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Dynamic Adaptive Event-Triggered Scheme for General Linear Multi-Agent Systems ⋆

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Abstract: This paper proposes a novel dynamic adaptive event-triggered scheme to deal with consensus problems in a class of general linear multi-agent systems. Firstly, a fully distributed event-triggered consensus protocol is proposed by assigning a time-varying coupling weight for each edge. Then by introducing an additional internal dynamic variable, a dynamic adaptive event condition is skillfully constructed and Zeno behavior is also successfully excluded. Based on this, the asymptotic consensus is eventually achieved and the frequency of communication among agents is significantly reduced. Finally, one simulation example is provided to verify the effectiveness of the proposed scheme.

Keywords: Consensus, event-triggered, multi-agent system, adaptive control.

1. INTRODUCTION

There is no doubt that consensus is a fundamental issue for coordination of multi-agent systems, since it underpins key functionalities ranging from formation Han et al. (2019), decision-making Xu et al. (2019a); Shen et al. (2020), distributed optimization Yang et al. (2017, 2019) to distributed Nash equilibrium seeking Gadjov and Pavel (2019). In the past decade, there is a substantial body of literature devoted to the development of consensus problems in various kinds of multi-agent systems. One key point for achieving consensus is the information sharing among agents, and thus different communication schemes have been proposed for scheduling information transmission among agents. Traditional communication schemes are generally dependent on time lapses and we call them as time-triggered communication schemes. Such a kind of schemes are always conservative and should be designed for the worst case. This possibly leads to the unnecessary waste of communication resources in practice.

Recently, an alternative kind of communication schemes, event-triggered schemes, have already become increasingly appealing, in which communication is triggered only when it is needed. Generally, an event condition is well designed to determine when agents need to communicate with their neighbors. The event-triggered schemes have advantages over the time-triggered schemes in terms of saving communication resources, especially when the resources are limited. In early work, constant research efforts have been made to the investigation of *static* event-triggered communication schemes with the hope of lowing communication frequency while guaranteeing consensus Cheng and Li (2019); Xu et al. (2019a); Dimarogonas et al. (2012). Recently, *dynamic* event-triggered schemes are proposed in Girard (2015); Xu et al. (2019b); He et al. (2019); Dolk and Heemels (2015); Yi et al. (2019); Li et al. (2020) by introducing an additional internal dynamic variable, which further improve the communication efficiency by ensuring a larger inter-event time than the static ones.

Based on the above discussion, we would like to further consider dynamic event-triggered schemes for a class of general linear multi-agent systems. It is worth noting that almost all existing event-triggered schemes for general linear multi-agent systems involve more or less some global information, such as the eigenvalue information of the Laplace matrix, or the total number of agents. As it is known, the involvement of global information could degrade the scalability of event-triggered schemes especially against when the network topology changes. Motivated by Li et al. (2013); Cheng and Li (2019); Yi et al. (2019), a dynamic adaptive event-triggered scheme is proposed, for the first time, which ensures the asymptotic consensus of general linear multi-agent systems and does not exhibit Zeno behavior. As compared with Cheng and Li (2019); He et al. (2019), the proposed event-triggered scheme not

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only successfully excludes the utilization of any global information, but also guarantees *asymptotic* consensus.

2. PRELIMINARIES

In this section, we recall a few known results to set the ground for the analysis in the rest of the paper.

2.1 Graph Theory

Consider an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a node set $\mathcal{V} = \{1, 2, \cdots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. If nodes i and j are connected by an edge, then nodes i and j are called as *neighbors*, and denote $(i, j) \in \mathcal{E}$. Let $\mathcal{N}_i$ mean a set including all neighbors of node i, and the total number of nodes in $\mathcal{N}_i$ is d_i. A path from node i to node j is a sequence of edges (i, j_1), (j_1, j_2), $\cdots$, (j_m, j) in graph $\mathcal{G}$ with distinct nodes j_k, $k = 1, 2, \cdots, m$. Then graph $\mathcal{G}$ is connected if and only if there exists a path from i to j for any two nodes i and j in $\mathcal{G}$. Moreover, define the Laplace matrix $\mathcal{L} = [l_{ij}]_{n\times n} \in \mathbb{R}^{n\times n}$ with

$$l_{ij} = \begin{cases} -1 & i \neq j \ \&\ (i,j) \in \mathcal{E}; \\ 0 & i \neq j \ \&\ (i,j) \notin \mathcal{E}; \\ -\sum\limits_{k\in\mathcal{N}_i} l_{ik} & i = j. \end{cases} \tag{1}$$

Specially, $l_{ij} = l_{ji}$ for an undirected graph $\mathcal{G}$.

Lemma 1. [Horn and Johnson (2012)] If graph $\mathcal{G}$ is undirected and connected, then the corresponding Laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 and all the other eigenvalues are positive.

Assumption 2. The communication graph $\mathcal{G}$ is undirected and connected.

2.2 Description of general linear multi-agent system

Consider a multi-agent system with general linear dynamics of agent i $(i = 1, 2, \cdots, N)$ being

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state of agent i at time instant t, and $u_i(t) \in \mathbb{R}^m$ means the control protocol of agent i at instant t. The matrices $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are constant matrices. The pair (A, B) is stabilizable.

The objectives of this paper are (i) to design a fully distributed event-triggered consensus protocol u_i such that $\lim_{t\to\infty} x_i(t) - x_j(t) = 0$ with $i, j = 1, 2, \cdots, N$, and meanwhile, (ii) to guarantee that the proposed event condition does not exhibit Zeno behavior.

3. MAIN RESULTS

In this section, we will propose a novel dynamic adaptive event-triggered scheme based on relative states for general linear multi-agent system such that all agents can asymptotically converge to the same trajectory. Both the event condition and the consensus protocol are designed and implemented in a fully distributed manner.

Event-Triggered Communication: Based on event-triggered communication schemes, agent i only transmits its information to all of its neighbors at time instant t_k^i $(k = 0, 1, 2, \cdots)$ instead of continuous communication. Here $\{t_k^i\}_{k\in\mathbb{N}}$ is the triggering time sequence for agent i, which will be designed subsequently.

Then, an event-triggered control protocol based on relative states is designed as

$$\begin{cases} u_i(t) = K\sum\limits_{j=1}^{N} c_{ij}(t)a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) \\ \hat{x}_i(t) = \mathbf{e}^{A(t-t_k^i)}x_i(t_k^i) \end{cases} \tag{3}$$

where the instant $t_k^i = \max\{t_s^i : t_s^i \leq t\}$, $c_{ij}(t)$ is a time-varying coupling weight to edge $(i, j) \in \mathcal{E}$ and the control gain matrix $K \in \mathbb{R}^{m\times n}$ is to determined later.

In addition, the coupling weight $c_{ij}(t)$ $(i, j \in \mathcal{E})$ has the following dynamics

$$\dot{c}_{ij}(t) = k_{ij}a_{ij}(\hat{x}_i(t) - \hat{x}_j(t))^T\Gamma(\hat{x}_i(t) - \hat{x}_j(t)) \tag{4}$$

where the initial values $c_{ij}(0) = c_{ji}(0) > 0$ and the weights $k_{ij} = k_{ji} > 0$ can be arbitrarily chosen. The positive semi-definite matrix Γ to be determined later.

Motivated by Li et al. (2013), the time-varying coupling weight $c_{ij}(t)$is used, which could be adjusted dynamically by (4) instead of being assigned a sufficiently large value. Different from Li et al. (2013), in this paper, agents communicate with their neighbors only at triggering instants, and thus $\dot{c}_{ij}(t)$ is based on the relative states at triggering instants. Recently, *static* event-triggered schemes with time-varying coupling weight were investigated in Cheng and Li (2019) to achieve *bounded* consensus. However, in this paper, our objective is to designed an effective *dynamic* event-triggered scheme to guarantee *asymptotic* consensus and meanwhile to exclude Zeno behavior.

A dynamic adaptive event-triggered scheme based on relative state is proposed as

$$\begin{aligned} t_{k+1}^i = \max_{t>t_k^i}\Bigg\{ & \beta_i\Bigg[\sum_{j=1}^{N} c_{ij}(t)a_{ij}e_i^T(t)\Gamma e_i(t) \\ & -\theta_i\sum_{j=1}^{N} a_{ij}[\hat{x}_i(t) - \hat{x}_j(t)]^T\Gamma[\hat{x}_i(t) - \hat{x}_j(t)]\Bigg] \leq \delta_i(t)\Bigg\} \end{aligned} \tag{5}$$

and

$$\begin{aligned} \dot{\delta}_i(t) = -\alpha_i\delta_i(t) - \mu_i & \Bigg[\sum_{j=1}^{N} c_{ij}(t)a_{ij}e_i^T(t)\Gamma e_i(t) \\ & -\theta_i\sum_{j=1}^{N} a_{ij}[\hat{x}_i(t) - \hat{x}_j(t)]^T\Gamma[\hat{x}_i(t) - \hat{x}_j(t)]\Bigg] \end{aligned} \tag{6}$$

for $j \in \mathcal{N}_i$ with $i = 1, \cdots, N$. Here the error function $e_i(t) = \hat{x}_i(t) - x_i(t)$, the coefficients $\alpha_i > 0$, $\beta_i > 0$, $\theta_i > 0$ and $\mu_i \in (0, 1)$. The initial value $\delta_i(0) > 0$.

Solving the following Algebraic Riccati Equation (ARE)

$$A^TP + PA - PBB^TP + I = 0 \tag{7}$$

to get a solution $P > 0$. Then let $K = B^TP$ and $\Gamma = PBB^TP$.

For the further analysis, define the track error $\xi_i(t) = x_i(t) - \frac{1}{N}\sum_{j=1}^{N} x_j(t)$ and $\hat{\xi}_{ij}(t) = \hat{x}_i(t) - \hat{x}_j(t)$. According to (2), one has

$$\dot{\xi}_i(t) = A\xi_i(t) + BK\sum_{j=1}^{N} c_{ij}(t)a_{ij}\hat{\xi}_{ji}(t) \tag{8}$$

where the matrices A, B, K, and Γ are given in (2) and (7).

Theorem 3. Consider a general linear multi-agent system (2). Suppose that Assumption 2 holds and $\alpha_i > \frac{1-\mu_i}{\beta_i}$ in (5) with P, K, and Γ are defined in (7). Then, for any initial condition, the control protocol (3) under dynamic adaptive event condition (5) guarantees all agents to reach consensus.

Proof. Consider a Lyapunov function candidate as

$$W(t) = \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)P\xi_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\frac{(c_{ij(t)} - \tilde{\alpha})^2}{8k_{ij}} + \sum_{i=1}^{N}\delta_i(t) \tag{9}$$

with the matrix P defined in (7), $c_{ij}(t)$ and k_{ij} given in (4) and a positive number $\tilde{\alpha}$ to be determined later. Then the derivative of $W(t)$ along the trajectory (2) can be obtained as

$$\begin{aligned}\dot{W}(t) =& \sum_{i=1}^{N}\xi_i^T(t)P[A\xi_i(t) + BK\sum_{j=1}^{N}c_{ij}(t)a_{ij}\hat{\xi}_{ji}(t)] \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\frac{c_{ij}(t)-\tilde{\alpha}}{4}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ &- \sum_{i=1}^{N}\alpha_i\delta_i(t) - \sum_{i=1}^{N}\mu_i\sum_{j=1}^{N}c_{ij}a_{ij}e_i^T(t)\Gamma e_i(t) \\ &+ \sum_{i=1}^{N}\mu_i\theta_i\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ =& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t) - \sum_{i=1}^{N}\alpha_i\delta_i(t) \\ &+ \sum_{i=1}^{N}\xi_i^T(t)PBK\sum_{j=1}^{N}c_{ij}(t)a_{ij}\hat{\xi}_{ji}(t) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\frac{c_{ij}(t)-\tilde{\alpha}}{4}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ &- \sum_{i=1}^{N}\mu_i\sum_{j=1}^{N}c_{ij}a_{ij}e_i^T(t)\Gamma e_i(t) \\ &+ \sum_{i=1}^{N}\mu_i\theta_i\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t).\end{aligned} \tag{10}$$

Graph $\mathcal{G}$ is undirected, and thus $a_{ij} = a_{ji}$. By (4), one has $c_{ij}(t) = c_{ji}(t)$ due to $c_{ij}(0) = c_{ji}(0)$. In addition, $\sum_{i=1}^{N}\xi_i^T(t)PBK\sum_{j=1}^{N}c_{ij}(t)a_{ij}\left[\hat{x}_j(t) - \hat{x}_i(t)\right] = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)a_{ij}[\xi_i(t)-\xi_j(t)]^T\Gamma\left[\hat{x}_j(t) - \hat{x}_i(t)\right]$. Therefore,

$$\begin{aligned}\dot{W}(t) =& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t) - \sum_{i=1}^{N}\alpha_i\delta_i(t) \\ &- \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)a_{ij}[\xi_i(t)-\xi_j(t)]^T\Gamma\hat{\xi}_{ij}(t) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\frac{c_{ij}(t)-\tilde{\alpha}}{4}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ &- \sum_{i=1}^{N}\mu_i\sum_{j=1}^{N}c_{ij}(t)a_{ij}e_i^T(t)\Gamma e_i(t) \\ &+ \sum_{i=1}^{N}\mu_i\theta_i\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t).\end{aligned} \tag{11}$$

According to (8), it is easy to obtain $\xi_i(t) - \xi_j(t) = x_i(t) - x_j(t) = \hat{x}_i(t) - e_i(t) - \hat{x}_j(t) + e_j(t) = \hat{\xi}_{ij}(t) - e_i(t) + e_j(t)$. Then

$$\begin{aligned}\dot{W}(t) =& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t) - \sum_{i=1}^{N}\alpha_i\delta_i(t) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(\mu_i\theta_i + \frac{c_{ij}(t)-\tilde{\alpha}}{4}\right)\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ &- \sum_{i=1}^{N}\mu_i\sum_{j=1}^{N}c_{ij}(t)a_{ij}e_i^T(t)\Gamma e_i(t) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)a_{ij}[e_i(t)-e_j(t)]^T\Gamma\hat{\xi}_{ij}(t).\end{aligned} \tag{12}$$

Note that $[e_i(t) - e_j(t)]^T\Gamma[\hat{x}_i(t) - \hat{x}_j(t)] \le \frac{1}{2}[e_i(t) - e_j(t)]^T\Gamma[e_i(t) - e_j(t)] + \frac{1}{2}[\hat{x}_i(t) - \hat{x}_j(t)]^T\Gamma[\hat{x}_i(t) - \hat{x}_j(t)]$, and thus

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)a_{ij}[e_i(t)-e_j(t)]^T\Gamma[\hat{x}_i(t) - \hat{x}_j(t)] \\ \le& \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ &+2\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)a_{ij}e_i^T(t)\Gamma e_i(t).\end{aligned} \tag{13}$$

Therefore,

$$\begin{aligned}\dot{W}(t) \le& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t) - \sum_{i=1}^{N}\alpha_i\delta_i(t) \\ &- \sum_{i=1}^{N}(\frac{\tilde{\alpha}}{4} - \theta_i)\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ &- \sum_{i=1}^{N}(1-\mu_i)\theta_i\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \\ &+ \sum_{i=1}^{N}(1-\mu_i)\sum_{j=1}^{N}c_{ij}a_{ij}e_i^T(t)\Gamma e_i(t).\end{aligned} \tag{14}$$

According to (5), one has

$$\beta_i\left[\sum_{j=1}^{N}c_{ij}(t)a_{ij}e_i^T(t)\Gamma e_i(t) - \theta_i\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)\right] \le \delta_i(t) \tag{15}$$

and thus

$$\begin{aligned}\dot{W}(t) \leq& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t)-\sum_{i=1}^{N}\kappa_i\delta_i(t)\\&-\sum_{i=1}^{N}\left(\frac{\tilde{\alpha}}{4}-\theta_i\right)\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)\\ \leq& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t)-\sum_{i=1}^{N}\kappa_i\delta_i(t)\\&-\left(\frac{\tilde{\alpha}}{4}-\theta\right)\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)\end{aligned} \tag{16}$$

with $\kappa_i = \alpha_i - \frac{1-\mu_i}{\beta_i}$ and $\theta = \max_i\{\theta_i\}$. Note that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}[x_i(t)-x_j(t)]^T\Gamma[x_i(t)-x_j(t)]\\&\leq 2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}[\hat{x}_i(t)-\hat{x}_j(t)]^T\Gamma[\hat{x}_i(t)-\hat{x}_j(t)]\\&\quad+2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}[e_i(t)-e_j(t)]^T\Gamma[e_i(t)-e_j(t)]\\&\leq 2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)+8\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}e_i(t)^T\Gamma e_i(t).\end{aligned} \tag{17}$$

By (4), one has $c_{ij}(t)$ is monotonously non-decreasing, i.e. $c_{ij}(t) \geq c_{ij}(0)$ for $t \geq 0$. Let $\tilde{c} = \min\{c_{ij}(0) : (i,j) \in \mathcal{E}\}$, by using (5), one has

$$\tilde{c}\sum_{j=1}^{N}a_{ij}e_i^T(t)\Gamma e_i(t)-\theta_i\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t) \leq \frac{\delta_i(t)}{\beta_i} \tag{18}$$

and thus

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}[x_i(t)-x_j(t)]^T\Gamma[x_i(t)-x_j(t)]\\&\leq 2\sum_{i=1}^{N}\left(1+\frac{4\theta_i}{\tilde{c}}\right)\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)\\&\quad+\sum_{i=1}^{N}\frac{8}{\beta_i\tilde{c}}\delta_i(t).\end{aligned} \tag{19}$$

Define $q_0 = \max\left\{2+\frac{8\theta}{\tilde{c}}, \frac{8(\frac{\tilde{\alpha}}{4}-\theta)}{\beta\tilde{c}q_1}\right\}$ with $\beta = \min_i\{\beta_i\}$ and $q_1 < \frac{1}{2}\left(\alpha_i - \frac{1-\mu_i}{\beta_i}\right)$, then one has

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}[x_i(t)-x_j(t)]^T\Gamma[x_i(t)-x_j(t)]\\&\leq q_0\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)+\frac{q_0q_1}{\frac{\tilde{\alpha}}{4}-\theta}\sum_{i=1}^{N}\delta_i(t),\end{aligned} \tag{20}$$

which implies that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)\\&\geq \frac{1}{q_0}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}[x_i(t)-x_j(t)]^T\Gamma[x_i(t)-x_j(t)]\\&\quad-\frac{q_1}{\frac{\tilde{\alpha}}{4}-\theta}\sum_{i=1}^{N}\delta_i(t).\end{aligned} \tag{21}$$

Thus

$$\begin{aligned}\dot{W}(t) \leq& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t)-\sum_{i=1}^{N}\frac{\kappa_i}{2}\delta_i(t)\\&-\left(\frac{\tilde{\alpha}}{4}-\theta\right)\frac{1}{q_0}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}[\xi_i(t)-\xi_j(t)]^T\\&\times\Gamma[\xi_i(t)-\xi_j(t)].\end{aligned} \tag{22}$$

Let $\xi = [\xi_1, \xi_2, \cdots, \xi_N]^T$, then

$$\begin{aligned}\dot{W}(t) =& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)(A^TP+PA)\xi_i(t)-\sum_{i=1}^{N}\frac{\kappa_i}{2}\delta_i(t)\\&-\left(\frac{\tilde{\alpha}}{2}-2\theta\right)\frac{1}{q_0}\xi^T(t)(\mathcal{L}\otimes PBB^TP)\xi(t)\\ \leq& \frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)\left(A^TP+PA-\hat{\lambda}PBB^TP\right)\xi_i(t)\\&-\sum_{i=1}^{N}\frac{\kappa_i}{2}\delta_i(t)\end{aligned} \tag{23}$$

with $\hat{\lambda} = \frac{(\tilde{\alpha}-4\theta)\lambda_2}{q_0}$. By choosing $\tilde{\alpha} > \frac{q_0}{\lambda_2} + 4\theta$, one has $\hat{\lambda} > 1$. According to (7), one has

$$\dot{W}(t) \leq -\frac{1}{2}\sum_{i=1}^{N}\xi_i^T(t)\xi_i(t)-\sum_{i=1}^{N}\frac{\kappa_i}{2}\delta_i(t). \tag{24}$$

Due to the condition $\alpha_i > \frac{1-\mu_i}{\beta_i}$, i.e., $\kappa_i > 0$, then $\dot{W} \leq 0$ and thus $W(t)$ is monotonously non-decreasing. Note that $\dot{W} = 0$ if and only if $\xi_i(t) = 0$ and $\delta_i(t) = 0$ for $i = 1, \cdots, N$. This implies $\lim_{t\to+\infty}\xi_i(t) = 0$ and $\lim_{t\to+\infty}\delta_i(t) = 0$, i.e. the asymptotic consensus can be achieved eventually.

Theorem 4. Under the same conditions in Theorem 3, Zeno behavior can be excluded in (5).

Proof. Consider the triggering sequences $\{t_k^i\}_{k\in\mathbb{N}}$ generated by (5). Assume that the Zeno behavior can not be excluded, then $\exists T_*^i > 0$ such that $\lim_{k\to+\infty} t_k^i = T_*^i$. In other word, for $\varepsilon > 0$, for all $k > N_0$, one has

$$t_k^i \in [T_*^i - \varepsilon, T_*^i]. \tag{25}$$

From (5), $\{t_k^i\}_{k\in\mathbb{N}^+}$ is monotone non-decreasing sequence. Thus, $\exists\ N_0' > 0$ such that $t_k^i \in [T_*^i - \varepsilon, T_*^i]$ for $k \geq N_0'$.

Firstly, according to (8) with (5), it is easy to obtain that $\delta_i(t) \geq \delta_i(t_0)\exp\left\{-\left(\alpha_i + \frac{\mu_i}{\beta_i}\right)(t-t_0)\right\}$.

According to (4), (6), (8), one obtains that $c_{ij}(t)$, $\delta_i(t)$ and $x_i(t)$ are continuous functions. This implies that $\exists\hat{c} > 0, B_1 > 0, B_2 > 0$ s.t. $|c_{ij}| \leq \hat{c}$, $\|\delta_i(t)\| \leq B_1$ and

$\|x_i(t) - x_j(t)\| \le B_2$ during $[0, T_*^i]$. It is not difficult to conclude that $\hat{\xi}_{ij}$ is bounded during $\in [T_*^i - \varepsilon, T_*^i]$.

One sufficient condition for event condition (5) is

$$\|e_i(t)\|^2 \le \frac{\delta_i(t_0)\exp\left\{-\left(\alpha_i + \frac{\mu_i}{\beta_i}\right)(t - t_0)\right\}}{\beta_i \hat{c} \|\Gamma\| d_i}. \tag{26}$$

By (3), one has the right-hand Dini derivative of $\|e_i(t)\|$ with

$$D^+\|e_i\| \le \|A\|\|e_i\| + \varrho_i \tag{27}$$

and ϱ_i is the upper bound of $\hat{c}\|BK\|\|\sum_{j=1}^N a_{ij}\hat{\xi}_{ij}^T(t)\Gamma\hat{\xi}_{ij}(t)\|$ during $\in [T_*^i - \varepsilon, T_*^i]$.

Let $\varepsilon < \ln\left(\frac{\|A\|\iota}{\varrho_i} + 1\right)$ with $\iota = \sqrt{\frac{\delta_i(t_0)}{\beta_i \hat{c}\|\Gamma\| d_i}} \exp\{-\frac{1}{2}(\alpha_i + \frac{\mu_i}{\beta_i})(T_*^i - t_0)\}$.

Define a function $\Phi(t)$ with

$$\dot{\Phi}(t) = \|A\|\Phi(t) + \varrho_i \tag{28}$$

with $\Phi(0) = 0$. Obviously, $\|e_i(t)\| \le \Phi(t - t_k^i)$ with $\Phi(t) = \frac{\varrho_i}{\|A\|}\left[\exp\{\|A\|t\} - 1\right]$. This implies that $t_{k+1}^i - t_k^i \ge \Delta$ and Δ is the time interval for $\Phi(t)$ evolving from 0 to ι. It is easy to obtain $\Delta = \ln\left(\frac{\|A\|\iota}{\varrho_i} + 1\right) > 0$. This contradicts the assumption of $t_{k+1}^i - t_k^i \le \varepsilon < \ln\left(\frac{\|A\|\iota}{\varrho_i} + 1\right) = \Delta$. Therefore, there is no Zeno behavior.

Remark 5. The proposed event-triggered scheme (5) depends on agents' states, which can be considered as a kind of *node* event-triggered scheme. Such an idea can be extended to design an *edge* event-triggered scheme, which will be detailedly analyzed in our near future works.

4. SIMULATION

In this section, a simulation example will be provided to verify the effectiveness of the proposed event-triggered scheme.

Example 6. Consider a multi-agent system with five agents and the dynamics of each agent is given in (2) with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{29}$$

The communication graph between five agents is given in Fig. 1.

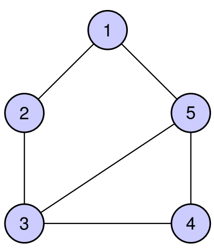

Fig. 1. Communication graph.

The Laplacian matrix $\mathcal{L}$ of the graph is given as

$$\mathcal{L} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}. \tag{30}$$

Solving the ARE (7) leads to

$$P = \begin{bmatrix} 2.3846 & 2.3504 & 0.9856 & 0.1691 \\ 2.3504 & 4.6632 & 2.3300 & 0.5212 \\ 0.9856 & 2.3300 & 2.2542 & 0.7608 \\ 0.1691 & 0.5212 & 0.7608 & 1.3938 \end{bmatrix} \tag{31}$$

and then one has

$$K = \begin{bmatrix} 0.9856 & 2.3300 & 2.2542 & 0.7608 \\ 0.1691 & 0.5212 & 0.7608 & 1.3938 \end{bmatrix} \tag{32}$$

$$\Gamma = \begin{bmatrix} 1.0000 & 2.3846 & 2.3504 & 0.9856 \\ 2.3846 & 5.7007 & 5.6488 & 2.4992 \\ 2.3504 & 5.6488 & 5.6601 & 2.7754 \\ 0.9856 & 2.4992 & 2.7754 & 2.5216 \end{bmatrix}. \tag{33}$$

Choose $\alpha_i = 2$, $\beta_i = 1$, $\theta_i = 0.1$, $\mu_i = 0.5$ for $1 \le i \le 5$ in (5) and $k_{ij} = 1$ for $(i, j) \in \mathcal{E}$ in (4). The initial values $x_i(0)$, $\delta_i(0)$ and $c_{ij}(0)$ are chosen randomly with $1 \le i \le 5$. According to Fig. 2, it is easy to see that the tracking error ξ_i of agents converge to zero, which implies that the consensus of the linear multi-agent system (2) can be achieved eventually. In addition, the coupling weights $c_{ij}(t)$ tends to constants, see Fig. 3, and the triggers of fives agents during $[0, 9]$ are 42, 30, 57, 54, and 43, respectively.

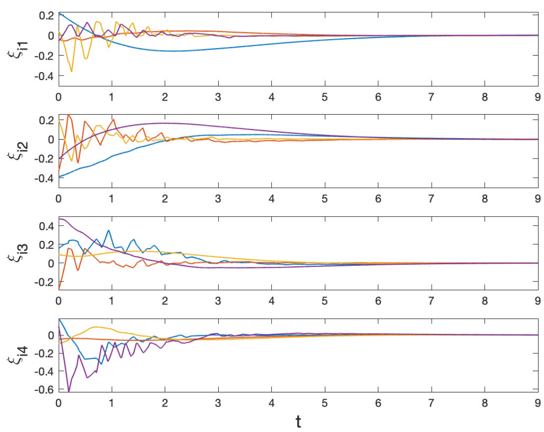

Fig. 2. The trajectory of the tracking error $\xi_i(t)$ with $i = 1, 2, 3, 4, 5$..

5. CONCLUSION

This paper has proposed a novel dynamic adaptive event-triggered scheme for general linear multi-agent systems, which effectively reduces the communication frequency and guarantees the asymptotic consensus. It is worth noting that the designed event condition and consensus protocol are fully distributed without using any global information. Meanwhile, there has been no Zeno behavior in our event-triggered scheme. Although this scheme is

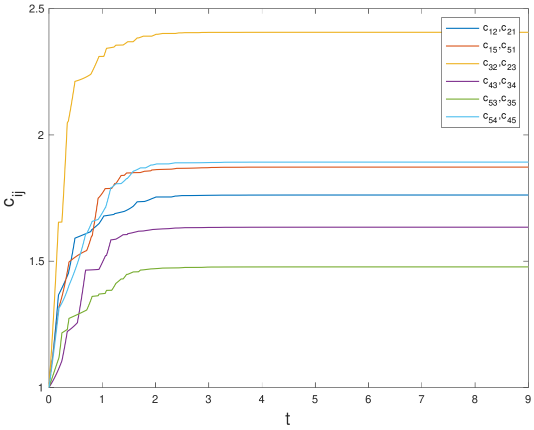

Fig. 3. The coupling weights $c_{ij}(t)$ in (4) for edge (i, j).

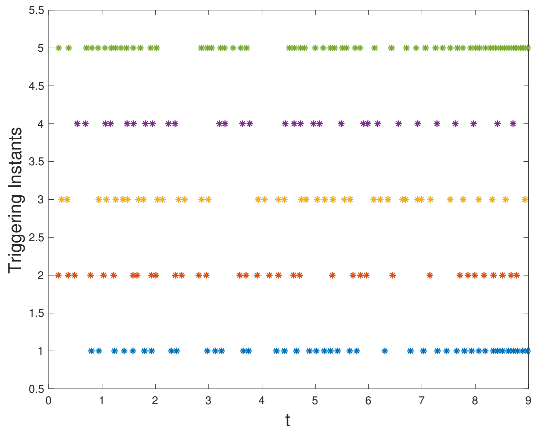

Fig. 4. The triggering instants for five agents.

currently designed based on the states of agents, it can be extended to deal with the cases of output feedback and edge event condition, which will be detailedly analyzed in our near future works.

REFERENCES

Cheng, B. and Li, Z. (2019). Fully Distributed Event-Triggered Protocols for Linear Multiagent Networks. *IEEE Transactions on Automatic Control*, 64(4), 1655–1662.

Dimarogonas, D.V., Frazzoli, E., and Johansson, K.H. (2012). Distributed event-triggered control for multi-agent systems. *IEEE Transactions on Automatic Control*, 57(5), 1291–1297. doi:10.1109/TAC.2011.2174666.

Dolk, V. and Heemels, W. (2015). Dynamic Event-Triggered Control Under Packet Losses: The Case With Acknowledgements. In *International Conference on Event-Based Control, Communication and Signal Processing*. doi:10.1109/EBCCSP.2015.7300651.

Gadjov, D. and Pavel, L. (2019). A Passivity-Based Approach to Nash Equilibrium Seeking Over Networks. *IEEE Transactions on Automatic Control*, 64(3), 1077–1092. doi:10.1109/TAC.2018.2833140.

Girard, A. (2015). Dynamic Triggering Mechanisms for Event-Triggered Control. *IEEE Transactions on Automatic Control*, 60(7), 1992–1997.

Han, Z., Guo, K., Xie, L., and Lin, Z. (2019). Integrated Relative Localization And Leader-Follower Formation control. *IEEE Transactions on Automatic Control*, 64(1), 20–34.

He, W., Xu, B., Han, Q.L., and Qian, F. (2019). Adaptive Consensus Control of Linear Multiagent Systems With Dynamic Event-Triggered Strategies. *IEEE Transactions on Cybernetics*, 1–13. doi:10.1109/tcyb.2019.2920093.

Horn, R.A. and Johnson, C.R. (2012). *Matrix Analysis*. Cambridge University Press.

Li, Q., Shen, B., Wang, Z., and Sheng, W. (2020). Recursive Distributed Filtering Over Sensor Networks On Gilbert–Elliott Channels: A Dynamic Event-Triggered Approach. *Automatica*, 113, 108681.

Li, Z., Ren, W., Liu, X., and Xie., L. (2013). Distributed Consensus Of Linear Multi-Agent Systems With Adaptive Dynamic Protocols. *Automatica*, 49(7), 1986–1995.

Shen, S.Y., Wang, Z., Shen, B., Alsaadi, F.E., and Alsaadi, F.E. (2020). Fusion Estimation For Multi-Rate Linear Repetitive Processes Under Weighted Try-Once-Discard Protocol. *Information Fusion*, 55, 281–291.

Xu, W., Ho, D.W., Zhong, J., and Chen, B. (2019a). Event/Self-Triggered Control for Leader-Following Consensus Over Unreliable Network With DoS Attacks. *IEEE Transactions on Neural Networks and Learning Systems*, 30(10), 3137–3149.

Xu, W., Hu, G., Ho, D.W.C., and Feng, Z. (2019b). Distributed Secure Cooperative Control Under Denial-of-Service Attacks From Multiple Adversaries. *IEEE Transactions on Cybernetics*. doi:10.1109/TCYB.2019.2896160.

Yang, S., Liu, Q., and Wang, J. (2017). A Multi-Agent System With A Proportional-Integral Protocol For Distributed Constrained Optimization. *IEEE Transactions on Automatic Control*, 62(7), 3461–3467.

Yang, S., Wang, J., and Liu, Q. (2019). Cooperative-Competitive Multiagent Systems for Distributed Minimax Optimization Subject to Bounded Constraints. *IEEE Transactions on Automatic Control*, 64(4), 1358–1372.

Yi, X., Liu, K., Dimarogonas, D.V., and Johansson, K.H. (2019). Dynamic Event-Triggered and Self-Triggered Control for Multi-agent Systems. *IEEE Transactions on Automatic Control*, 64(8), 3300–3307.