Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Parameter Identification of Block-Oriented Nonlinear Systems in the Frequency Domain

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Abstract: A problem of parameter identification of nonlinear dynamic systems of manufacturing processes on the set of block-oriented models in the frequency domain is considered. Method of parameter identification in steady state at the input sinusoidal influences is proposed. The solution of the problem of parameter identification is reduced to the solution of the systems of algebraic equations by using the Fourier approximation. The parameters estimations are received by the least squares method**.** The identification method is investigated by theoretical analysis and computer modelling.

Keywords**:** nonlinear system, block-oriented model, identification, parameter, dynamic, sinusoidal.

1. INTRODUCTION

Basis of automatic control of any system of manufacturing processes is the presence of the information on a condition of the system in the form of mathematical model.

The construction of the system mathematical model by using the system identification methods is connected with the solution of different problems depending on the a priori information about the system (Eykhoff 1974). The construction of the system's optimal model in many respects depends on successfully solving the parameter identification problem at known model structure.

Identification of systems is based basically on the linear stationary models, which are widely applied to manufacturing processes. At the same time, many of the current dynamic processes in manufacturing systems bear the nonlinear character. At the research of the nonlinear systems principally new events appear, which are not observed in the linear systems. Application of linear or linearized models during formalization of the regularity of the process proceeding in nonlinear systems, is possible only in the limited area of change of variables. The research of physical events and their features in the nonlinear systems can be examined and adequately characterized only by using the nonlinear dynamic models.

The nonlinear systems are generally represented by block-oriented models consisting of different modifications of the Hammerstein and Wiener models (Haber and Keviczky 1976) or general models, in particular, the Volterra (Volterra 1959) and Wiener (Wiener 1958) series and the Kolmogorov-Gabor (Kolmogorov 1941; Gabor et al. 1961) continuous and discrete polynomials.

Different modifications of the Hammerstein and Wiener models consist of different combinations of connections of linear dynamic and nonlinear static blocks. Such models are successfully used in many fields of the industrial processes to identify systems of mining and smelting, ore dressing, chemical, mechanical, biological processes and etc.

At the representation of nonlinear systems by the block-oriented models, most of the existing developed methods of parameter identification are developed for the simple Hammerstein and Wiener models (e.g. Bottegal *et al*. 2018; Giri and Bai (eds) 2010; Mattsson and Wigren 2016; Salukvadze and Shanshiashvili 2013). Comparatively small quantity of works are devoted to the identification of parameters of other block-oriented models (e.g. Brouri *et al*. 2017); Giordano and Sjöberg 2018; Schoukens and Tiels 2017). This can be explained by the fact that the majority of such models, except for the Hammerstein models (simple and generalized) are nonlinear relative to the parameters, and also because of the large number number of estimated parameters. So, for example, the number of estimated parameters of the simple Hammerstein-Wiener cascade model with the nonlinear elements in the form of polynomial functions of n_1 and n_2 degree and linear dynamic element described by the differential equation of m order, is equal to $m+n_1+n_2+3$. Therefore, the solution of the problem of parameter identification is analytically possible only for some block-oriented low order models.

In this work the problem of parameter identification is considered on the set of models, elements of which are simple and generalized Hammerstein and Wiener models, Wiener-Hammerstein and Hammerstein-Wiener cascade models with nonlinear elements in the form of polynomial functions of second degree and linear elements described by the ordinary differential equation of first and second order. Despite their simplicity, such models are widely used for the modelling of manufacturing processes.

It should be noted that the problem of parameter identification considered in this work can be connected directly with the problem of structure identification using the experimental data, received for solving the problem of structure identification based on a posteriori information application (Shanshiashvili and Prangishvili 2017).

2. CLASSES OF MODELS AND INPUT SIGNALS

2.1. Class of models

The problem parameter identification is considered on the following class of continuous block-oriented models (Fig. 1):

$$L = \{s_i | i = 1, 2, \ldots, 6\}, \qquad (1)$$

where s_1 and s_2 - the simple and generalized Hammerstein models, s_3 and s_4 - the simple and generalized Wiener models, s_5 - the simple Wiener- Hammerstein cascade model, s_6 - the simple Hammerstein-Wiener cascade model.

The models of the set L are described by the following equations:

- Simple Hammerstein model

$$y(t) = c_0 W(0) + c_1 W(p) u(t) + c_2 W(p) u^2(t); \qquad (2)$$

- Generalized Hammerstein model

$$y(t) = c_0 + W_1(p) u(t) + W_2(p) u^2(t); \qquad (3)$$

- Simple Wiener model

$$y(t) = c_0 + c_1 W(p) u(t) + c_2 [W(p) u(t)]^2; \qquad (4)$$

- Generalized Wiener model

$$y(t) = c_0 + W_1(p) u(t) + [W_2(p) u(t)]^2; \qquad (5)$$

- Simple Wiener-Hammerstein cascade model

$$y(t) = c_0 W(0) + c_1 W_1(p) W_2(p) u(t) + c_2 W_2(p) [W_1(p) u(t)]^2; \qquad (6)$$

- Simple Hammerstein -Wiener cascade model

$$y(t) = d_0 + c_0 d_1 W(0) + c_0^2 d_2 W^2(0) + [c_1 d_1 W(p) + 2c_0 c_1 d_2 \times \times W(0)] W(p)] u(t) + [c_2 d_1 W(p) + c_1^2 d_2 W^2(p) + 2c_0 c_2 d_2 \times \times W(0) W(p)] u^2(t) + 2c_1 c_2 d_2 W^2(p) u^3(t) + c_2^2 d_2 W^2(p) u^4(t). \qquad (7)$$

In the formulas (2) - (7) $u(t)$ and $y(t)$ are input and output variables, correspondingly. The nonlinear static elements forming the models are described by the second order polynomial functions:

$$f_1[x(t)] = c_0 + c_1 x(t) + c_2 x^2(t), \qquad (8)$$

$$f_2[x(t)] = d_0 + d_1 x(t) + d_2 x^2(t), \qquad (9)$$

where c_i, d_i $(i = 0,1,2)$ - constant coefficients, $W_i(p)$ $(i = 1,2)$ - transfer functions of linear dynamic systems in the operational form, i.e. $p \equiv d/dt$.

The linear dynamic elements are included in the class of block-oriented models are assumed to be steady, i.e. the roots of their characteristic equations are placed in left half plane of the roots plane.

2.2. Class of input signals

To solve the problem of the parameter identification of the nonlinear systems, based on active experiment, it is supposed, that input variable of the system $u(t)$ is a sinusoidal function:

$$u(t) = A\cos\omega t. \qquad (10)$$

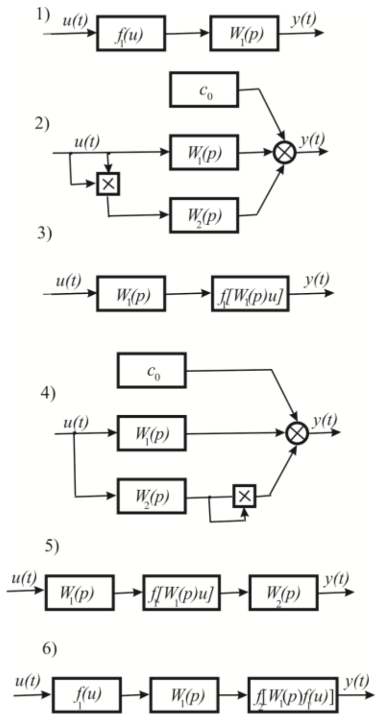

Fig.1. The block-oriented models: 1) simple Hammerstein model; 2) generalized Hammerstein model; 3) simple Wiener model; 4) generalized Wiener model; 5) simple Wiener-Hammerstein cascade model; 6) simple Hammerstein - Wiener cascade model.

3. MATHEMATICAL DESCRIPTION OF FORCED OSCILLATIONS

When a sinusoidal signal acts on the input of the linear element, then at the output of the element in the steady state after the termination of the transient process the sinusoidal signal with same frequency and with the amplitude and phase, which depends on the frequency, is obtained. In this case, the periodic signal is obtained on the output of nonlinear static element, which is the sum of the sinusoidal signals with multiple frequencies. These facts were used to determine analytical expressions of the forced oscillations on the output of models.

Let's consider the case when the transfer functions of the model's linear dynamic parts are defined by the expression

$$W_i(p)=\frac{1}{T_{0i}p^2+T_ip+1}\quad (i=1,2),\qquad (11)$$

where $T_{0i}>0\,(i=1,2)$ has a dimension of time square, and $T_i>0\,(i=1,2)$ - a dimension of time.

The forced oscillations, obtained at the output of the models in the steady state are determined by the expressions:

- Simple Hammerstein model

$$y(t)=c_0+\frac{1}{2}A^2+\frac{c_1A\left(1-\omega^2T_{01}\right)}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\cos\omega t+$$
$$+\frac{c_1A\omega T_1}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\sin\omega t+\frac{c_2A^2\left(1-4\omega^2T_{01}\right)}{2\left[\left(1-4\omega^2T_{01}\right)^2+4\omega^2T_1^2\right]}\times$$
$$\times\cos 2\omega t+\frac{c_2A^2\omega T_1}{\left(1-4\omega^2T_{01}\right)^2+4\omega^2T_1^2}\sin 2\omega t\,;\qquad (12)$$

- Generalized Hammerstein model

$$y(t)=c_0+\frac{1}{2}A^2+\frac{A\left(1-\omega^2T_{01}\right)}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\cos\omega t+$$
$$+\frac{A\omega T_1}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\sin\omega t+\frac{A^2\left(1-4\omega^2T_{02}\right)}{2\left[\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2\right]}\times$$
$$\times\cos 2\omega t+\frac{A^2\omega T_2}{\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2}\sin 2\omega t\,;\qquad (13)$$

- Simple Wiener model

$$y(t)=c_0+\frac{c_2A^2}{2\left[\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2\right]}+\frac{c_1A\left(1-\omega^2T_{01}\right)}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\times$$
$$\times\cos\omega t+\frac{c_1A\omega T_1}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\sin\omega t+$$
$$+\frac{c_2A^2\left[\left(1-\omega^2T_{01}\right)^2-\omega^2T_1^2\right]}{2\left[\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2\right]^2}\cos 2\omega t+$$
$$+\frac{c_2A^2\left(1-\omega^2T_{01}\right)\omega T_1}{\left[\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2\right]^2}\sin 2\omega t\,;\qquad (14)$$

- Generalized Wiener model

$$y(t)=c_0+\frac{A^2}{2\left[\left(1-\omega^2T_{02}\right)^2+\omega^2T_2^2\right]}+\frac{A\left(1-\omega^2T_{01}\right)}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\times$$
$$\times\cos\omega t+\frac{A\omega T_1}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\sin\omega t+$$
$$+\frac{A^2\left[\left(1-\omega^2T_{02}\right)^2-\omega^2T_2^2\right]}{2\left[\left(1-\omega^2T_{02}\right)^2+\omega^2T_2^2\right]^2}\cos 2\omega t+$$
$$+\frac{A^2\left(1-\omega^2T_{02}\right)\omega T_2}{\left[\left(1-\omega^2T_{02}\right)^2+\omega^2T_2^2\right]^2}\sin 2\omega t\,;\qquad (15)$$

- Simple Wiener-Hammerstein cascade model

To simplify the calculations, we will suppose that in expression (8): $c_0=c_1=0,\ c_2=c$ and in (11): $T_{0i}=0$ $(i=1,2)$, then after calculation we will receive:

$$y=\frac{cA^2}{2(1+\omega^2T_1^2)}+\frac{cA^2\left(1-\omega^2T_1^2-4\omega^2T_1T_2\right)}{2(1+\omega^2T_1^2)^2(1+4\omega^2T_2^2)}\cos 2\omega t+$$
$$+\frac{cA^2\left[\left(1-\omega^2T_1^2\right)\omega T_2+\omega T_1\right]}{2(1+\omega^2T_1^2)^2(1+4\omega^2T_2^2)}\sin 2\omega t\,;\qquad (16)$$

- Simple Hammerstein -Wiener cascade model

After supposing that in expressions (8), (9): $c_0=c_1=0$, $c_2=1$ $d_0=d_1=0,\ d_2=1$ and in (11): $T_{0i}=0$ $(i=1)$, $T_1=T$, then we will receive:

$$y=\frac{A^4}{4}+\frac{A^4+4A^4\omega^2T^2}{8\left(1+4\omega^2T^2\right)^2}+\frac{A^4}{2\left(1+4\omega^2T^2\right)}\cos 2\omega t+$$
$$+\frac{A^4\omega T}{1+4\omega^2T^2}\sin 2\omega t+\frac{A^4-4A^4\omega^2T^2}{8\left(1+4\omega^2T^2\right)^4}\cos 4\omega t+$$
$$+\frac{A^4\omega T}{2\left(1+4\omega^2T^2\right)^2}\sin 4\omega t\,.\qquad (17)$$

4. PARAMETER IDENTIFICATION

Let's consider the features for the parameters estimation of models by the method using the Fourier approximation by the method of the least squares.

4.1. Simple Hammerstein model

The application of the Fourier approximation (Hamming 1987) for the output periodic signal of the system enables to obtain the estimates of the Fourier coefficients $a_0/2$ a_k, b_k, $(k=1,2)$. Equating such estimates with their theoretical values we'll get:

$$\frac{\hat{a}_0}{2} = c_0 + \frac{1}{2} c_2 A^2 , \tag{18}$$

$$\hat{a}_1 = \frac{c_1 A\left(1-\omega^2 T_{01}\right)}{\left(1-\omega^2 T_{01}\right)^2 + \omega^2 T_1^2} , \tag{19}$$

$$\hat{b}_1 = \frac{c_1 A \omega T_1}{\left(1-\omega^2 T_{01}\right)^2 + \omega^2 T_1^2} , \tag{20}$$

$$\hat{a}_2 = \frac{c_2 A^2 \left(1-4\omega^2 T_{01}\right)^2}{2\left[\left(1-4\omega^2 T_{01}\right)^2 + 4\omega^2 T_1^2\right]} , \tag{21}$$

$$\hat{b}_2 = \frac{c_2 A^2 \omega T_1}{\left(1-4\omega^2 T_{01}\right)^2 + 4\omega^2 T_1^2} . \tag{22}$$

Using the expressions (19) – (22) at different frequencies $\omega = \omega_i \ \left(i = 1,2,\ldots,n\right)$, we obtain the dependence between the parameters T_{01} and T_1

$$\omega_i^2 T_{01} + \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i T_1 + \varepsilon_{1i} = 1 \ \left(i = 1,2,\ldots,n\right), \tag{23}$$

$$4\omega_i^2 T_{01} + 2\frac{\hat{a}_{2i}}{\hat{b}_{2i}} \omega_i T_1 + \varepsilon_{2i} = 1 \ \left(i = 1,2,\ldots,n\right), \tag{24}$$

where $\hat{a}_{1i}, \hat{b}_{1i}, \hat{a}_{2i}, \hat{b}_{2i} \ \left(i = 1,2,\ldots,n\right)$ - values of the Fourier coefficients at the frequency ω_i, $\varepsilon_{1i}, \varepsilon_{2i} \ \left(i = 1,2,\ldots,n\right)$ - errors of measurements and caused by noise.

Let's consider the features for T_{01} and T_1 parameters estimation by the method of least squares using the expression (23).

The error squared sum is

$$S = \sum_{i=1}^{n} \varepsilon_{1i}^2 = \sum_{i=1}^{n} \left(1 - \omega_i^2 T_{01} - \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i T_1\right)^2 . \tag{25}$$

Now we'll determine the values of the estimates $\hat{T}_{01}$ and $\hat{T}_1$ so that their substitution for T_{01} and T_1 should give the minimal value S in the equation (25). For that purpose differentiating (25) at first by T_{01} and then by T_1, and equating the received results to zero, we'll obtain the following expressions for estimating $\hat{T}_{01}$ and $\hat{T}_1$:

$$\begin{aligned} &\sum_{i=1}^{n} \omega_i^4 T_{01} + \sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 T_1 = \sum_{i=1}^{n} \omega_i^2 , \\ &\sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 T_{01} + \sum_{i=1}^{n} \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2 T_1 = \sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i . \end{aligned} \tag{26}$$

The solution of the system of equations (26) allows obtaining the estimates of the parameters $\hat{T}_{01}$ and $\hat{T}_1$ using the method of the least squares:

$$\hat{T}_{01} = \frac{\left(\sum_{i=1}^{n} \omega_i^2\right)\left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2\right) - \left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i\right)\left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3\right)}{\left(\sum_{i=1}^{n} \omega_i^4\right)\left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2\right) - \left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3\right)^2} , \tag{27}$$

$$\hat{T}_1 = \frac{\left(\sum_{i=1}^{n} \omega_i^4\right)\left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i\right) - \left(\sum_{i=1}^{n} \omega_i^2\right)\left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3\right)}{\left(\sum_{i=1}^{n} \omega_i^4\right)\left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2\right) - \left(\sum_{i=1}^{n} \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3\right)^2} . \tag{28}$$

Estimates $\hat{T}_{01}$ and $\hat{T}_1$ can be also obtained by using expression (24). If we know the values of estimates $\hat{T}_{01}$ and $\hat{T}_1$ it is possible to obtain estimates of parameters $\hat{c}_0$, $\hat{c}_1$ and. $\hat{c}_2$ by using the expressions (18) – (22).

4.2. Generalized Hammerstein model

Equating the estimates $\hat{a}_0/2$, $\hat{a}_k$, $\hat{b}_k$, $\left(k = 1,2\right)$ of the Fourier coefficients, determining by using Fourier approximation for the periodic signal (13), with their theoretical values we obtain the equations for determining the estimates of the parameters c_0, T_{01}, T_1, T_{02} and T_2:

$$\hat{c}_0 = \frac{1}{2}\left(\hat{a}_0 - A^2\right), \tag{29}$$

$$\omega_i^2 T_{01} + \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i T_1 + \varepsilon_{1i} = 1 \ \left(i = 1,2,\ldots,n\right), \tag{30}$$

$$4\omega_i^2 T_{02} + 2\frac{\hat{a}_{2i}}{\hat{b}_{2i}} \omega_i T_2 + \varepsilon_{2i} = 1 \ \left(i = 1,2,\ldots,n\right). \tag{31}$$

The $\hat{T}_{01}$ and $\hat{T}_1$ estimates by the method of the least squares, obtained using (30), are determined by the expressions (27) and (28), and the $\hat{T}_{02}$ and $\hat{T}_2$ obtained using (31), look like:

$$T_{02} = \frac{1}{4} \frac{\left(\sum_{i=1}^{n} \omega_i^2\right)\left(\sum_{i=1}^{n} \frac{a_{2i}^2}{b_{2i}^2} \omega_i^2\right) - \left(\sum_{i=1}^{n} \frac{a_{2i}}{b_{2i}} \omega_i\right)\left(\sum_{i=1}^{n} \frac{a_{2i}}{b_{2i}} \omega_i^3\right)}{\left(\sum_{i=1}^{n} \omega_i^4\right)\left(\sum_{i=1}^{n} \frac{a_{2i}^2}{b_{2i}^2} \omega_i^2\right) - \left(\sum_{i=1}^{n} \frac{a_{2i}}{b_{2i}} \omega_i^3\right)^2} , \tag{32}$$

$$T_2 = \frac{1}{2} \frac{\left(\sum_{i=1}^{n} \omega_i^4\right)\left(\sum_{i=1}^{n} \frac{a_{2i}}{b_{2i}} \omega_i\right) - \left(\sum_{i=1}^{n} \omega_i^2\right)\left(\sum_{i=1}^{n} \frac{a_{2i}}{b_{2i}} \omega_i^3\right)}{\left(\sum_{i=1}^{n} \omega_i^4\right)\left(\sum_{i=1}^{n} \frac{a_{2i}^2}{b_{2i}^2} \omega_i^2\right) - \left(\sum_{i=1}^{n} \frac{a_{2i}}{b_{2i}} \omega_i^3\right)^2} . \tag{33}$$

4.3. Simple Wiener model

From expression (14) we receive:

$$\frac{\hat{a}_0}{2} = c_0 + \frac{c_2 A^2}{2\left[\left(1-\omega^2 T_{02}\right)^2 + \omega^2 T_2^2\right]}. \quad (34)$$

The coefficients a_1 and b_1 in that case are determined by the expressions (19), (20) and

$$\hat{a}_2 = \frac{c_2 A^2\left[\left(1-\omega^2 T_{01}\right)^2 - \omega^2 T_1^2\right]}{2\left[\left(1-\omega^2 T_{01}\right)^2 + \omega^2 T_1^2\right]^2},\ \hat{b}_2 = \frac{c_2 A^2\left(1-\omega^2 T_{01}\right)\omega T_1}{\left[\left(1-\omega^2 T_{01}\right)^2 + \omega^2 T_1^2\right]^2}. \quad (35)$$

$\hat{T}_{01}$ and $\hat{T}_1$ estimates by the method of the least squares are determined by the expressions (27) and (28), and the estimates of the parameters $\hat{c}_0$, $\hat{c}_1$ and. $\hat{c}_2$ - by the expressions (19), (20), (34) and (35).

4.4. Generalized *Wiener model*

From expression of the Fourier coefficients $\hat{a}_0/2$ we obtain the estimate of coefficient c_0:

$$\hat{c}_0 = \frac{\hat{a}_0}{2} - \frac{A^2}{2\left[\left(1-\omega^2 T_{02}\right)^2 + \omega^2 T_2^2\right]}. \quad (36)$$

In that case $\hat{T}_{01}$ *and* $\hat{T}_1$ *estimates* are determined by the expressions (27) and (28).

Using the expressions of $\hat{a}_2$ and $\hat{b}_2$, determining from (15), it is possible to get the system of equations linear with respect to T_{02}, T_2, T_{02}^2, T_2^2, $T_{02}T_2$:

$$2\omega_i^2 T_{02} + 2\omega_i \frac{a_{2_i}}{b_{2_i}} T_2 - \omega_i^4 T_{02}^2 - 2\omega_i^2 \frac{a_{2_i}}{b_{2_i}} T_{02} T_2 + \omega_i^2 T_2^2 = 1 \ \left(i = 1, 2, \ldots, n\right) \quad (37)$$

and using it to receive the estimates of the parameters $\hat{T}_{02}$ and $\hat{T}_2$ obtained by the method of the least squares.

4.5. Simple Wiener-Hammerstein cascade model

Using expression of estimate of the Fourier coefficient $\hat{a}_0/2$ of the output periodic signal (16), after series of operations according to the least squares method and transformations we'll get the estimates of the parameters $\hat{c}$ and $\hat{T}_1$:

$$\hat{c} = \frac{\left(\sum_{i=1}^n \hat{a}_{0i}\right)\left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^4\right) - \left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^2\right)^2}{A^2 n\left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^4\right) - A^2\left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^2\right)}, \quad (38)$$

$$\hat{T}_1 = \sqrt{\frac{n\left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^2\right) - \left(\sum_{i=1}^n \hat{a}_{0i}\right)\left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^2\right)}{n\left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^4\right) - \left(\sum_{i=1}^n \hat{a}_{0i}\omega_i^2\right)}}. \quad (39)$$

After receiving estimate $\hat{T}_1$ using expression of ratio $\hat{a}_2/\hat{b}_2$ it is possible to obtain the estimate of the parameter T_2 by the least squares method:

$$\hat{T}_2 = \frac{\sum_{i=1}^n \left(\hat{a}_{2i}\omega_i^3 \hat{T}_1^2 - 4b_{2i}\omega_i^2 \hat{T}_1\right) H_i}{\sum_{i=1}^n \left(\hat{a}_{2i}\omega_i^3 \hat{T}_1^2 - 4\hat{b}_{2i}\omega_i^2 \hat{T}_1\right)}, \quad (40)$$
$$H_i = \left(\hat{a}_{2i} - \hat{b}_{2i} + \hat{a}_{2i}\omega_i \hat{T}_1 + \hat{b}_{2i}\omega_i^2 \hat{T}_1^2\right)$$

4.6. Simple Hammerstein-Wiener cascade model

Using expression of estimate of $\hat{a}_2$ from (17) we obtain another estimate $\hat{T}$:

$$\hat{T} = \sqrt{\frac{2\sum_{i=1}^n \hat{a}_{2i}^2 \omega_i^2 - A^4 \sum_{i=1}^n \hat{a}_{2i}\omega_i^2}{8\sum_{i=1}^n \hat{a}_{2i}^2 \omega_i^4}}. \quad (41)$$

Using expression of estimate of Fourier coefficient $\hat{b}_2$ from (17) we obtain another estimate $\hat{T}$:

$$\hat{T} = \frac{\left(\sum_{i=1}^n \hat{b}_{2i}\omega_i\right)\left(\sum_{i=1}^n \hat{b}_{2i}^2 \omega_i^4\right) - \left(\sum_{i=1}^n \hat{b}_{2i}^2 \omega_i^3\right)\left(\sum_{i=1}^n \hat{b}_{2i}^2 \omega_i^2\right)}{A^4\left[\left(\sum_{i=1}^n \omega_i^2\right)\left(\sum_{i=1}^n \hat{b}_{2i}^2 \omega_i^4\right) - \left(\sum_{i=1}^n \hat{b}_{2i}^2 \omega_i^3\right)^2\right]}. \quad (42)$$

5. INVESTIGATION OF THE IDENTIFICATION METHOD ON ACCURACY

The algorithms of parameter estimation of nonlinear systems have been designed in accordance with the developed method of parameter identification. In order to use of the designed algorithms of parameter identification in the industrial conditions under noise and disturbances, it is necessary to investigate identification method on accuracy.

The identification method is investigated by theoretical analysis and computer modelling. The reliability of the received results, at the parameter identification of nonlinear systems of industrial processes conditions in the presence of noise and errors, depends on the measurement accuracy of the systems' input and output signals and on the mathematical processing of the experimental data. Besides, as it is known, that used method of the least squares is noiseless.

The investigation of the algorithms of the parameter identification of nonlinear systems was carried out by means of the computer modelling based on using MATLAB.

We used both, the tool of package Simulink-toolbox for the system modelling and tool Symbolic Math Toolbox for the solution of the equations. For example, result of computer modelling is given here for Generalized Hammerstein model.

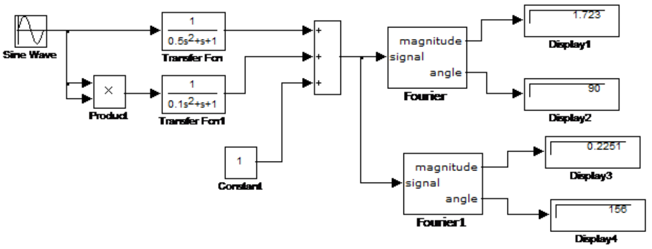

Fig. 2. Scheme for Generalized Hammerstein model

When investigating the algorithms of parameter identification the programs corresponding to such algorithms were designed. Using such programs, the diagrams of output variable models and the estimations of unknown parameters have been obtained. As an example, the results obtained for the Generalized Hammerstein model are given below. The experiments were carried out at values of the parameters $c_0 = 2,\ T_{01} = 1{,}5,\ T_1 = 2,\quad T_{02} = 0{,}5,\ T_2 = 1$. Following estimations of the unknown parameters are obtained: c0 =2, T01 = 1.4709, T1 = 1.9528, T02 = 0.4882, T2 =0.9729.

6. CONCLUSION

In this work the problem of parameter identification is considered on the set of models, elements of which are simple and generalized Hammerstein and Wiener models, Wiener-Hammerstein and Hammerstein-Wiener cascade models with nonlinear elements in the form of polynomial functions of second degree and linear elements described by the ordinary differential equation of first and second order. Despite their simplicity such models are widely used for the modelling of manufacturing processes.

The solution of the problem of parameter identification for the majority of block - oriented models is complicated due to the nonlinearity of such models relative to the sought parameters.

Proposed method of parameter identification in steady state based on the observation of the system's input and output variables at the input sinusoidal influences is proposed. The solution of the problem of parameter identification is reduced to the solution of the algebraic equations systems by using the Fourier approximation. The parameters estimations are received by the least squares method. Reliability of the received results depends on the accuracy of the measurement of system output signals and mathematical processing of the experimental data.

Developed parameter identification method can be used for the modelling of nonlinear manufacturing processes when the model structure is known a priori. As the estimations of parametres are received by the least squares method, which is noiseless, it can be used in the industrial conditions in the presence of the noise and measurement errors. The specification of the method of identification allows to use Fourier coefficients of various harmonics to estimate the parametres and compare the received results.

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