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# Active constraint switching with the generalized split range control structure using the baton strategy

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**Abstract:** Split range control is used to extend the steady-state operating range for a single output (controlled variable, CV) by using more than one input (manipulated variable, MV). In the context of optimal operation, this advanced control structure can be used for active constraint switching, also in combination with selectors. The generalized split range control structure analyzed in this paper overcomes the limitations of standard split range control in terms of tuning by using multiple independent controllers with the same setpoint. By using the *baton strategy*, this structure avoids undesired switching between the controllers. In this contribution, we implement in this novel control structure in a simulation case study of a mixing process in which we must switch the MV used to control a high priority CV due to MV saturation.

*Keywords:* split range control, control structure, PID, tuning, anti-windup, baton, multiple input, constraint switching, optimal operation

## 1. INTRODUCTION

We can use three alternative classical control structures when we need more than one manipulated variable (MV, $u_i$, input) to cover the steady-state operating range for a single controlled variable (CV, $y$, output):

(1) (Standard) split range control
(2) One controller for each input, each with a different setpoint for the output
(3) Input (valve) position control

In the context of optimal operation, these structures can be used for active constraint switching, namely, *MV to MV constraint switching* and *MV to CV constraint switching* (Reyes-Lúa and Skogestad, 2020). This is further discussed in Section 2.

With valve position control we cannot utilize the full steady-state range of the primary input as it requires a back-off. If we use more than one controller with different setpoints, the difference between setpoints ($\Delta y^{sp}$) should be large enough to assure that only one controller is active at a time [1]. Therefore, split range control is often the chosen alternative.

In this paper we focus on split range control, although the structure that we are using has more than one independent controller. Split range control has been in use for at least 75 years (Eckman, 1945) [2], and it is still applied in industry (Sun et al., 2015). However, except for descriptions and examples of applications (see Stephanopoulos (1984); Hägglund (1997); Marlin (2000); Bequette (2002); Seborg et al. (2003); Smith (2010)), there are almost no academic studies.

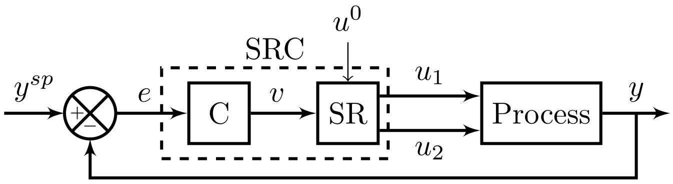


Fig. 1. Standard implementation of split range control (SRC) with two inputs ($u_i$) and one output ($y$). An SR-block is shown in Fig. 2. $u_0$ contains information about maximum and minimum values for both physical inputs.

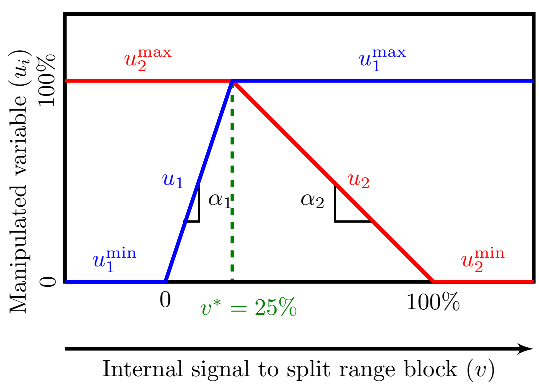


Fig. 2. Typical split range block for Fig.1, with $v^* \neq 50\%$.

[1] Having a setpoint deviation ($\Delta y^{sp}$) may be optimal in some cases Reyes-Lúa and Skogestad (2019b).
[2] Eckman (1945) called it "dual control agent".

In standard split range control (Fig. 1) the common controller ($C(s)$) computes the internal signal ($v$) to the split range block (SR), which assigns the value (e.g the valve opening) for each of the inputs ($u_i$). As can be observed from the split range block in Fig. 2, the resulting controller from $y$ to each $u_i$ is $\alpha_i C(s)$.

In Reyes-Lúa et al. (2019), we proposed a systematic procedure to design the standard (classical) split range controller in which we select $\alpha_i$ (or equivalently $v^*$) such that $\alpha_i K_C = K_{C,i}$. However, since we only have one design parameter for each MV ($\alpha_i$), we cannot make $\alpha_i C(s) = C_i(s)$ for all MVs. Thus, we must have a common integral time ($\tau_I$) for all MVs, which is a compromise for dynamic performance. This is an intrinsic limitation for standard split range control in terms of tuning

To overcome this limitation, in Reyes-Lúa and Skogestad (2019a), we proposed a generalized split range control structure, which allows using multiple independent controllers with the same setpoint. Undesired switching between the controllers is avoided by using a *baton strategy*, and only one controller (and one MV) is active at a time. This is the structure that we use in this paper.

This paper is structured as follows. In Section 2 we describe in which cases we can use split range control for active constraint switching. In Section 3, we detail the generalized split range control structure recently introduced in Reyes-Lúa and Skogestad (2019a) and used in this paper. In Section 4 we apply the generalized split range control structure in a mixing process that requires active constraint switching for optimal operation. We conclude the paper in Section 5.

## 2. SPLIT RANGE CONTROL FOR ACTIVE CONSTRAINT SWITCHING

*Active constraints* are variables that should *optimally* be kept at their limiting value. These can be either *manipulated variable* (MV, input) constraints or *controlled variable* (CV, output) *constraints*. When a disturbance occurs, the set of active constraints may change and we might need to update the pairing. Split range control is one of the advanced control structures that can be used for active constraint switching (Reyes-Lúa et al., 2018).

*MV to MV constraint switching* refers to the case in which the primary MV saturates and one or more extra MVs are added to cover the required steady-state range and maintain control of the CV (Reyes-Lúa and Skogestad, 2020). This is the most extended application of split range control, and corresponds to Fig. 1.

*MV to CV constraint switching* refers to the case in which there are the same number of MVs and CVs (any MV may be used to control any CV) and one of the MVs is likely to saturate. As we loose one degree of freedom, we must give up controlling one CV. In this case there are two possibilities:

(1) The *input saturation pairing rule* (Minasidis et al., 2015) was followed. This means that, compared to other CVs, it is less important to control the CV ($y_1$) paired with the MV that is likely to saturate ($u_1$). Then, when we loose one degree of freedom we should give up controlling $y_1$ (the least important CV).

(2) The *input saturation pairing rule* was *not* followed. This means that there are other CVs that are less important to control compared to $y_1$, and we should not give up controlling $y_1$. Thus, when $u_1$ saturates, we need to find another MV ($u_2$) to control $y_1$. In this case, we give up controlling $y_2$ (the CV previously controlled by $u_2$). To do this, we can implement an *MV to MV* switching strategy, such as split range control, in combination with a *min/max* selector (Reyes-Lúa et al., 2018), as shown in Fig. 3.

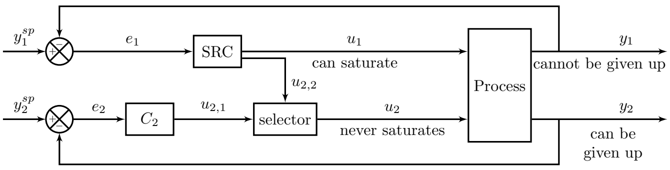


Fig. 3. MV to CV switching for the case when the input saturation rule is *not* followed; so control of $y_1$ cannot be given up.

In Fig. 3, the SRC block can be either the standard split range structure in Fig. 1 or the generalized split range structure in Fig. 4. It should be pointed out, that it is sensible to implement this scheme only if it is possible to leave the low priority CV uncontrolled. For this reason, it is important to prioritize CVs when input saturation (loss of degrees of freedom) may happen.

## 3. GENERALIZED SPLIT RANGE CONTROLLER USING THE BATON STRATEGY

Fig. 4 depicts the generalized split range control structure proposed in Reyes-Lúa and Skogestad (2019a). Each input has its own controller $C_i(s)$, which can be any type of controller, but is commonly a PI controller. Each controller produces a suggested input $u_i'$ and the *baton strategy block* selects and computes the actual physical inputs ($u_i$), based on a predefined sequence.

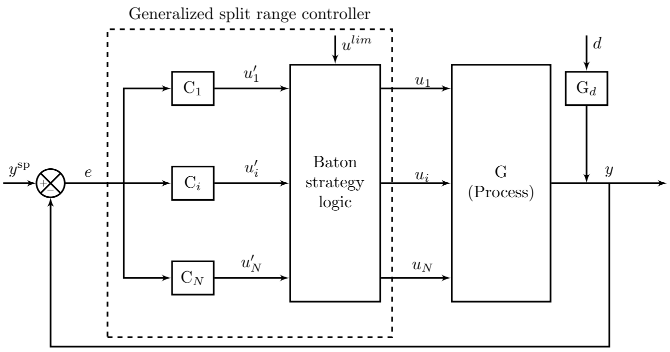


Fig. 4. Generalized split range control structure using the baton strategy. Note here that $u_i$ contains the bias information (maximum and minimum values for each input).

Importantly, at any given time, *only one input* ($u_i$) *is actively controlling the output* ($y$) and the other inputs are required to be at fixed values, $u_i^{lim}$, which may be $u_i^{min}$ or $u_i^{max}$. We call this *baton strategy* because we let the active input ($k$) decide when to switch to another input (*pass the baton*). The *active input* remains active as long as its not saturated ($u_k^{min} < u_k < u_k^{max}$) and will only *pass the baton* to another input once it becomes saturated (reaches $u_k^{min}$ or $u_k^{max}$).

As with standard split range control (Reyes-Lúa et al., 2019), the first step to design the control structure is to:

- Define the minimum and maximum values for every MV ($u_i^{\min}$, $u_i^{\max}$) and the desired (least expensive) operating point.
- Define the sequence in which we want to use the MVs. This is done considering the effect of each MV on the CV and economics (for MVs with the same effect, we use the least expensive MV first).

This sequence can be illustrated using a split range block such as in Fig. 2, but for the generalized split range controller the slopes have no significance.

### *3.1 Design of the baton strategy block*

Consider that input $k$ is the active input (*has the baton*). The baton strategy is then:

B.1 Controller $C_k$ computes $u_k$, which is the suggested value for the input $k$.
B.2 If $u_k^{min} < u'_k < u_k^{max}$
  (a) keep $u_k$ active, with $u_k \leftarrow u'_k$
  (b) keep the remaining inactive inputs at their corresponding constant values, $u_i^{lim}$ ($u_i^{min}$ or $u_i^{max}$).
B.3 If $u'_k \leq u_k^{min}$ or $u'_k \geq u_k^{max}$
  (a) Set $u_k = u_k^{min}$ or $u_k = u_k^{max}$ and *pass the baton* to the new active input $j$. The new active input is selected according to the predefined sequence, depending on which bound of $k$ is reached ($j = k+1$ or $j = k-1$).
  (b) Set $k = j$ and go to step B.1.

We need to decide how to initialize the new active controllers and avoid windup. There are several alternatives. A simple strategy is to set all the states of the non active controllers to zero. For a PI controller (Eq. (1)), this means that the integral action starts at the time of the switching ($t_b$) and not from 0.

$$u'_k(t) = u_k^0 + K_{C,k}\left(e(t) + \frac{1}{\tau_{I,k}}\int_{t_b}^{t} e(t)\right) \tag{1}$$

The value of the bias $u_k^0$ is the input value just before receiving the baton, that is, either $u_k^{max}$ or $u_k^{min}$. Another alternative is to implement bumpless transfer (Åström and Hägglund, 2006).

## 4. CASE STUDY: MIXING OF AIR AND METHANOL

In a formaldehyde production process, air and methanol (MeOH) are mixed in a vaporizer. Here we consider that air is fed using a blower with limited capacity. The high priority CV is the methanol molar fraction at the outlet of the vaporizer ($y_1 = x_{MeOH}$) which should be kept at 0.10 (desired), and with a minimum value of 0.08 (more important), such that the reaction can take place. Additionally, we want to control the total mass flow ($y_2 = \dot{m}_{tot}$), and preferably maximize it. This process is also described in Reyes-Lúa and Skogestad (2020), and the model can be found in Appendix A.

The controlled variables (CVs) are:

- $y_1 = x_{MeOH}$: MeOH molar fraction
- $y_2 = \dot{m}_{tot}$: total mass flow

The two manipulated variables (MVs) are:

- $u_1 = \dot{m}_{air}^{sp}$: mass flow of air
- $u_2 = \dot{m}_{MeOH}^{sp}$: mass flow of methanol

Note that the physical MVs are the air blower rotational speed ($\dot{\omega}_{air}$) and the MeOH valve opening ($z_{MeOH}$), but we use a (lower) regulatory control layer with flow controllers which follow $u_1 = \dot{m}_{air}^{sp}$ and $u_2 = \dot{m}_{MeOH}^{sp}$.

Table 1 shows the constraints and nominal operating conditions. Note that the valve for $u_2 = \dot{m}_{MeOH}$ is not limited, and only $y_1 = x_{MeOH}$ and $u_1 = \dot{m}_{air}$ have relevant boundaries.

Table 1. Maximum and nominal values for case study.

| Variable | Units | Maximum | Nominal |
|---|---|---|---|
| $y_1 = x_{MeOH}$ | kmol/kmol | 0.10 | 0.10 |
| $y_2 = \dot{m}_{tot}$ | kg/h | - | 26860 |
| $u_1 = \dot{m}_{air}$ | kg/h | 25800 | 23920 |
| $u_2 = \dot{m}_{MeOH}$ | kg/h | - | 2940 |

As the main CV is $y_1 = x_{MeOH}$, it has a higher priority to keep

$$x_{MeOH} = x_{MeOH}^{sp}; \quad \text{setpoint for } y_1 \tag{2}$$

compared to keeping

$$\dot{m}_{tot} = \dot{m}_{tot}^{sp}; \quad \text{setpoint for } y_2 \tag{3}$$

At the nominal operating point (defined in Table 1), we are able to satisfy all the constraints. Due to upstream plant conditions, we pair $u_1 = \dot{m}_{air}$ with $y_1 = x_{MeOH}$ and $u_2 = \dot{m}_{MeOH}$ with $y_2 = \dot{m}_{tot}$. As $u_1 = \dot{m}_{air}$ is likely to saturate, we are *not following the input saturation pairing rule* with this pairing.

When $u_1 = u_{air}$ reaches its maximum value ($u_1 = u_1^{max}$) we lose a degree of freedom, and we must give up controlling $y_2$ (constraint (3)) to keep controlling $y_1$ (constraint (2)). To do this, we must implement an MV to CV switching strategy such as the one in Fig. 3. The solution using split range control with a *min* selector in this process is shown in Fig. 5.

The split range block (SRC) in Fig. 5 can be a standard split range controller (Fig. 1) or a generalized split range controller (Fig. 4).

For both alternatives, we realize that available inputs have opposite effects on $y_1 = x_{MeOH}$ (see $K_{p,i}$ in Table A.1); that is,

- increasing $u_1 = \dot{m}_{air}^{sp}$ decreases $x_{MeOH}$
- increasing $u_2 = \dot{m}_{MeOH}^{sp}$ increases $x_{MeOH}$

Considering the pairing in Fig. 5, we use first use $u_1 = \dot{m}_{air}^{sp}$ to control $y_1 = x_{MeOH}$. When $u_1 = \dot{m}_{air}^{max}$, we start using $u_2 = \dot{m}_{MeOH}^{sp}$ to control $y_1 = x_{MeOH}$ and we give up controlling $y_2 = \dot{m}^{tot}$.

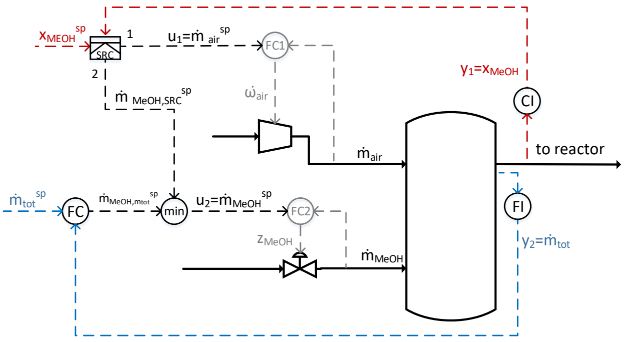


Fig. 5. Control structure for mixing of MeOH and air when *not* following the *input saturation pairing rule* using split range control (SRC) with a *min* selector.

The value of $u_2 = \dot{m}_{MeOH}$ at the time of the switch (when $u_1 = u_1^{max} = \dot{m}_{air}^{max}$) is not fixed, and it depends on the setpoint for $x_{MeOH}$ ($x_{MeOH}^{sp}$). Therefore, to improve the dynamic performance, the bias for $\dot{m}_{MeOH}$, $u_{MeOH}^{0}$, should be updated to the value of $\dot{m}_{MeOH}$ at the time of the switch. This value can also be obtained from the steady-state mass balance, as described in Appendix B.

The desired PI tuning parameters ($K_{C,i}$ and $\tau_{I,i}(s)$) for both MVs to control $y_1$ are in Table 2. These are obtained using the transfer functions in Table A.1 and the SIMC tuning rules (Skogestad, 2003), where $\tau_c$ is the desired closed loop time constant.

Table 2. Desired PI tuning parameters for $u_1$ and $u_2$ to control $y_1$.

| Controller (Fig. 8) | MV ($u_i$) | $\tau_{c,i}(s)$ | $K_{C,i}$ | $\tau_{I,i}(s)$ |
|---|---|---|---|---|
| $C_1$ | $u_1 = \dot{m}_{air}$ | $\theta_{air}$ | -74360 | 2.83 |
| $C_2$ | $u_2 = \dot{m}_{MeOH}$ | $2\theta_{MeOH}$ | 10736 | 1.26 |

### *4.1 Standard split range controller*

Fig. 6 shows the configuration of the standard split range controller (SRC) for the control structure in Fig. 5. Using the procedure introduced in Reyes-Lúa et al. (2019), we obtain the slopes $\alpha = [-29874;\ 4313]$ for the split range block in Fig. 7 and the tuning parameters for the common PI controller ($C$ in Fig. 6) $K_C = 2.5$, $\tau_I = 2.83$ s.

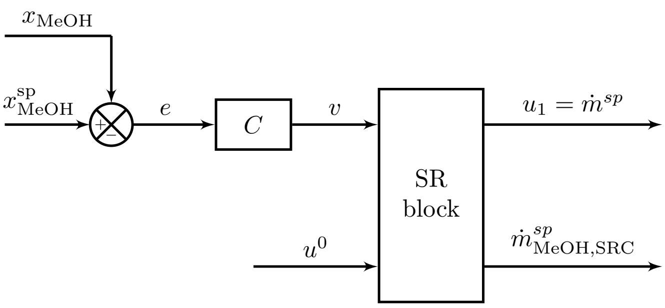


Fig. 6. Standard split range control solution for mixing air and MeOH. This structure can be used in the SRC block in Fig. 5. The SR-block is shown in Fig. 7.

We should note that in this case, we do not need input tracking (anti-windup) for the common controller ($C$ in Fig. 6) because $y_1 = x_{MeOH}$ is always being controlled; that is, the selected signal in the split range controller will always be active. Anti-windup is necessary for the flow controller for $y_2 = \dot{m}_{tot}$, as it will wind up during the period in which it is not selected and we give-up controlling $y_2 = \dot{m}_{tot}$.

Fig. 7 shows the split range block for the standard split range configuration for mixing of air and MeOH. The MVs are not scaled, and we can see the opposite effects of $u_1$ and $u_2$ on $y_1$. Note that $u_2^{max} = \dot{m}_{MeOH}^{max}$ is not a fixed value, as the value of $\dot{m}_{MeOH}$ at the time of the switching will depend on the setpoint coming from the controller for the total mass flow ($\dot{m}_{tot}$), FC in Fig. 5. Thus, as shown in Fig. 6 this bias ($u^0$) should be added to $v$ to make $u_2$ "jump" to the current value at the time $u_2$ is chosen from the SR block.

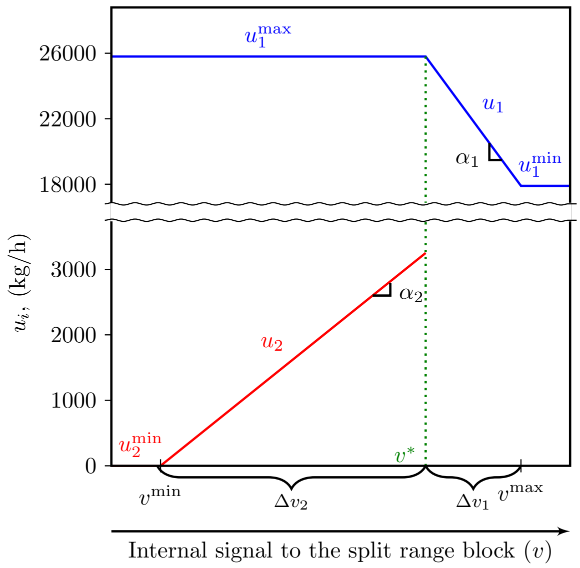


Fig. 7. Split range block for standard implementation of mixing of air ($u_1$) and MeOH ($u_2$). This is the SR-block in Fig. 6.

### *4.2 Generalized split range controller*

Fig. 8 shows the block diagram for the generalized split range controller. $C_1$ and $C_2$ are PI controllers (Eq. (1)), and the tunings are in Table 2. We consider Fig. 7 to define the sequence for the inputs. The baton strategy logic is written in Table 3.

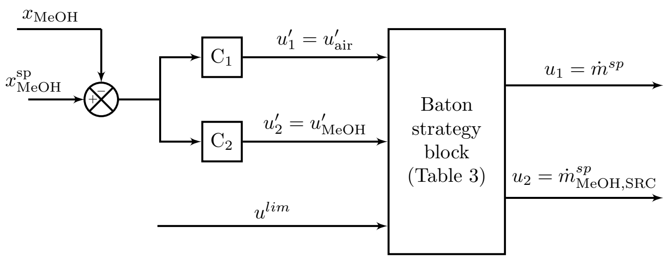


Fig. 8. Generalized split range control solution for mixing of air and MeOH. This structure can be used in the SRC block in Fig. 5.

Table 3. Baton strategy logic for mixing of air and MeOH; $u_k$ is the active MV (the one that has the *baton*).

| Value of $u_k$ | $u_k = u_1 = \dot{m}_{air}$ | $u_k = u_2 = \dot{m}_{MeOH}$ |
|---|---|---|
| $u'_k \geq u_k^{max}$ | baton to $u_2$<br>$u_1 \leftarrow u_1^{max}$<br>$u_2^0 = u_2^{max}$ | baton to $u_1$<br>$u_1^0 = u_1^{max}$<br>$u_2 \leftarrow u_2^{max}$ |
| $u_k^{min} < u'_k < u_k^{max}$ | Keep $u_1$ active<br>$u_1 \leftarrow u'_1$<br>$u_2 \leftarrow u_2^{max}$ | Keep $u_2$ active<br>$u_2 \leftarrow u'_2$<br>$u_1 \leftarrow u_1^{max}$ |

When the input *receives the baton* (at time $t = t_b$), the integrator of its corresponding PI controller is reset (see Eq. (1)), and the initial value for $u_k$ will be the proportional term plus the bias.

Note that in Table 3, when $u_2 = \dot{m}_{MeOH}$ receives the baton, the initial value is $u_2^0 = u_2^{max}$. This is a generalization. However, as explained before and in Appendix B, this value is actually not fixed, and a better dynamic response is obtained if $u_2^0$ is set to the value of $u_2$ at the time of the switch.

As in standard split range control, anti-windup is implemented for the flow controller for $y_2 = \dot{m}_{tot}$.

*Simulations* We test the generalized split range structure and the standard structure with a step change in $x_{MeOH}^{sp}$ of $-0.005$ (from 0.1 to 0.095) $t = 10$s. at $t = 30$s, $m_{tot}^{sp}$ is increased by 10% (from 26860 kg/h to 29546 kg/h). Finally, at $t = 70$s $m_{tot}^{sp}$ is brought back to its initial value.

Fig. 9 shows the simulation results for both structures. We observe that both structures bring $y_1 = x_{MeOH}$ to its set point at steady state. When $y_1 = \dot{m}_{air}$ saturates and $y_1 = x_{MeOH}$ is controlled using $u_2 = \dot{m}_{MeOH}$ and we give up controlling $y_2 = \dot{m}_{tot}$

However, at $t = 30$ s, when $u_1 = \dot{m}_{air}$ saturates, the response of $y_1 = x_{MeOH}$ is clearly better with the generalized structure, with a lower input usage for $u_2$. Likewise, when we can use again $u_1$ to control $y_1$, the generalized structure keeps $y_1$ closer to $y_1^{sp}$.

Table 4 shows that, with an improved dynamic response due to better tunings, the integral absolute error (IAE) for the high priority CV $y_1 = x_{MeOH}$ decreases when using the generalized structure. The IAE for $y_2 = \dot{m}_{tot}$ is expected to be high, as we give up controlling $\dot{m}_{tot}$ when $\dot{m}_{air}$ saturates ($\dot{m}_{air} = \dot{m}_{air}^{max}$).

Table 4. Comparison of IAE with standard and generalized split range control for mixing of air and MeOH.

| Case | IAE $x_{MeOH}$ (mol/mol) | IAE $\dot{m}_{tot}$ (kg/h) |
|---|---|---|
| Standard SRC | 0.1623 | 59754 |
| Generalized SRC | 0.1082 | 60274 |

## 5. FINAL REMARKS

The generalized split range control structure using the baton strategy in Figs. 4 and 8 can be used in the same applications as standard split range control (Fig. 1). In this novel structure, each MV has its own controller, but only one MV (the one *with the baton*) is active at a time. This approach has the obvious advantage that once that the *baton strategy logic*, such as the one in Table 3, is implemented one can independently adjust the tunings for each MV and obtain the desired dynamic performance, without affecting the performance of the other the MVs.

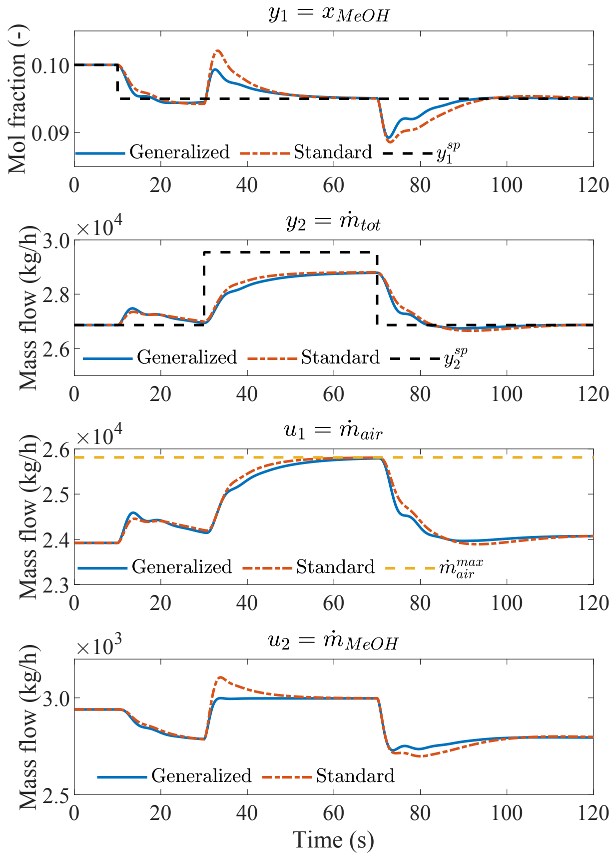


Fig. 9. Comparison of generalized and standard split range control for mixing of MeOH and air.

In this paper, we implemented this structure in a mixing process that requires an *MV to CV constraint switching strategy* (split range control with *min* selector) to maintain control of an important CV. We compared simulation results with standard split range control, and showed that by having independent tunings for each MV, we can better handle switches in active constraints.

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## Appendix A. MODEL FOR MIXING OF METHANOL AND AIR

Eq. (A.1) and (A.2) describe the steady-state mass and molar balance for this system.

$$\dot{m}_{tot} = \dot{m}_{air} + \dot{m}_{MeOH} \tag{A.1}$$

which corresponds to $y_1 = u_1 + u_2$.

$$x_{MeOH} = \frac{\dot{m}_{air}/MW_{air}}{\dot{m}_{air}/MW_{air} + \dot{m}_{MeOH}/MW_{MeOH}} \tag{A.2}$$

where $\dot{m}_i$ are the air and MeOH inlet mass flow rates (kg/h), $\dot{m}_{tot}$ is the outlet total mass flow rate (kg/h), and $x_{MeOH}$ is the methanol molar concentration (kmol/kmol), and $MW_i$ are the methanol and air (average) molecular weights (kg/kmol).

Taking into account the dynamics of the actuators and the measurements, the dynamic responses can be approximated to the first order transfer functions with time delay $\left(G_i = \frac{K_{p,i}e^{-\theta_i s}}{\tau_i s+1}\right)$ in Table A.1, which were identified using step-tests with simulation results for each possible pairing.

Table A.1. First order transfer functions for mixing of MeOH and air.

| $\text{MV}_i$ | $\text{CV}_i$ | $K_{p,i}$ | $\tau_i$ (s) | $\theta_i$ (s) |
|---|---|---|---|---|
| $\dot{m}_{air}$ | $x_{MeOH}$ | -3.43E-05 | 2.83 | 0.37 |
| $\dot{m}_{air}$ | $\dot{m}_{tot}$ | 1.1229 | 2.90 | 0.56 |
| $\dot{m}_{MeOH}$ | $x_{MeOH}$ | 2.94E-04 | 1.26 | 0.20 |
| $\dot{m}_{MeOH}$ | $\dot{m}_{tot}$ | 9.14 | 3.80 | 1.15 |

## Appendix B. BIAS CALCULATION FOR METHANOL FLOW

When $\dot{m}_{air}^{sp} = \dot{m}_{air}^{max}$, the value of $\dot{m}_{MeOH}$ that will be required to get $x_{MeOH} = x_{MeOH}^{sp}$ is a function of the current value of $x_{MeOH}^{sp}$. From the mass balance (Eq. (A.1)), when $\dot{m}_{air} = \dot{m}_{air}^{max}$, the $\dot{m}_{MeOH}$ that satisfies the mass balance is:

$$\dot{m}_{MeOH}^{0} = \dot{m}_{air}^{max}\left(\frac{MW_{MeOH}}{MW_{air}}\right)\left(\frac{x_{MeOH}^{sp}}{1 - x_{MeOH}^{sp}}\right) \tag{B.1}$$

Updating $u_2^0 = \dot{m}_{MeOH}^0$ improves the dynamic response of the system.

The bias update can also be done by setting the bias in the split range controller equal to the current value of $\dot{m}_{MeOH}$ at the moment when $u_2 = \dot{m}_{MeOH}$ *receives the baton* and starts controlling $y_1 = x_{MeOH}$.