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Unscented stochastic model predictive perimeter control of uncertain two-regions urban traffic

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Abstract: In this article, Stochastic Model Predictive Control (SMPC) is employed for optimal perimeter control of traffic flow with uncertain Macroscopic Fundamental Diagram (MFD), traffic accumulation and traffic demand of two regions. Two regions urban traffic networks are described through the MFD. The MFD is a fundamental relation between average flow (production) and density (accumulation) in urban regions. Although the MFD is often assumed as a simple deterministic curve, possible heterogeneity in urban regions results in large scattering of the MFD pattern. Traffic accumulation is considered uncertain due to limited sources of measurements. Moreover, traffic demand is based on the stochastic nature of drivers. The stochastic uncertainty is modeled through appropriate probability distribution functions for MFD, traffic accumulation and demand. Simulation results show the superiority of the proposed method compared to deterministic MPC in the presence of model mismatch.

Keywords: Macroscopic fundamental diagram, Predictive control, Uncertainty, Unscented transform, Traffic control, Probabilistic models.

1. INTRODUCTION

Perimeter control of urban traffic regions have been proposed to increase mobility and reduce travel times in urban areas by controlling traffic flow through boundaries of different urban regions. Macroscopic traffic models are utilized for modeling and control of urban traffic. However, performance of model-based control methods rely on accurate description of model. Perimeter control is achieved by manipulating allowed traffic flow between regions, an example of this is reducing traffic flow by reducing effective green light time.

Macroscopic fundamental diagram (MFD), described in Daganzo (2005), is considered as a main tool for macroscopic modeling of the relation of large-scale traffic parameters such as density (accumulation) and flow or speed (production). The relation has been used as a basis of macroscopic modeling in traffic flow management in various studies such as Geroliminis et al. (2007), Ramezani et al. (2015), Keyvan Ekbatani et al. (2015). However, Geroliminis and Sun (2011) suggests that in a large-scale scenario it is unlikely that such a relation will remain constant across the network. Network clustering is used in different traffic zones to minimize the effect of heterogeneity. However, traffic zone clustering might depend on origin-destination (O-D) pairs as shown in Saeedmanesh and Geroliminis (2017) and adaptive behavior of drivers as shown in Aboudolas and Geroliminis (2013). As a result, the inherent heterogeneity of traffic flow which is the result of stochastic behavior of traffic flow suggests further consideration of stochastic methods.

Previously robust worst-case perimeter control was considered for linear perimeter control problem without consideration of optimality. In Ampountolas et al. (2017), a worst-case control is achieved by solving a linear matrix inequality (LMI). Moreover, set-based describing function method is employed in Haddad and Shraiber (2014) for robust control of traffic flow. On the other hand, optimal controllers (nominal MPC) as suggested in Geroliminis et al. (2012), Kouvelas et al. (2017a) are not robust to uncertainties. We propose the Stochastic MPC (SMPC) to achieve constrained, optimal control subject to the uncertainty defined by probabilistic variables.

In this paper, we extend the MFD traffic flow model, proposed in Geroliminis et al. (2012) to include uncertainty. We assume uncertainty in MFD, measured traffic accumulation and traffic demand between regions. We consider fixed given uncertainty distributions. This assumption may be relaxed by exploiting a Kalman filter as previously introduced to estimate traffic flow variables in Kouvelas et al. (2017b). The contribution of the paper is proposing a new stochastic MPC that achieves higher trip completion rate by perimeter control that takes into account uncertainty in traffic variables.

Fundamental to SMPC is the choice of uncertainty propagation method. There are different methods to propagate a nonlinear model subject to uncertainty. For example, Bayesian sampling and Monte Carlo methods are applied in Scenario-MPC and Polynomial chaos expansion is effective for chance constrained stochastic MPC Harati and Noubari (2016). In this paper, we consider

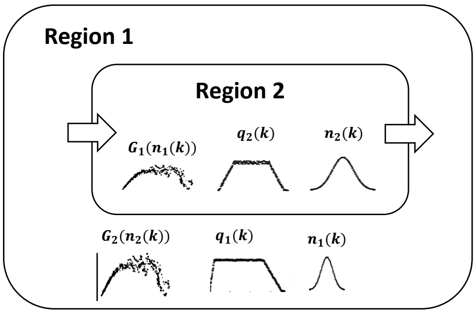

Fig. 1. R-region perimeter control method is defined for MFD (θ_i), demand (q_i) and accumulation (n_i).

the unscented transform Julier and Uhlmann (1997) for uncertainty propagation. Unscented transform is chosen for efficient SMPC by formulating the problem in the first two moments of the uncertainty distribution. Further review on SMPC is provided in Mesbah (2016).

The paper is organized as follows: In Section 2, we introduce the problem of traffic modeling in two-regions urban traffic networks and optimal traffic control problem. In Section 3, we provide the background material on deterministic MPC of the traffic flow problem. The method for considering probabilistic uncertainty in traffic flow is presented in Section 4. Unscented transformation is shown in Section 5 for Unscented based SMPC. Section 6 shows the simulation results of the proposed methodology as applied to the two-regions perimeter control problem. Finally, the paper is concluded in Section 7.

2. PROBLEM DEFINITION

Consider two urban regions with traffic accumulation in region i denoted by $n_i(k)$ for $(i = 1, 2)$. Traffic flow is transferred within the boundary at time kT_s, where T_s is the sampling time. For macroscopic modeling of transfer flows, the MFD $G_i(n_i(k))$ is defined as the trip completion rate (production) for region i at $n_i(k)$. We assume average trip length in region i to be constant L_i. The accumulation $n_i(k)$ (veh) is further divided by vehicles at region i with final destination in the source region i (n_{ii}) or destination region j (n_{ij}), i.e. $n_1 = n_{11} + n_{12}$. The perimeter control problem is to design $u_{ij}(k)$ ($u_{lb} \leq u_{ij}(k) \leq u_{ub}$), for $j = 1, 2$, by manipulating the transfer flows $q_{ij} = u_{ij}(k)\frac{n_{ij}(k)}{n_i(k)}G_i(n_i(k)$, where the value u_{ij} corresponds to control inputs modifying the permissible percentage of vehicles that can pass the region boundaries. Moreover, $q_i^I = q_{ii}$, $q_i^O = \frac{n_{ii}(k)}{n_i(k)}G_i(n_i(k)$. For simplicity, we will remove the index term (k) for accumulation $n_{ij}(k)$, $q_{ij}(k)$ unless we wish to emphasize the time of consideration. We arrive at

$$\begin{aligned}
n_{11}(k+1) &= n_{11} + T_s(q_{11} - \frac{n_{11}}{n_1}G_1(n_1) + u_{21}\frac{n_{21}}{n_2}G_2(n_2)),\\
n_{12}(k+1) &= n_{12} + T_s(q_{12} - u_{12}\frac{n_{12}}{n_1}G_1(n_1)), \qquad (1)\\
n_{22}(k+1) &= n_{22} + T_s(q_{22} - \frac{n_{22}}{n_2}G_2(n_2) + u_{12}\frac{n_{12}}{n_1}G_1(n_1)),\\
n_{21}(k+1) &= n_{21} + T_s(q_{21} - u_{21}\frac{n_{21}}{n_2}G_2(n_2)),
\end{aligned}$$

Hereafter, we consider the dynamics of (1) compactly as $n(k+1) = f(n(k), u(k), q(k), \theta(k))$ where $n = [n_{11}, n_{12}, n_{21}, n_{22}]$, $q = [q_{11}, q_{12}, q_{21}, q_{22}]$ and $u = [u_{21}, u_{12}]$, and θ corresponds to the MFD parameters defined as

$$G_i(n_i) = \theta_{i,3}n_i^3 + \theta_{i,2}n_i^2 + \theta_{i,1}n_i^1, \quad \text{for } i = 1, 2. \qquad (2)$$

Notice that the MFD is a simplification of a complex phenomenon. The MFD is approximated through a nonlinear 3^{rd} order polynomial, i.e. The MFD is depends on the heterogeneity of regions and adaptive behavior of drivers, etc. To account for uncertainty, we consider uncertain values for the polynomial coefficients in (2). The uncertainty is introduced by assuming

$$\tilde{\theta}_i(k) = \theta_i(k) + \xi_i(k). \qquad (3)$$

where for each region ξ_i is a three dimensional vector.

2.1 Estimation and Control

In practice, uncertainty originates from stochastic behavior of drivers, heterogeneity in traffic flow and restricted sources for measurement of traffic variables. The MFD parameters θ are considered stochastic by the inherent nature of traffic flow. On the other hand, optimal control problem formulation requires estimated values of the traffic demand q, and the accumulation n which are perturbed by stochastic disturbance. We use $\tilde{\theta}$ for uncertain θ, $\hat{n}$ for n and $\hat{q}$ for q to stress that. We consider that the values of q_{ij} given by

$$\hat{q}_{ij}(k) = q_{ij}(k) + \eta_{ij}(k). \qquad (4)$$

Moreover, in the assumed plant model it is reasonable to assume uncertainty on the estimated number of vehicles inside a region n_i. Consequently, the estimated values of n_{ij} are given by

$$\hat{n}_{ij}(k) = n_{ij}(k) + \zeta_{ij}(k). \qquad (5)$$

Similar to n and q the parameters $\eta_{ij}, \zeta_{ij}(k)$ are compactly written as η, ζ. As a result, the corresponding uncertain traffic flow dynamics is given by $\hat{n}(k+1) = f(\hat{n}(k), u(k), \hat{q}, \tilde{\theta})$. Notice that for each realization of unknown values $[\eta, \zeta, \xi]$ the model f is completely deterministic and given by (1).

We consider constant and known distributions Gaussian prior distribution for uncertain terms, estimation methods such as Kouvelas et al. (2017b) may further be combined for more realistic definition of uncertain parameters. The uncertain terms are defined by Gaussian distribution with zeros mean value and known variance, such as for demand $\eta = \mathcal{N}(0, \Sigma_q)$, MFD parameters $\xi = \mathcal{N}(0, \Sigma_\theta)$ and measured accumulation by $\zeta = \mathcal{N}(0, \Sigma_n)$. More compactly written, uncertain terms are combined as $x = [\hat{n}, \hat{q}, \tilde{\theta}]$ that is distributed according to a distribution given by $x \sim \mathcal{D}_x$.

We consider the optimal control problem (OCP) by maximizing the TTD from the start time (0) to the final time (T_f) subject to uncertainty. The OCP is defined to maximize Total Traveled Distance (TTD) defined as follows

$$\begin{aligned}
\max_u TTD &= T_s \sum_{k=0}^{T_f} \sum_{i=1}^{2} L_i G_i(n_i(k))),\\
subject\ to: \hat{n}(k+1) &= f(\hat{n}(k), u(k), \hat{q}, \tilde{\theta})\\
u_{lb} &\leq u \leq u_{ub} \qquad (6)
\end{aligned}$$

3. MODEL PREDICTIVE CONTROL OF TRAFFIC FLOW

In this section, we approximately solve the OCP using model predictive control by considering the nominal model (equivalently $\hat{n} = n$, $\hat{q} = q$ and $\hat{\theta} = \theta$). A performance index (J) is maximized over hypothetical future inputs resulting in predicted future outputs. The optimal control is fed to the plant and the MPC will be formulated at the next sampling time with the arrival of new measured states.

The MPC approximates the OCP problem (6) at sampling time k defined by

$$\begin{aligned} &\max_u J(k) = PTD_k^{(k+H)} \\ &subject\ to: u_{lb} \leq u \leq u_{ub} \\ &\qquad n(k+1) = f(n(k), u(k), q(k), \theta(k)) \end{aligned} \tag{7}$$

where H is the prediction horizon and Predicted Traveled Distance (PTD) is defined as

$$PTD_k^{(k+H)} = T_s \sum_{\tau=k}^{k+H} \sum_{i=1}^{2} L_i G_i(n_i(\tau)). \tag{8}$$

In this work, the nonlinear MPC problem is approximated by linear time-variant MPC in Harati (2011). The Linear Time-Variant MPC (LTV-MPC) is a convex approximate solution to the original non-convex OCP. In this manner, the MFD is linearized about the trajectory of the system. This is more accurate than previously suggested methods such as Kouvelas et al. (2017a) where macroscopic fundamental diagram is approximated with a pre-defined piecewise linear function. The optimization problem is defined in Appendix A.

The following proposition is fundamental for development of proposed Stochastic MPC from nominal MPC.

Proposition 1. *The LTV-MPC approximates* (7) *as a linear programming problem, and the performance index can be written as* $J = PTD_k^{(k+H)} = y^T u$, *where* y *is a linear function of* $x = [n, q, \theta]$.

Proof. See appendix A. □

Although the nominal deterministic MPC yields near to optimal results when there is no model mismatch between the model and the true physical plant, the controller performs poorly in the case of model mismatch (Kouvaritakis and Cannon (2016)). In the next section, we introduce the proposed method to account for the uncertainty in MPC formulation.

4. STOCHASTIC MODEL PREDICTIVE CONTROL OF TRAFFIC FLOW

Consider that the uncertain parameter $x = [n, q, \theta]$ may take any distribution in $\mathcal{D}_x$ with known mean $\mathbb{E}(x)$ and variance $Var(x)$ at time $k + i$. The Stochastic MPC (SMPC) problem consider the probability distribution of uncertain variables in performance function or constraints. Motivated by probabilistic constraints in Camacho and Bordons (2016), we consider maximizing the worst predicted traveled distance $PTD_k^{(k+H)}$ that might happen with probability $(1-\epsilon)$ as

$$\begin{aligned} &\max_u J_\epsilon \\ &subject\ to: Prob\{PTD_k^{(k+H)}(x) \geq J_\epsilon\} \geq 1-\epsilon \\ &\qquad u_{lb} \leq u \leq u_{ub} \\ &\qquad \hat{n}(k+1) = f(\hat{n}(k), u(k), \hat{q}(k), \tilde{\theta}(k)) \\ &\qquad x := [\hat{n}, \hat{q}, \tilde{\theta}] \sim \mathcal{D}_x \end{aligned} \tag{9}$$

The first two lines states that we are minimizing the worst case lower bound of PTD that happens with probability of $1-\epsilon$, it is followed by definition of model and distribution of uncertainty. The SMPC problem may be formulated efficiently, considering the following theorem on distributionally robust chance constraint Calafiore and El Ghaoui (2006).

Theorem 1. *The distributionally robust chance constraint*

$$\begin{aligned} &Prob\{y^T u \geq J\} \geq 1-\epsilon \\ &subject\ to: y \sim \mathcal{D}_y \end{aligned} \tag{10}$$

is equivalent to the convex second-order cone constraint for any $\epsilon \in (0, 1)$.

$$\mathbb{E}(y^T u) + k_\epsilon Var(y^T u) \geq J, k_\epsilon = \sqrt{(1-\epsilon)/\epsilon} \tag{11}$$

Notice that y is a function of aggregated uncertain traffic variables x with a modified distribution but known mean $\mathbb{E}(y)$ and variance $Var(y)$ known as $\mathcal{D}_y$. The proposed method for defining the Stochastic Model Predictive Control follows immediately since the previously defined deterministic model predictive control problem is formulated as a linear programming problem which follows from Proposition 1 and Theorem 1.

Corollary 1. *The problem SMPC* (9) *is equivalent to*

$$\begin{aligned} &\max_u J(k) = \mathbb{E}(PTD)_k^{(k+H)} + k_\epsilon Var(PTD)_k^{(k+H)} \\ &subject\ to: u_{lb} \leq u \leq u_{ub} \\ &\qquad n(k+1) = f(n(k), u(k), q(k), \theta(k)) \end{aligned} \tag{12}$$

As a result, the stochastic model predictive control corresponds to the maximization of upper $(1-\epsilon)$ quantile of the probability distribution for the cost of nominal MPC with various realizations of uncertainty. In other words, the joint maximization of the expected value and variance of the PTD is equivalent to maximization of the $(1-\epsilon)$ quantile of uncertain parameters. In this direction, we will consider computation of $\mathbb{E}(G_i)$ and $Var(G_i)$ which requires propagation of the probability distribution of the uncertainty.

In this paper, we consider the Unscented Transform in Julier and Uhlmann (1997) to determine the first two moments of stochastic variations of traffic flow. We are interested in predicting $\mathbb{E}M_{ij}(k)$ and $Var(M_{ij}(k))$ to arrive at the performance index for the SMPC (9). Unscented transform is used since it accurately propagates the first two moments (mean and variance) of a random variable.

5. UNSCENTED TRANSFORM BASED STOCHASTIC MPC

Uncertainty propagation is fundamental problem in nonlinear dynamic models and it is important for SMPC algorithms. We have shown that the problem of SMPC requires the information of the first two moments of uncertainty in future times. Unscented transform estimates the first two moments of a random variable analytically by propagating a set of realizations of uncertain variables known as Sigma points. The advantage of the method is that the set of realizations of uncertainty is very few as compared to scenario based MPC.

Consider the mean of accumulated variable $\bar{x} = \mathbb{E}(x)$ with variance $\mathbb{E}((x-\bar{x})^2) = P_x$. We have $x = \mathcal{N}(0, P_x)$ where $P_x = diag(\Sigma_n, \Sigma_q, \Sigma_\theta)$, and $diag$ returns a block-diagonal matrix with given matrices on the diagonal. We calculate $2l+1$ sigma points with l being the length of x. The Sigma points are defined as in Haykin (2001)

$$\mathcal{X}_0 = \mathbb{E}x,$$

$$\mathcal{X}_i = \begin{cases} \mathbb{E}x + [\sqrt{(l+\lambda)P_x}]_i & i = 1, \cdots, l \\ \mathbb{E}x - [\sqrt{(l+\lambda)P_x}]_i & i = l+1, \cdots, 2l \end{cases} \tag{13}$$

where $\lambda = \rho^2(l+\kappa) - l$ is a scaling parameter and κ is assumed zero in this work, ρ determines the spread of sigma points around $\bar{x}$ and $\beta = 2$ can be modified for prior assumption of distribution of x. Furthermore, $[M]_i$ is the i^{th} column of a matrix M and $\sqrt{.}$ of a matrix is calculated using Cholesky factorization.

Moreover, we define the following one dimensional weight variables

$$\mathcal{W}_0^{(m)} = \lambda/(\lambda + l), \tag{14}$$

$$\mathcal{W}_0^{(c)} = \mathcal{W}_0^{(m)} + (1 + \rho^2 + \beta),$$

$$\mathcal{W}_i^{(m)} = \mathcal{W}_i^{(c)} = \frac{1}{2\lambda + 2l}, \quad i = 1, \cdots, 2l.$$

We may separate the variable $\mathcal{X}_i = [\mathcal{X}_i^n, \mathcal{X}_i^q, \mathcal{X}_i^\theta]$, with $\mathcal{X}_i^n$,$\mathcal{X}_i^q$ and $\mathcal{X}_i^\theta$ having respectively the same length as n, q and θ.

We define the 1-step ahead predictor of the sigma points by $\mathcal{X}_i^n(k+1) = f(\mathcal{X}_i^n(k), u(k), \mathcal{X}_i^q, \mathcal{X}_i^\theta)$, 2-step predictor by $\mathcal{X}_i^n(k+2) = f(\mathcal{X}_i^n(k+1), u(k), \mathcal{X}_i^q, \mathcal{X}_i^\theta)$, etc. Finally, we arrive at the expectation

$$\mathbb{E}G_i(\mathcal{X}_i^n(k)) = \sum_{i=0}^{2l} \mathcal{W}_i^{(m)} G_i(\mathcal{X}_i^n(k)) \tag{15}$$

and the estimated variance by

$$Var(G_i) = \sum_{i=0}^{2l} \mathcal{W}_i^{(c)} (G_i(\mathcal{X}_i^n(k)) - \mathbb{E}G_i(\mathcal{X}_i^n(k))) \\ (G_i(\mathcal{X}_i^n(k)) - \mathbb{E}G_i(\mathcal{X}_i^n(k)))^T \tag{16}$$

Consequently, we have proposed a transformed MPC problem that was based on solving probabilistic optimization problem defined in infinite dimensional probability space. The transformed problem is deterministic and defined on $2l+1$ times the original dimension of MPC problem.

6. SIMULATION RESULTS

In this section, we provide the simulation results of the proposed method for optimal distribution control of two regions urban traffic control. The area under consideration is down-town San Francisco. The Simulation of Urban Mobility (SUMO) environment is used for micro-simulation. The region is divided into two parts: a central part that

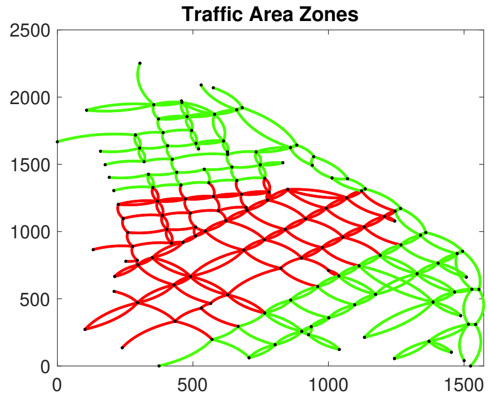

Fig. 2. San Francisco downtown is used for macroscopic modeling with a scale of 1km.

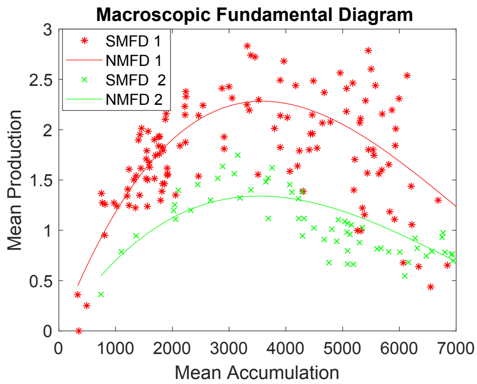

Fig. 3. Stochastic and Nominal MFD of the two regions are shown.

attracts more vehicles and the outer region that less demand is observed. The area is shown in Fig. 2. The mico-simulation is used to arrive at the MFDs of the central and suburb region. Notice that the MFD is only dependent on the configuration of the network and it is not dependent on the demand. By simulating the network, we arrive at two different MFDs as shown in Fig. 3.

We assume at initial time $n_{11} = 2500$, $n_{12} = 2000$, $n_{21} = 1800$, $n_{22} = 2500$. As a result, the initial accumulation of two regions are $n_1 = n_{11} + n_{12} = 4500$ and $n_2 = n_{21} + n_{22} = 4300$. It can be seen from the MFD of Fig. 3 that in both regions, we are slightly congested initially. Noticing that perimeter control corresponds to reducing average allowed flow passing the boundary of regions, we assume $u_{lb} = 0.1, u_{ub} = 0.9$. Moreover, we assume $\epsilon = 0.05$. Model predictive control parameters are assumed as Control Horizon of $H_c = 3$, prediction horizon $N = H_p = 50$. For Stochastic MPC, P_x is considered as identity for n and q for θ, we consider $\xi = (\mathcal{N}(0,1)[2.69e{-}8; 3.01e{-}4; -0.416])$

Accumulation of the two regions for later times is driven by demand. Deterministic demand is defined to model the number of vehicles that request a trip in the specified duration of time. However true demand does not follow deterministic curve and the uncertain demand is further considered for simulation study as shown in Fig. 4.

We simulate the model predictive controller and stochastic model predictive control for various conditions. First, the nominal model that matches exactly the MPC model is

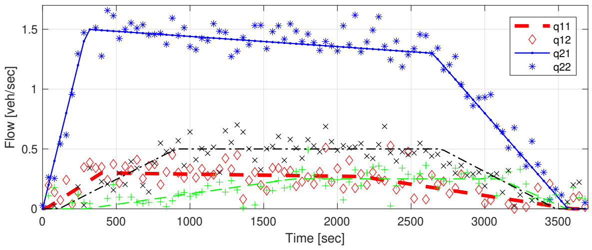

Fig. 4. Regional traffic demands are assumed uncertain to describe the stochastic nature of drivers.

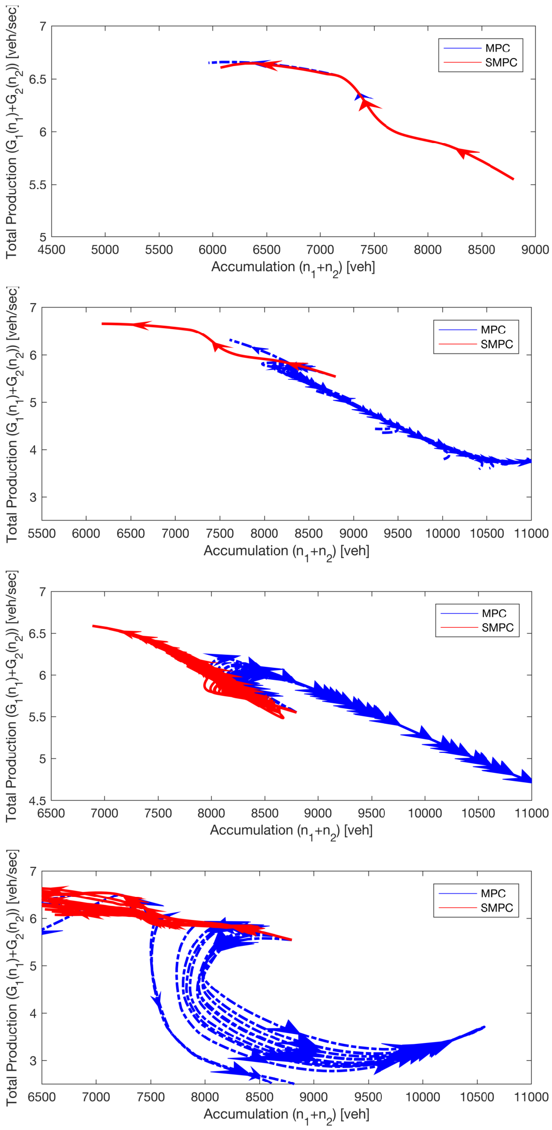

Fig. 5. Comparison of Total Production for nominal model (top) and uncertainty in demand (second), uncertainty in the MFD (third) and uncertainty in accumulation (bottom). The arrows show direction of time.

used in simulation. Second, we consider the known demand is disturbed with the uncertain variable given by the distribution, and the nominal controller is not consistent with the assumed demand profile. The simulation study continues with consideration of not knowing the exact MFD. The perturbed MFD is assumed given by a distribution. Finally, accumulation measurement uncertainty is considered.

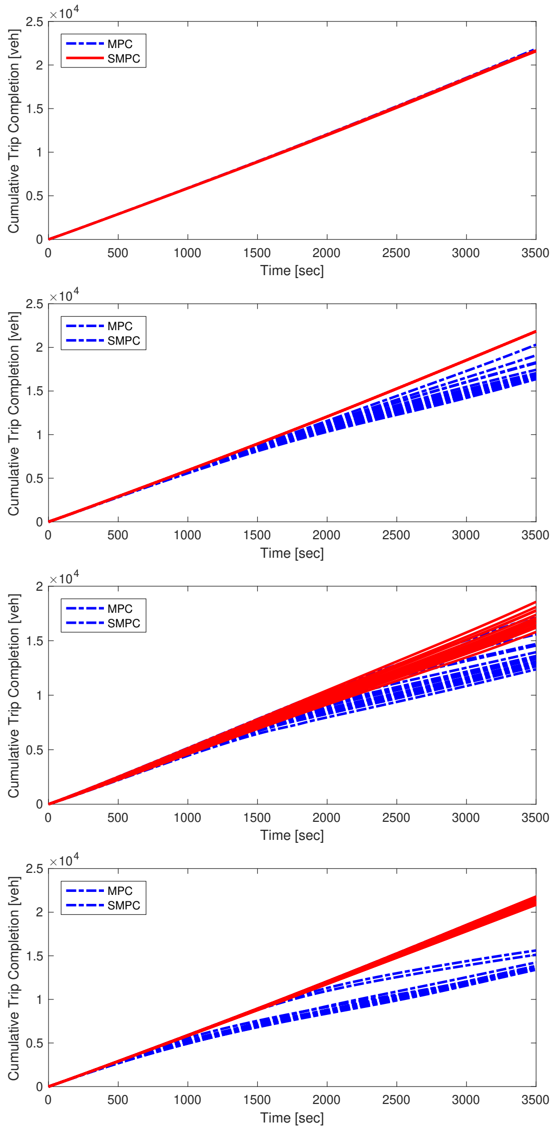

Fig. 6. Comparison of Cumulative Trip Completion for nominal model (top) and uncertainty in demand (second), uncertainty in the MFD (third) and uncertainty in accumulation (bottom).

One measure to compare the performance of a controller is total production versus accumulation curve. Total production rate describes the rate at which vehicle reach destination and accumulation measures the number of vehicles inside a region given the demand and initial conditions of both controllers are the same. Fig. 5 shows the performance of both controllers in different conditions. Moreover, the trip completion rate can be summed cumulatively by $CTR(k) = \sum_{\tau=0}^{k} \sum_{i \in \mathcal{R}} G_i(n_i(\tau))$. The cumulative trip completion rates of different scenarios are shown in Fig. 6. It can be observed that the SMPC arrives at similar results to nominal MPC controller for exact matching condition of controller and simulation model. One the other hand, mismatch in the model results in congestion for nominal MPC. On the other hand, SMPC can robustly counteract the mismatch and arrive at higher trip completion rate.

Simulation results shows that the nominal MPC is unable to counteract the model mismatch and arrives at heavily congested regions. It can be observed accordingly that SMPC outperforms nominal MPC by higher trip completion rates. Cumulative trip completion rate is higher in SMPC in all cases and the simulation will result in less congested regions.

7. CONCLUSION

Stochastic predictive perimeter control was introduced for uncertain two-regions urban traffic. Optimal trip completion rate was previously achieved by using model predictive perimeter control, however, the controller was very sensitive to model imperfections. Stochastic predictive perimeter control is robust to uncertainty in Macroscopic Fundamental Diagram, traffic demand and traffic accumulation in different regions. Simulation results show that the cumulative trip completion rate is almost similar for the nominal case. Moreover, in the presence of uncertainty, the proposed method increases the performance considerably. It is suggested to consider estimation techniques to describe the uncertainty more accurately. Moreover, probabilistic constraints is left for future study.

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Appendix A. MPC: LINEAR PROGRAMMING FORMULATION

In the following, we derive the MPC formulation by considering the state space as in 1 and Ψ, Θ given in Harati (2011). The MPC optimization problem maximizes the following objective function for all regions as

$$J = \sum_{\tau=k}^{k+H} \sum_{i\in\mathcal{R}} L_i y_i(n_i(\tau)). \tag{A.1}$$

The problem can be formulated as a linear programming problem as follows

$$\begin{aligned} J =& T_s[L_i \cdots L_i][y_i(n_i(k+1)) \cdots y_i(n_i(k+H))]^T = \\ & S\Psi n(k) + R^T \Theta U(k) \end{aligned} \tag{A.2}$$

where L_i is the average trip length for region i, $S = T_s L_i[1 \cdots 1]$ and $R = T_s \bar{L}_i[1 \cdots 1]$.