{k\overline{q}_i^*}{b_i^*} - \frac{\epsilon b_i^* g(\overline{q}_i^*)}{2}, \\ f_{i-}^e &= -\frac{k\overline{q}_{i-1}^*}{b_{i-1}^*} - \frac{\epsilon b_{i-1}^* g(\overline{q}_{i-1}^*)}{2}. \end{aligned} \tag{9}$$

With these auxiliary variables, the control input (6) can be explicitly written as

$$u_i = \begin{cases} \sum_{j\in\mathcal{N}_i} u_{ij} + f_{T(i)}^e, & \text{if } i \in S, \\ \sum_{j\in\mathcal{N}_i} u_{ij}, & \text{otherwise,} \end{cases} \tag{10}$$

where $T : S \to \mathbb{N}_m$ is a mapping operator satisfying $T(s_i) = i$.

3.2 Stability analysis

We present the first main result as follows.

Theorem 1. For the swarm robots (2), under the control law (10), there exists an equilibrium point rendering $b = b^*$ and $\overline{q} = \overline{q}^*$, if $\epsilon < \frac{2k||\overline{q}_i^*||}{b_i^*(b_i^*-1)}, \ \forall i \in \mathbb{N}_m$.

Proof. We first consider those robots who receive external control inputs. Combining (4), (8), and (10), one has

$$\begin{aligned} u_{s_i} =& -k(q_{s_i} - q_{s_i+1}) + \frac{\epsilon}{2} g(q_{s_i} - q_{s_i+1}) \\ &+ k(q_{s_i-1} - q_{s_i}) + \frac{\epsilon}{2} g(q_{s_i-1} - q_{s_i}) \\ &+ \frac{k\overline{q}_i^*}{b_i^*} - \frac{\epsilon b_i^* g(\overline{q}_i^*)}{2} - \frac{k\overline{q}_{i-1}^*}{b_{i-1}^*} - \frac{\epsilon b_{i-1}^* g(\overline{q}_{i-1}^*)}{2} \\ =& -k\big(z_{s_i} - d_{s_i}^* g(\overline{q}_i^*)\big) + \frac{\epsilon}{2}\big(g(z_{s_i}) - g(\overline{q}_i^*)\big) \\ &+ k\big(z_{s_i-1} - d_{s_i-1}^* g(\overline{q}_{i-1}^*)\big) + \frac{\epsilon}{2}\big(g(z_{s_i-1}) - g(\overline{q}_{i-1}^*)\big), \end{aligned}$$

where $z_i \triangleq q_i - q_{i+1} \in \mathbb{R}^2$ denotes the relative position and $d_{s_i}^* \in \mathbb{R}$ and $d_{s_i-1}^* \in \mathbb{R}$ are defined by

$$\begin{aligned} d_{s_i}^* &\triangleq \frac{||\overline{q}_i^*||}{b_i^*} - \frac{(b_i^* - 1)\epsilon}{2k} \\ d_{s_i-1}^* &\triangleq \frac{||\overline{q}_{i-1}^*||}{b_{i-1}^*} + \frac{(b_{i-1}^* - 1)\epsilon}{2k}. \end{aligned} \tag{11}$$

Let $z_{s_i}^* = d_{s_i}^* g(\overline{q}_i^*), z_{s_i-1}^* = d_{s_i-1}^* g(\overline{q}_{i-1}^*)$. It can be easily checked that $d_{s_i-1}^*$ is a positive constant. If $\epsilon < \frac{2k||\overline{q}_i^*||}{b_i^*(b_i^*-1)}, \forall i \in \mathbb{N}_m$, $d_{s_i}^*$ is also a positive constant. Therefore, we have

$$g(z_{s_i}^*) = g(\overline{q}_i^*), \ \ g(z_{s_i-1}^*) = g(\overline{q}_{i-1}^*). \tag{12}$$

Then, the control input u_{s_i} can be written in the following form

$$\begin{aligned} u_{s_i} =& -k\big(z_{s_i} - z_{s_i}^*\big) + \frac{\epsilon}{2}\big(g(z_{s_i}) - g(z_{s_i}^*)\big) \\ &+ k\big(z_{s_i-1} - z_{s_i-1}^*\big) + \frac{\epsilon}{2}\big(g(z_{s_i-1}) - g(z_{s_i-1}^*)\big). \end{aligned} \tag{13}$$

Next, we consider the robots who are free of external control inputs. It is assumed that robot j lies between robot s_i and s_{i+1}, i.e.,

$$0 < (j - s_i) \mod n < b_i^*.$$

Combining (4) and (8), one has

$$\begin{aligned} u_j =& -k(q_j - q_{j+1}) + \frac{\epsilon}{2} g(q_j - q_{j+1}) \\ &+ k(q_{j-1} - q_j) + \frac{\epsilon}{2} g(q_{j-1} - q_j) \\ =& -k\big(z_j - d_j^* g(\overline{q}_i^*)\big) + \frac{\epsilon}{2}\big(g(z_j) - g(\overline{q}_i^*)\big) \\ &+ k\big(z_{j-1} - d_{j-1}^* g(\overline{q}_i^*)\big) \\ &+ \frac{\epsilon}{2}\big(g(z_{j-1}) - g(\overline{q}_i^*)\big), \end{aligned}$$

where $d_j^* \in \mathbb{R}$ is given by

$$d_j^* \triangleq \frac{||\overline{q}_i^*||}{b_i^*} - \frac{(b_i^* - 1 - 2l)\epsilon}{2k} \tag{14}$$

with $l = (j - s_i) \bmod n$ and $0 \le l < b_i^*$. An auxiliary variable z_j^* is introduced

$$z_j^* \triangleq d_j^* g(\overline{q}_i^*). \tag{15}$$

In view of (11) and (14), there holds $d_j^* > d_{(j-1)}^* \ge d_{s_i}^* > 0$. Then, one has

$$g(z_j^*) = g(z_{j-1}^*) = g(\overline{q}_i^*). \tag{16}$$

Similarly, the control input u_j can be written as

$$\begin{aligned} u_j =& -k(z_j - z_j^*) + \frac{\epsilon}{2}\big(g(z_j) - g(z_j^*)\big) \\ &+ k(z_{j-1} - z_{j-1}^*) + \frac{\epsilon}{2}\big(g(z_{j-1}) - g(z_{j-1}^*)\big), \ \ j \in \mathcal{V} - S. \end{aligned} \tag{17}$$

Hence, one can conclude that

$$z_i = z_i^*, \ \ \forall i \in \mathcal{V} \tag{18}$$

is an equilibrium point of the closed-loop system (2) and (10). By considering (14), (15), and (18), one has

$$\begin{aligned} \overline{q}_i =& q_{s_{i+1}} - q_{s_i} \\ =& \sum_{i=s_i}^{s_{i+1}-1} z_i^* \\ =& \sum_{l=0}^{b_i^*-1} \Big(\frac{||\overline{q}_i^*||}{b_i^*} - \frac{(b_i^* - 1 - 2l)\epsilon}{2k}\Big) g(\overline{q}_i^*) \\ =& ||\overline{q}_i^*|| g(\overline{q}_i^*) \\ =& \overline{q}_i^*. \end{aligned} \tag{19}$$

From (7), one has

$$b_i = (s_{i+1} - s_i) \mod n = b_i^*. \tag{20}$$

This completes the proof.

□

To facilitate the following analysis, we introduce a function $h : \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ as follows:

$$h(e_i, e_j, z_j^*) = e_i^T \big(g(e_j + z_j^*) - g(z_j^*)\big). \tag{21}$$

The upper bound of the function $h(e_i, e_j, z_j^*)$ is given in the following lemma.

Lemma 2. The function $h(e_i, e_j, z_j^*)$ defined in (21) satisfies

$$|h(e_i, e_j, z_j^*)| \leq \pi \frac{||e_i|| \cdot ||e_j||}{||z_j^*||}, \ \forall e_i \in \mathbb{R}^2, e_j \in \mathbb{R}^2, z_j^* \in \mathbb{R}^2.$$

Proof. Let $\theta_1 \in [0, \pi]$ and $\theta_2 \in [0, \pi]$ be, respectively, the angle between e_i and z_j^*, and the angle between $e_j + z_j^*$ and z_j^*. Then, the angle between e_i and $e_j + z_j^*$ is $\theta_1 + \theta_2$ or $|\theta_1 - \theta_2|$. Consequently, one has

$$\begin{aligned} h(e_i, e_j, z_j^*) &= ||e_i||(\cos(\theta_1 \pm \theta_2) - \cos\theta_2) \\ &= \mp 2||e_i|| \sin\frac{2\theta_1 \pm \theta_2}{2} \sin\frac{\theta_2}{2}. \end{aligned} \tag{22}$$

Therefore, we have

$$|h(e_i, e_j, z_j^*)| \leq 2||e_i|| \sin\frac{\theta_2}{2} \leq ||e_i||\theta_2. \tag{23}$$

When $||e_j|| > ||z_j^*||$, we know $\theta_2 \in [0, \pi]$. By combining (23), it yields

$$|h(e_i, e_j, z_j^*)| \leq ||e_i||\pi \leq \pi \frac{||e_i|| \cdot ||e_j||}{||z_j^*||}. \tag{24}$$

When $||e_j|| \leq ||z_j^*||$,we know $\theta_2 \in [0, \arcsin\frac{||e_j||}{||z_j^*||}]$. This leads to

$$|h(e_i, e_j, z_j^*)| \leq ||e_i|| \arcsin\frac{||e_j||}{||z_j^*||} \leq \pi \frac{||e_i|| \cdot ||e_j||}{||z_j^*||}. \tag{25}$$

In light of (24) and (25), we can conclude that

$$|h(e_i, e_j, z_j^*)| \leq \pi \frac{||e_i|| \cdot ||e_j||}{||z_j^*||}.$$

Theorem 3. The swarm robots can be globally asymptotically stabilized at the equilibria (18), if $\epsilon < \frac{4k||z_i^*||}{\pi n^2}, \ \forall i \in \mathbb{N}_n$.

Proof. In view of (13) and (17), the control input (10) can be rewritten as

$$\begin{aligned} u_i =& -k(z_i - z_i^*) + \frac{\epsilon}{2}\big(g(z_i) - g(z_i^*)\big) \\ &+ k(z_{i-1} - z_{i-1}^*) + \frac{\epsilon}{2}\big(g(z_{i-1}) - g(z_{i-1}^*)\big), i \in \mathcal{V}. \end{aligned} \tag{26}$$

$e_i \in \mathbb{R}^2$ is used to denote the error of the relative position associated with ith edge in $\mathcal{G}$, defined as

$$e_i \triangleq z_i - z_i^*. \tag{27}$$

Substituting (27) into (26), we get

$$\begin{aligned} u_i =& -ke_i + \frac{\epsilon}{2}\big(g(e_i + z_i^*) - g(z_i^*)\big) \\ &+ ke_{i-1} + \frac{\epsilon}{2}\big(g(e_{i-1} + z_{i-1}^*) - g(z_{i-1}^*)\big). \end{aligned} \tag{28}$$

The Lyapunov function candidate is chosen as

$$V = \frac{1}{2}\sum_{i\in\mathcal{V}} ||e_i||^2.$$

Note that V is globally positive definite and radially unbounded with respect to e_i. In addition, $V = 0$ implies $z_i = z_i^*, \forall i \in \mathcal{V}$. The derivative of V satisfies

$$\begin{aligned} \dot{V} =& \sum_{i\in\mathcal{V}} e_i^T \dot{e}_i \\ =& -k\sum_{i\in\mathcal{V}} ||e_{i+1} - e_i||^2 \\ &+ \frac{\epsilon}{2}\sum_{i\in\mathcal{V}} e_i^T \big(g(e_{i-1} + z_{i-1}^*) - g(z_{i-1}^*)\big) \\ &- \frac{\epsilon}{2}\sum_{i\in\mathcal{V}} e_i^T \big(g(e_{i+1} + z_{i+1}^*) - g(z_{i+1}^*)\big) \\ =& -k\sum_{i\in\mathcal{V}} ||e_{i+1} - e_i||^2 \\ &+ \frac{\epsilon}{2}\sum_{i\in\mathcal{V}} h(e_i, e_{i-1}, z_{i-1}^*) - h(e_i, e_{i+1}, z_{i+1}^*). \end{aligned}$$

From Lemma 2, one has

$$\begin{aligned} \dot{V} \leq& -k\sum_{i\in\mathcal{V}} ||e_{i+1} - e_i||^2 \\ &+ \frac{\epsilon\pi}{2}\sum_{i\in\mathcal{V}}\Big(\frac{||e_i|| \cdot ||e_{i-1}||}{||z_{i-1}^*||} + \frac{||e_i|| \cdot ||e_{i+1}||}{||z_{i+1}^*||}\Big). \end{aligned} \tag{29}$$

Let $||e_{max}|| = \max\{||e_0||, ||e_1||, \cdots, ||e_{n-1}||\}$. Without lose of generality, it is assumed that $e_{max} = e_0$. Combining (19) and (27), one has

$$\begin{aligned} e_0^T \sum_{i=0}^{n-1} e_i =& e_0^T \sum_{i=0}^{n-1}(z_i - z_i^*) \\ =& e_0^T \Big(\sum_{i=0}^{n-1}(q_i - q_{i+1}) - \sum_{i=0}^{m-1}(\overline{q}_i^*)\Big) \\ =& e_0^T \mathbf{0} \\ =& 0. \end{aligned}$$

Therefore, there exists $e_s, s \in \mathcal{V}$, satisfying

$$e_0^T e_s \leq 0.$$

Consequently, there holds

$$\begin{aligned} \sum_{i=0}^{n-1} ||e_{i+1} - e_i||^2 =& (||e_1 - e_0||^2 + ... + ||e_s - e_{s-1}||^2) \\ &+ (||e_{s+1} - e_s||^2 + ... + ||e_0 - e_{n-1}||^2) \\ \geq& \frac{||e_0 - e_s||^2}{s} + \frac{||e_0 - e_s||^2}{n-s} \\ \geq& \frac{4||e_0 - e_s||^2}{n} \\ =& \frac{4(||e_0||^2 + ||e_s||^2 - 2e_0^T e_s)}{n} \\ \geq& \frac{4||e_0||^2}{n}. \end{aligned} \tag{30}$$

Let $||z_{min}^*|| = \min\{z_0^*, z_1^*, \cdots, z_{n-1}^*\}$. From (30), we get

$$\dot{V} \leq \Big(\frac{-4k}{n} + \frac{\epsilon\pi n}{||z_{min}^*||}\Big)||e_0||^2.$$

It can be seen that $\dot{V} \leq 0$, if ϵ satisfies

$$\epsilon < \frac{4k||z_{min}^*||}{\pi n^2}.$$

And $\dot{V} = 0$ only when $e_i = 0, \ \forall i \in \mathcal{V}$. Then one can conclude from the LaSalle Invariance Principle (LaSalle (1960)), that the equilibrium point (18) is globally asymptotically stable. This completes the proof. □

Remark 4. In fact, in the situation where $\epsilon = 0$, the proof of Theorem 1 and 3 will be much easier and the control strategy is still valid. However, $\epsilon > 0$ is a necessary condition for the self-organized control strategy to be presented in the next section.

4. SELF-ORGANIZED CONTROL STRATEGY

In Section 3, although the desired formation can be achieved through a few external regulation, the vertex robots should be carefully chosen to satisfy condition (7). In this section, we will propose a self-organized control strategy which is free of the choice of vertex robots. By communicating with neighbors, the external influence can be smoothly transferred and finally stabilized at "right position". In the process of influence transition, there may be two adjacent robots affected by the external regulation. In this paper, every single robot is only allowed to be influenced by one external input at a time.

Let $\Gamma = [\gamma_{ij}] \in \mathbb{R}^{n\times m}$ be the *influence incidence matrix* with $0 \leq \gamma_{ij} \leq 1$ and $\sum_{i\in\mathcal{V}} \gamma_{ij} = 1$. γ_{ij} indicates the coupling strength between robot i and the jth external control input. The larger γ_{ij} is, the more influence is exerted to robot i from the jth external input. $\gamma_{ij} = 0$ means that the motion of agnet i is independent of f_j^e.

Then the control input for robot i is designed as

$$u_i = \sum_{j\in\mathcal{N}_i} u_{ij} + \sum_{j\in\mathbb{N}_m} \gamma_{ij} f_j^e. \tag{31}$$

Recall that robots in the $S = \{s_0, s_1, \cdots, s_{m-1}\}$ are those who are exerted by external inputs. The set S is organized such that

$$(s_i - s_0) \mod n > (s_{i-1} - s_0) \mod n, i \in \mathbb{N}_m - \{0\}.$$

At the initial stage, without loss of generality, we assume the external control input f_i^e is applied to robot s_i, i.e., $\gamma_{s_i i} = 1, i \in \mathbb{N}_m$. As the swarm formation evolves, the external control input moves along the virtual "elastic string", and hence γ_{ij} changes continuously. Let variables $\overline{\gamma}_j$ denote the positions of external control inputs, written as

$$\overline{\gamma}_j = \begin{cases} i, & \text{if } \gamma_{ij} = 1, \\ i + \gamma_{(i+1)j}, & \text{if } \gamma_{ij} > 0 \text{ and } \gamma_{(i+1)j} > 0. \end{cases} \tag{32}$$

To derive the dynamics of $\overline{\gamma}_j$, we introduce an auxiliary variable $\delta_j \in \mathbb{R}, j \in \mathbb{N}_m$.

1) If $\gamma_{ij} = 1$, δ_j is defined as

$$\delta_j \triangleq (||u_{i(i-1)}|| - ||f_{j-}^e||) - (||u_{i(i+1)}|| - ||f_{j+}^e||). \tag{33}$$

2) If $\gamma_{ij} > 0$ and $\gamma_{(i+1)j} > 0$, δ_j is defined as

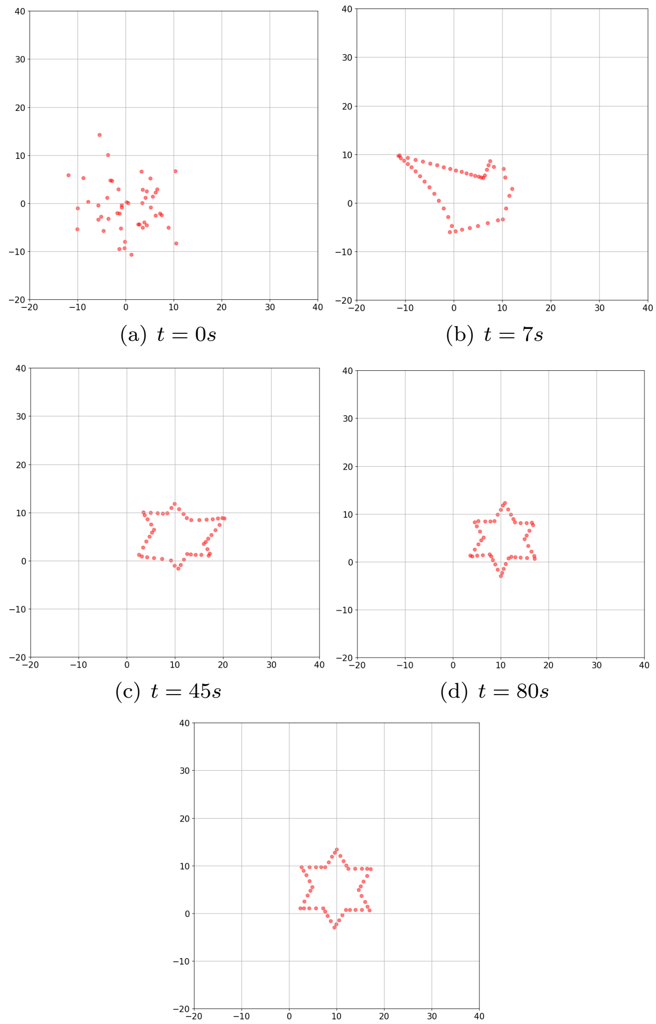
$$\begin{aligned} \delta_j \triangleq & (||u_{i(i+1)} + \gamma_{ij} f_j^e|| - ||f_{j-}^e||) \\ & - (||u_{(i+1)i} + \gamma_{(i+1)j} f_j^e|| - ||f_{j+}^e||) + \epsilon. \end{aligned} \tag{34}$$

Then the updating law of $\overline{\gamma}_j$ is given by

$$\dot{\overline{\gamma}}_j = \alpha\big(\delta_j + \beta\gamma_{ij}\gamma_{(i+1)j}\text{sgn}(\delta_j)\big), \tag{35}$$

where α and β are two positive constants.

In this proposed control method, not only the sensing is carried out in a local manner, but also the interaction



(a) $t = 0s$ (b) $t = 7s$ (c) $t = 45s$ (d) $t = 80s$ (e) $t = 200s$

Fig. 2. Formation evolution of 48 robots using controller (31)

between neighbors and external control inputs is implemented locally. In addition, consider that the external regulation is transferred automatically based on the dynamics of $\overline{\gamma}_j$. Therefore, the control strategy is self-organized and distributed. Due to the complexity of the control strategy, we do not have rigorous proof for global convergence to the desired equilibrium. But convergence to the desired polygon formation can be validated through the following simulation results.

5. SIMULATIONS

This section presents simulation results to verify the effectiveness of the proposed self-organized control strategy. Consider a swarm of 48 robots with random initial positions. The desired polygon formation $(b^*, \overline{q}^*)$ is a dodecagon. The vector b^* is set as $b^* = [4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4]^T$. The nominal configuration $\overline{q}^*$ is given by

$$\overline{q}^* = \begin{bmatrix} -5 & -2.5 & -2.5 & -5 & 2.5 & -2.5 & 5 & 2.5 & 2.5 & 5 & -2.5 & 2.5 \\ 0 & 4 & -4 & 0 & -4.5 & -4.5 & 0 & -4 & 4 & 0 & 4.5 & 4.5 \end{bmatrix}.$$

The set of robots who are influenced by external inputs is set as $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. The control variables are choosed as $k = 8$, $\epsilon = 2$, $\alpha = 0.4$, and $\beta = 4$.

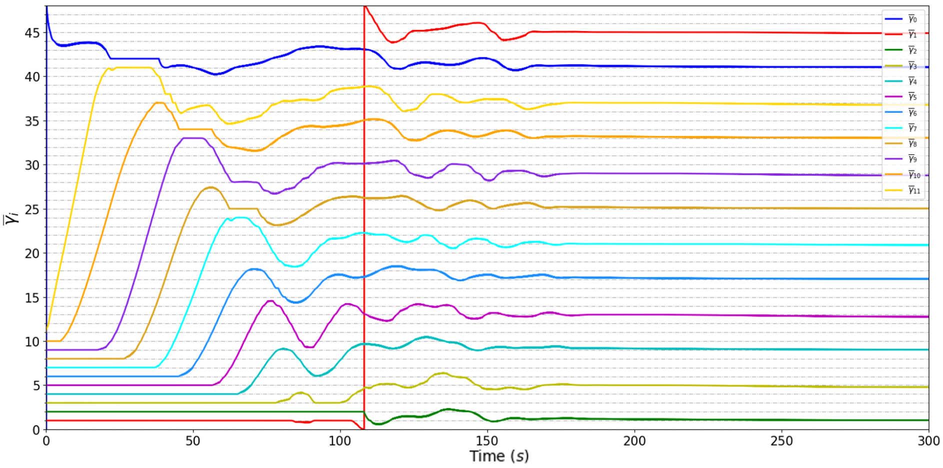


Fig. 3. Position changes of external control inputs

Fig. 2 shows the formation shape of the swarm robots at $t = \{0, 7, 45, 80, 200\}s$. It can be seen that the robots from random initial positions finally converge to the desired polygon formation using the self-organized control strategy. The position transfers of external control inputs are shown in Fig. 3, which implies that the external control inputs will eventually stop at the "right" robots.

6. CONCLUSION AND FUTURE WORK

In this paper, we study the formation control problem of swarm robots under cyclic topologies. First, inspired by elastic strings, we have proposed a distributed control strategy to achieve polygon formation shape with a small number of external control inputs. Then to relax the constraint that the external influences need to be applied to appropriate robots, we have designed an adaptive influence-transition mechanism to allow the external influence to transfer between neighboring robots. By utilizing this mechanism, we have proposed a self-organized control strategy, within which the external regulation can be initially exerted to arbitrary robots. Future work includes extending the current control strategy to more general topologies rather than only cyclic topologies.

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