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A Continuous-discrete Adaptive Observer Design for Nonlinear Systems Subject to Sensor Nonlinearities

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Abstract: In this paper, we address the problem of nonlinear continuous-discrete adaptive observer design for a class of system with sampled data measurements subject to sensor nonlinearities. The main difficulty of the considered class of system is coming from the fact that the output equation contains unknown parameters which renders the design of a classical sampled data observer difficult. To overcome this difficulty, we propose a new online continuous-discrete adaptive observer which ensures a simultaneous exponential convergence of both states and parameters. Comparing to other observer structures, our design is characterized by a simpler structure thanks to the introduction of a parametric adaptation law. To show the efficiency of our proposed approach, numerical simulations have been performed for different values of sampling time. In addition, the delayed-sampled measurements case is also illustrated.

Keywords: nonlinear systems, continuous-discrete, nonlinear adaptive observer, Lyapunov stability, online parameter estimation.

1. INTRODUCTION

Nowadays, the problem of controlling industrial systems is facing two major drawbacks. The first one is coming from the fact that the system variables are partly accessibles for sensor measurements. The second one is that, in addition to the system variables, the system parameters can also be unknown a priori. In addition to the problem of controlling industrial systems, the field of system fault diagnosis is also concerned by this problem. Indeed, comparing to the fault free case, a faulty system behaviour usually modifies the system structure via the states variables or the system unknown parameters. To counteract these problems, it is crucial to develop an adaptive observer which can estimate simultaneously the state variables and the unknown parameters of the system.

The problem of designing adaptive observers has been, for many decades, an active research area with special emphasis on observers with exponentially stable error dynamics. The first contributions go back to the seventies and concern time-invariant linear systems where the uncertain parameters come in linearly (e.g. Luders and Narendra (1973), Kreisselmeier (1977), Ioannou and Kokotovic (1983), Narendra and Annaswamy (2005), Ioannou and Sun (2012), Rueda-Escobedo and Moreno (2017)).

These previous works have been widely extended to nonlinear cases (see Bastin and Gevers (1988), Khalil (2002), Rajamani (2017a) and Rajamani (2017b)). In Krener and Isidori (1983), Krener and Respondek (1985), Back et al. (2006) and Boutat et al. (2007), the authors addressed the problem of designing adaptive observers for nonlinear systems through linearization.

In Zhang (2002), the authors have adopted a specific design approach to deal with general time-varying systems. The main novel idea in Zhang (2002) is a design based on a new coordinate transformation that makes the parameter adaptive law design decoupled from the state estimation law. In Besançon et al. (2006), the authors show how this design can even be equivalent to a Kalman-like adaptive observer for state affine time-varying systems.

Despite the success of these adaptive observers, a common characteristic of these approaches is coming from the fact that they were designed in continuous time. In fact, the implementation of these observers in digital platforms requires the measured system outputs to be sampled. Therefore, the observer convergence is no longer ensured due to this sampling process. Meanwhile, the need of developing adaptive observer structures which deals with the problem of sampled data output measurements has become an important issue to overcome this difficulty.

Sampled-data adaptive observer design can be classified following two differents techniques. The first one is the so called continuous-adaptive design approach. In Ahmed-Ali et al. (2009), an adaptive observer based on this approach is developed. In this work the authors have considered a class of state affine systems where the unknown parameters enter the system through solely the state equations. Following the principle of this approach, the observer structure has two components. The first one is a predictor which copies the system dynamic between two sampling times. The second one perform a correction at each sampling time. The correction part is fulfilled by an observer which corrects the estimate state trajectories by using the output error. The second technique is based upon the idea of the use of an output predictor introduced by the authors in Karafyllis and Kravaris (2009). The main advantage of this observer is that the estimated states remain continuous while the update process which concerns only the predictor is re-initialized at each sampling time. Following this way, the authors in Hann and Ahmed-Ali (2014) propose an adaptive sampled-data observer which ensures that system states as well as parameters remain continuous.

As stated previously, most of the proposed observers assumes that the unknown parameters enter system via the state equations. Li et al. (2011) have treated the case of unknown parameters in both state and output equations but only in the linear case. However, if we are brought to consider sensor nonlinearities, this can lead to take into account a model with unknown parameters only on the output equation like in Giri et al. (2014). Consequently, it is clearly impossible in this case to use an inter-sample output predictor as in Massieu et al. (2017), Kahelras et al. (2018) or Karafyllis and Kravaris (2009).

In this paper, we propose a continuous-discrete adaptive observer for a class of system subject to sensor nonlinearities like in Ahmed-Ali et al. (2019). Comparing to this work, our proposed observer is characterized by a simpler structure in terms of parameter estimation law. So, in addition to simplifying the implementation, this allows us to deal with the case of sampled and delayed measurements in a simpler way.

Our paper is organized as follows. In section 2, the class of the nonlinear system considered is presented. Section 3 presents our continuous-discrete adaptive observer design and the analysis of states and parameters errors which leads to the main result of this paper. Section 4 presents an extension to the delayed sampled-measurements case. A simulation example is used in section 5 to illustrate the efficiency of our proposed design method. Some concluding remarks end the paper in section 6.

2. PROBLEM STATEMENT

We are considering the class of continuous-discrete systems described by the following equations:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B[u(t)] \\ y(t) = Cx(t_k) + \phi[y(t_k)]\theta \\ \quad t \in \; [t_k \;\; t_{k+1}), \quad k \; \in \; \mathbb{N} \end{cases} \tag{1}$$

$A \in \mathbf{R}^{n\times n}; B \in \mathbf{R}^n; C \in \mathbf{R}^{1\times n}; \phi \in \mathbf{R}^{1\times m}, \theta \in \mathbf{R}^m$. A, B and ϕ are $\mathcal{C}^1$ functions and ϕ is supposed to be bounded. u and y respectively denote the bounded input and output and $x \in \mathbf{R}^n$ is the state vector with $x(0)$ arbitrary chosen.

All quantities in the model are known or accessible to measurements, except the state vector x and the constant parameter vector θ. The problem at hand is precisely to design an observer that provides simultaneous online estimation of x and θ, based only on the measurements of the output y, only accessible at sampling instants t_k .

Remark 1. The class of continuous-discrete systems (1) we consider is almost the same than in Ahmed-Ali et al. (2019). However, the design of our observer is simpler than in this work in the sense that there is one less ODE to solve.

Remark 2. The input signal $u(t)$ must be a persistent excitation in order to guarantee the uniform stability of system (1). Details can be found in Besançon et al. (1996).

3. OBSERVER DESIGN AND ANALYSIS

To get simultaneous online estimates of both state vector $\hat{x}$ and parameter vector $\hat{\theta}$, we propose the following continuous-discrete adaptive observer, composed of two interconnected parts (the state observer (2-3) and the parameter vector estimator (4-7):

$$\begin{aligned} \dot{\hat{x}}(t) &= A(t)\hat{x}(t) + B[u(t)] \\ &\quad - S(t)^{-1}C^T \left[C\hat{x}(t_k) + \phi[y(t_k)]\hat{\theta}(t_k) - y[t_k] \right] \end{aligned} \tag{2}$$

$$\begin{aligned} \dot{S}(t) &= -\Delta \; S(t) - S(t) \; A(t) - A^T(t)S(t) + C^TC \\ &\quad S(0) > 0, \; \Delta > 0 \end{aligned} \tag{3}$$

$$\hat{\theta}(t_k) = \hat{\theta}(t_{k-1}) - \frac{F^T(t_k)}{1 + \|F(t_k)\|^2} \; \tilde{y}(t_k) \tag{4}$$

$$F(t_k) = C\lambda(t_k) + \phi[y(t_k)] \tag{5}$$

$$\tilde{y}(t_k) = C\hat{x}(t_k) + \phi[y(t_k)]\hat{\theta}(t_k) - y(t_k) \tag{6}$$

$$\begin{aligned} \dot{\lambda}(t) &= A(t)\lambda(t) - S(t)^{-1}C^TC\lambda(t_k) \\ &\quad - S(t)^{-1}C^T\phi[y(t_k)] \\ & t \in [t_k \quad t_{k+1}), k \in \mathbb{N} \end{aligned} \tag{7}$$

The analysis will mainly consist in proving that our observer is a global exponential observer for system (1). This will be done in two steps. Foremost, we study state observer convergence. Then, we focus on parameters estimation. At last, we state the theorem which guarantees the global exponential convergence of both states and parameters.

3.1 States observer

The following definitions (Gauthier and Kupka (1994) and Batista et al. (2017)) introduce the concepts of observability Gramian and uniform complete observability:

Definition 1. (Observability Gramian). The observability Gramian associated with the pair $(A(t), C(t))$ on $[t_0, t_f]$ is defined as:

$$\mathcal{W}_0(t_0, t_f) = \int_{t_0}^{t_f} \left(C\Upsilon(s, t_0)\right)^T \left(C\Upsilon(s, t_0)\right) ds$$

where$\Upsilon(t, t_0)$ is the state transition matrix associated with $A(t)$ from t_0 to t.

Definition 2. (Uniform Complete Observability). The pair $(A(t), C(t))$ is uniformly completely observable if there exist positive constants $\alpha > 0$ and $\beta > 0$ such that, for all $t \geq t_0$:

$$\mathcal{W}_0(t, t+\beta) \geq \alpha \mathbf{I}$$

Suppose uniform observability as Definition 2 is checked for system (1), let us now consider the following errors system:

$$\begin{aligned} \tilde{x}(t) &= \hat{x}(t) - x(t) \\ \tilde{\theta}(t_k) &= \hat{\theta}(t_k) - \theta \\ t &\in [t_k \quad t_{k+1}), k \in \mathbb{N} \end{aligned} \tag{8}$$

Subtracting (1) from (2), one gets:

$$\dot{\tilde{x}}(t) = A(t)\tilde{x}(t) - S(t)^{-1}C^T\left[C\tilde{x}(t_k) + \phi[y(t_k)]\tilde{\theta}(t_k)\right] \tag{9}$$

Let us now introduce the following decoupling change of variable (Zhang (2002)):

$$\eta(t) = \tilde{x}(t) - \lambda(t)\tilde{\theta}(t_k) \tag{10}$$

Differentiating (10) with respect to time yields:

$$\dot{\eta}(t) = \dot{\tilde{x}}(t) - \dot{\lambda}(t)\tilde{\theta}(t_k) \tag{11}$$

$$\begin{aligned} \dot{\eta}(t) = {} & A(t)\tilde{x}(t) - S(t)^{-1}C^TC\tilde{x}(t_k) \\ & - S(t)^{-1}C^T\phi[y(t_k)]\tilde{\theta}(t_k) - \dot{\lambda}(t)\tilde{\theta}(t_k) \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{\eta}(t) = {} & A(t)\left[\eta(t) + \lambda(t)\tilde{\theta}(t_k)\right] \\ & - S(t)^{-1}C^TC\left[\eta(t_k) + \lambda(t_k)\tilde{\theta}(t_k)\right] \\ & - S(t)^{-1}C^T\phi[y(t_k)]\ \tilde{\theta}(t_k) - \dot{\lambda}(t)\tilde{\theta}(t_k) \end{aligned} \tag{13}$$

We derive that

$$\begin{aligned} \dot{\eta}(t) = {} & A(t)\eta(t) - S(t)^{-1}C^TC\eta(t_k) \\ & + \big[A(t)\lambda(t) - S(t)^{-1}C^TC\lambda(t_k) \\ & -S(t)^{-1}C^T\phi[y(t_k)] - \dot{\lambda}(t)\Big]\ \tilde{\theta}(t_k) \end{aligned} \tag{14}$$

Replacing $\dot{\lambda}(t)$ by its expression in (7), one gets:

$$\dot{\eta}(t) = A(t)\eta(t) - S(t)^{-1}C^TC\eta(t_k) \tag{15}$$

The exponential stability of (15) is ensured under the two following conditions (Proof of Part 1, eq. 48) in Ahmed-Ali et al. (2019):

$$\begin{cases} \Delta - T_eK(\Delta) > 0 & (16) \\ 1 - T_e\ [\lambda_{\text{MAX}}(C^TC) + T_eK(\Delta) + \Delta] > 0 & (17) \end{cases}$$

with $K(\Delta) = \max\left[K_1(\Delta), K_2(\Delta)\right]$ such that:

- $K_1(\Delta) = \dfrac{\left(2\sup\|A(t) - S(t)^{-1}C^TC\|\right)^2}{\alpha_S(\Delta)}$
- $\alpha_S(\Delta)\ \mathbf{I} \leq S(t)$ and $\alpha_S(\Delta)$ is a decreasing function of Δ (Besançon et al. (1996))
- $K_2(\Delta) = 2(\alpha_S^{-1}(\Delta)\|C^TC\|)^2$

Under conditions (16)-(17), $\eta(t)$ tends exponentially to the origin as $t \to \infty$.

3.2 Parameters estimation

In this section, the estimation of the unknown vector parameter is done only at each sampling time $t_k, k \in \mathbb{N}$.

$$\begin{cases} y(t_k) &= Cx(t_k) + \phi[y(t_k)]\theta \\ \hat{y}(t_k) &= C\hat{x}(t_k) + \phi[y(t_k)]\hat{\theta}(t_k) \end{cases} \tag{18}$$

Let

$$\tilde{y}(t_k) = C\tilde{x}(t_k) + \phi[y(t_k)]\tilde{\theta}(t_k) \tag{19}$$

Using (10) leads to:

$$\tilde{y}(t_k) = C\Big[\eta(t_k) + \lambda(t_k)\tilde{\theta}(t_k)\Big] + \phi[y(t_k)]\tilde{\theta}(t_k) \tag{20}$$

$$\tilde{y}(t_k) = C\eta(t_k) + \Big[C\lambda(t_k) + \phi[y(t_k)]\Big]\tilde{\theta}(t_k) \tag{21}$$

$$\tilde{y}(t_k) = C\eta(t_k) + F(t_k)\tilde{\theta}(t_k) \tag{22}$$

with $F(t_k) = C\lambda(t_k) + \phi[y(t_k)]$

The estimate of the unknown vector parameter θ can be expressed under the following well-known regressor form (Anderson et al. (1986)):

$$\hat{\theta}(t_k) = \hat{\theta}(t_{k-1}) - \frac{F^T(t_k)}{1 + \|F(t_k)\|^2}\tilde{y}(t_k) \tag{23}$$

3.3 Main result

Assumption 1. Let $\lambda(t) \in \mathbf{R}^{n\times m}$ be the solution of (7), the matrix $\lambda(t)$ is persistently excited so that there exist $k \in \mathbf{N}, \beta > 0$ such that:

$$\sum_{k=1}^{\infty} F(t_k)^TF(t_k) \geq \beta\ \mathbf{I}_n$$

Theorem 1. Under Assumption 1, for $0 < T_e < T_{e\text{MAX}}$, system (2-7) is a continuous-discrete adaptive observer for system (1) with the following properties:

i) The auxiliary variable $\lambda(t)$ is bounded.
ii) the parameter estimation $\hat{\theta}$ converges exponentially toward θ.
iii) $\hat{x}(t)$ exponentially converges to $x(t)$.

Proof.

Part i) Choosing $\lambda(t)$ as in (7), $\eta(t)$ can be expressed by (15).Then, it's easy to find a Lyapunov function, as in Appendix (Proof of Part 1) in Ahmed-Ali et al. (2019), to prove stability of (15). Thereby, $\eta(t)$ tends exponentially to the origin as t tends to infinity. It implies that $\lambda(t)$ stays bounded under the following additional conditions (Proof of Part 2) in Ahmed-Ali et al. (2019):

$$\begin{cases} \frac{\Delta}{2} - T_eK_3(\Delta) > 0 & (24) \\ 1 - T_e\ [\lambda_{\text{MAX}}(C^TC) + T_eK_3(\Delta) + \Delta] > 0 & (25) \end{cases}$$

with $K_3(\Delta) = \max\left[\dfrac{3K_1^2(\Delta)}{\alpha_S(\Delta)}, 3K_2^2(\Delta)\right]$

Inequations (16)-(17) and (24)-(25) make it possible to obtain $T_{e\text{MAX}}$ the upper bound of T_e:

$$T_{e\text{MAX}} \leq \min\{(16), (17), (24), (25)\} \tag{26}$$

Part ii) In this part, we will show the exponential convergence of the parametric error $\tilde{\theta}(t_k)$ toward zero. Combining (8) and (23), for $t \in [t_k \quad t_{k+1})$, we have:

$$\tilde{\theta}(t_k) = \tilde{\theta}(t_{k-1}) - \frac{F^T(t_k)}{1 + \|F(t_k)\|^2}\ \tilde{y}(t_k) \tag{27}$$

Replacing $\tilde{y}(t_k)$ by its expression in (22), we derive:

$$\tilde{\theta}(t_k) = \left[I - \frac{F^T(t_k)F(t_k)}{1 + \|F(t_k)\|^2}\right]\tilde{\theta}(t_{k-1}) - \frac{F^T(t_k)C\eta(t_k)}{1 + \|F(t_k)\|^2} \tag{28}$$

From i), $\eta(t)$ converges to the origin and we know that $\lambda(t)$ is bounded and as ϕ is also bounded, then the non-homogenous part of (28) vanishes.

With respect to Assumption 1, the homogenous part of (28) ensures an exponentially convergence to the origin (for more details, see Ahmed-Ali et al. (2009)).

Part iii) Equation (10) allows to write $\|\tilde{x}(t)\| \leq \|\eta(t)\| + \|\lambda(t)\| \; \|\tilde{\theta}(t_k)\|$ which implies that $\tilde{x}(t)$ is exponentially vanishing because parts i)-ii) of proof. □

4. EXTENSION TO DELAYED SAMPLED-MEASUREMENTS

In this section, we will assume that in addition to the fact that state variables, which are available for measurements at only sampling times t_k, are also received within a delay τ (Hespanha et al. (2007)). For $t \in [t_k \quad t_{k+1})$, $k \in \mathbb{N}$, the initial system (1) becomes:

$$\begin{cases} \dot{x}(t) &= A(t)x(t) + B[u(t)] \\ y(t) &= Cx(tk) + \phi[y_d(t_k))].\theta \\ y_d(t) &= y(t-\tau) \end{cases} \tag{29}$$

In the sequel, we will design a continuous discrete observer for system (29) which provides simultaneous estimation of the state x and the parameter vector θ based only on the delayed sampled-measurements.

The whole observer will be composed of the three following components:

- An observer (30)-(31) which provides the estimation of the delayed state vector $x(t-\tau)$,
- A predictor (32) which provides estimation of state vector $x(t)$, at each sampling time,
- An estimator (33)-(36) which delivers the value of the unknown parameter vector θ, at each sampling time.

$$\dot{z}(t) = A(t-\tau)z(t) + B[u(t-\tau)] - S(t)^{-1}C^T \times \Big[Cz(t_k) + \phi[y(t_k-\tau)]\hat{\theta}(t_k) - y[t_k-\tau]\Big] \tag{30}$$

$$\dot{S}(t) = -\Delta S(t) - S(t)A(t-\tau) - A^T(t-\tau)S(t) + C^T C \qquad S(0) > 0, \Delta > 0 \tag{31}$$

$$\hat{x}(t) = z(t) + \int_{t-\tau}^{t} (A(s)\hat{x}(s) + B[u(s)])\,\mathrm{d}s \qquad \hat{x}(s) = 0, s \in [-\tau, 0] \tag{32}$$

$$\hat{\theta}(t_k) = \hat{\theta}(t_{k-1}) - \frac{F^T(t_k)}{1 + \|F(t_k)\|^2}\, \tilde{y}(t_k - \tau) \tag{33}$$

$$F(t_k) = C\lambda(t_k) + \phi[y(t_k - \tau)] \tag{34}$$

$$\tilde{y}(t_k - \tau) = Cz(t_k) + \phi[y(t_k-\tau)]\hat{\theta}(t_k) - y(t_k - \tau) \tag{35}$$

$$\dot{\lambda}(t) = A(t-\tau)\lambda(t) - S(t)^{-1}C^T C\lambda(t_k) - S(t)^{-1}C^T\phi[y(t_k-\tau)] \qquad t \in [t_k \quad t_{k+1}), k \in \mathbb{N} \tag{36}$$

Due to the lack of space, only a sketch of proof is given in this paper. Inspired from Ahmed-Ali et al. (2016), the proof of the convergence of our proposed observer will consists in demonstrating the following properties:

i) The auxiliary variable $\lambda(t)$ is bounded.
ii) the parameter estimation $\hat{\theta}$ converges exponentially toward θ.
iii) $z(t)$ exponentially converges toward the delayed state $x(t-\tau)$.
iv) $\hat{x}(t)$ exponentially converges to $x(t)$.

5. SIMULATION EXAMPLE

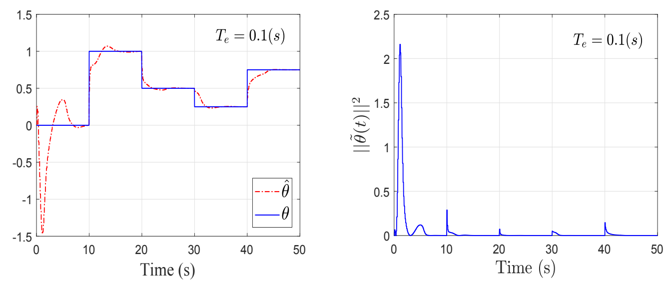
This section is dedicated to the illustration of the proposed observer by means of a second order system (37) which belongs to the class of system described in (1) expanded to the case of a delayed output.

$$\begin{cases} \dot{x}_1(t) &= -x_1(t) + \Big(2 + \cos(t)\Big)\, x_2(t) \\ \dot{x}_2(t) &= -x_1(t) + \sin(0.5\,t) \\ y(t) &= x_1(t) + \cos\Big(y(t_k)\Big)\,\theta \\ & t \in [t_k \quad t_{k+1}), \qquad k \in \mathbb{N} \\ y_d(t) &= y(t-\tau) \end{cases} \tag{37}$$

The simulations were conducted with τ a constant delay, the parameter θ has been chosen constant by steps of 10 seconds with variations not too large, system states are initialized such as $x(0) = [1; -1]$, initial estimated states are $\hat{x}(0) = [0; 0]$, unknown estimated parameter $\hat{\theta}$ is initialized such as $\hat{\theta}(0) = 1$ and observer (2)-(7) and (30)-(36) parameters are chosen such as $\Delta = 2$, $S(0) = I_{2\times 2}$, $\lambda(0) = [1; 1]$. In this paper we presents two simulations cases. The first one without delay (τ=0) and the second one with a non-zero constant delay.

Case 1: without delay

These simulations have been done in the case where $T_e = 0.1$ (s). The reference profile chosen for θ is depicted in Fig. 1a. On the same figure, the estimated value of θ appears in dotted-line. After a short transient, the estimated $\hat{\theta}$ clearly converges to the real value θ. To highlight this result, Fig. 1b describes the sum of squares of $\tilde{\theta}$ for the same reference profile. We can clearly see that after the transient, the sum of squares of $\tilde{\theta}$ is approaching nearly to zero.



(a) θ and $\hat{\theta}$. (b) Sum of squares of $\tilde{\theta}$.

Fig. 1. Parameter and his estimate (a) and parameter estimation errors (b).

While considering the same reference profile, Fig. 2 shows the state errors versus time. The initial transient seems important because initial conditions are far from zero, but then for each next step with θ, the transient is very smaller and shorter. Results are of course depending on the choice of T_e.

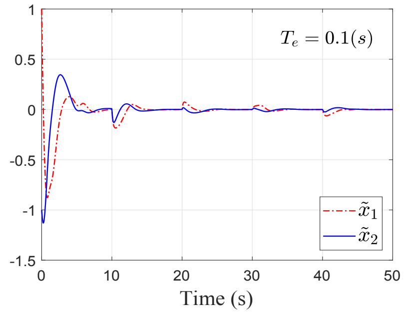
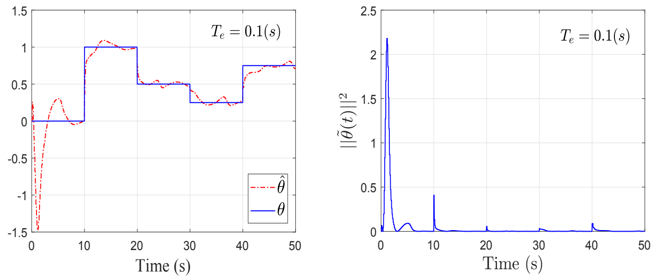


Fig. 2. Evolution of the state errors.

Case 2: non-zero constant delay

In this case, we present the extension to the non-zero constant delay $\tau = T_e = 0.1\ s$ on the output.



(a) θ and $\hat{\theta}$.

(b) Sum of squares of $\tilde{\theta}$.

Fig. 3. Parameter and his estimate (a) and parameter estimation errors (b).

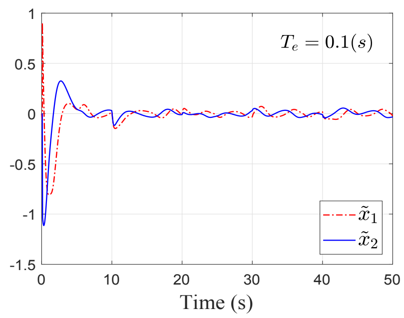


Fig. 4. Evolution of the state errors.

Looking at Fig. 3a-4, we can say that the observer (30)-(36) still performs in terms of parameter and states estimations.

In the case of many practical applications, it turns out critical to be able to work with a great sampling period. Moreover, the output can be delayed by, for example, a network. Then, it is interesting to explore the behavior of our observer for different values of $T_e < T_{e\text{MAX}}$ and τ. To this end, let us introduce the following criteria:

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left[\tilde{\theta}(t_i)^2 + \tilde{x}_1(t_i)^2 + \tilde{x}_2(t_i)^2\right] \tag{38}$$

Application of (38) for different values of τ and T_e leads to results plotted in Fig. 5.

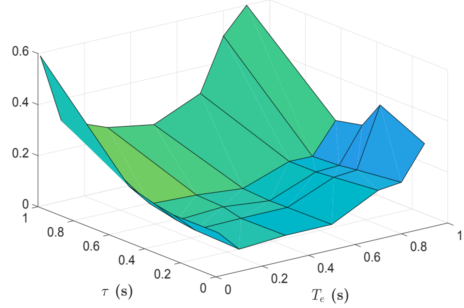


Fig. 5. MSE versus τ and T_e.

It's clear that the MSE is growing fast with the value of T_e but stays unsignificant for small T_e. Until a quite large enough sampling period, our design allows a correct behavior in spite of a deterioration of the observer performances. For a too large sampling period (greater than $T_{e\text{MAX}}$), the loss of convergence occurs. The same behaviour is observed with τ. Considering the two cumulating effects, the worst case obviously appears for great values of τ and T_e, but even for small values of T_e, the performances fall down for big delay.

6. CONCLUDING REMARKS

In this paper, a new continuous-discrete adaptive nonlinear observer for time-varying nonlinear systems with unknown parameters in output equation has been designed. The main advantage of the proposed observer lies in its simplicity. Proof of convergence of both parameter and state vectors is given and ensured that it remains valid for high sampling periods, less than $T_{e\text{MAX}}$ which is explicited in the main result part. As the good simulations results suggest, future work will focus on the extension to NCS (Hespanha et al. (2007)).

REFERENCES

Ahmed-Ali, T., Giri, F., Krstic, M., and Lamnabhi-Lagarrigue, F. (2016). Adaptive Observer for a Class of Output-Delayed Systems with Parameter Uncertainty - A PDE Based Approach. *12th IFAC International Workshop on Adaptation and Learning in Control and Signal*, 49(13), 158–163. doi:10.1016/j.ifacol.2016.07.944.

Ahmed-Ali, T., Postoyan, R., and Lamnabhi-Lagarrigue, F. (2009). Continuous-discrete adaptive observers for state affine systems. *Automatica*, 45(12), 2986–2990. doi:10.1016/j.automatica.2009.09.005.

Ahmed-Ali, T., Tiels, K., Schoukens, M., and Giri, F. (2019). Sampled-data adaptive observer for state-affine systems with uncertain output equation. *Automatica*, 103, 96–105. doi:10.1016/j.automatica.2019.01.006.

Anderson, B.D.O., Bitmead, R., Johnson, C., Kokotovic, P., Kosut, R., Mareels, I., Praly, L., and Riedle, B.

(1986). *Stability of Adaptive Systems: Passivity and Averaging Analysis*. MIT Press, Cambridge, MA, USA.

Back, J., Yu, K.T., and Seo, J.H. (2006). Dynamic observer error linearization. *Automatica*, 42(12), 2195 – 2200. doi:10.1016/j.automatica.2006.07.009.

Bastin, G. and Gevers, M. (1988). Stable adaptive observers for nonlinear time-varying systems. *IEEE Transactions on Automatic Control*, 33, 650—658.

Batista, P., Petit, N., Silvestre, C., and Oliveira, P. (2017). Relaxed conditions for uniform complete observability and controllability of ltv systems with bounded realizations. *IFAC-PapersOnLine*, 50(1), 3598 – 3605. doi:10.1016/j.ifacol.2017.08.701. 20th IFAC World Congress.

Besançon, G., León-Morales, J.D., and Huerta-Guevara, O. (2006). On adaptive observers for state affine systems. *International Journal of Control*, 79(6), 581–591. doi:10.1080/00207170600552766.

Besançon, G., Bornard, G., and Hammouri, H. (1996). Observer synthesis for a class of nonlinear control systems. *European Journal of Control*, 2(3), 176 – 192. doi:10.1016/S0947-3580(96)70043-2.

Boutat, D., Benali, A., and Hammouri, H. (2007). Geometrical conditions for observer error linearization with a diffeomorphism on the outputs. *IFAC Proceedings Volumes*, 40(12), 870 – 875. doi:10.3182/20070822-3-ZA-2920.00144. 7th IFAC Symposium on Nonlinear Control Systems.

Gauthier, J. and Kupka, I. (1994). Observability and observers for nonlinear systems. *SIAM Journal on Control and Optimization*, 32(4), 975–994. doi:10.1137/S0363012991221791.

Giri, F., Radouane, A., Brouri, A., and Chaoui, F.Z. (2014). Combined frequency-prediction error identification approach for wiener systems with backlash and backlash-inverse operators. *Automatica*, 50(3), 768 – 783. doi:10.1016/j.automatica.2013.12.030.

Hann, C.A. and Ahmed-Ali, T. (2014). Continuous adaptive observer for state affine sampled-data systems. *International Journal of Robust and Nonlinear Control*, 24(4), 669–681. doi:10.1002/rnc.2912.

Hespanha, J.P., Naghshtabrizi, P., and Xu, Y. (2007). A survey of recent results in networked control systems. *Proceedings of the IEEE*, 95(1), 138–162. doi:10.1109/JPROC.2006.887288.

Ioannou, P.A. and Kokotovic, P.V. (1983). *Adaptive Systems with Reduced Models*. Springer-Verlag, Berlin, Heidelberg.

Ioannou, P.A. and Sun, J. (2012). *Robust Adaptive Control*. Dover Books on Electrical Engineering, Reprint of Prentice-Hall, Inc., Upper Saddle River, NJ, USA.

Kahelras, M., Ahmed Ali, T., Giri, F., and Lamnabhi-Lagarrigue, F. (2018). Sampled-data chain-observer design for a class of delayed nonlinear systems. *International Journal of Control*, 91(5), 1076–1090.

Karafyllis, I. and Kravaris, C. (2009). From continuous-time design to sampled-data design of observers. *IEEE Transactions on Automatic Control*, 54(9), 2169–2174. doi:10.1109/TAC.2009.2024390.

Khalil, H.K. (2002). *Nonlinear systems; 3rd ed.* Prentice-Hall, Upper Saddle River, NJ.

Kreisselmeier, G. (1977). Adaptive observers with exponential rate of convergence. *IEEE Transactions on Automatic Control*, 22(1), 2–8. doi:10.1109/TAC.1977.1101401.

Krener, A. and Respondek, W. (1985). Nonlinear observers with linearizable error dynamics. *SIAM Journal on Control and Optimization*, 23(2), 197–216. doi:10.1137/0323016.

Krener, A.J. and Isidori, A. (1983). Linearization by output injection and nonlinear observers. *Systems & Control Letters*, 3(1), 47 – 52. doi:10.1016/0167-6911(83)90037-3.

Li, X., Zhang, Q., and Su, H. (2011). An adaptive observer for joint estimation of states and parameters in both state and output equations. *International Journal of Adaptive Control and Signal Processing*, 25(9), 831–842. doi:10.1002/acs.1244.

Luders, G. and Narendra, K. (1973). An adaptive observer and identifier for a linear system. *IEEE Transactions on Automatic Control*, 18(5), 496–499. doi:10.1109/TAC.1973.1100369.

Massieu, J.F., Dorléans, P., and Van-Assche, V. (2017). Sampled-data observer design with time-varying scalar gain for sensorless control of induction motors. *International Journal of Control*, 90(4), 825–833. doi:10.1080/00207179.2016.1215530.

Narendra, K.S. and Annaswamy, A. (2005). *Stable Adaptive Systems*. Dover Books on Electrical Engineering, Reprint of Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Rajamani, R. (2017a). Observers for nonlinear systems: Introduction to part 1 of the special issue. *IEEE Control Systems Magazine*, 37(3), 22–24. doi:10.1109/MCS.2017.2674400.

Rajamani, R. (2017b). Observers for nonlinear systems: Part 2: An overview of the special issue. *IEEE Control Systems Magazine*, 37(4), 30–32. doi:10.1109/MCS.2017.2696758.

Rueda-Escobedo, J.G. and Moreno, J.A. (2017). Fixed-time convergent adaptive observer for lti systems. *IFAC-PapersOnLine*, 50(1), 11639 – 11644.

Zhang, Q. (2002). Adaptive observer for multiple-input-multiple-output (mimo) linear time-varying systems. *IEEE Transactions on Automatic Control*, 47(3), 525–529. doi:10.1109/9.989154.