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Robust adaptive control of voltage-type MLS based on wavelet neural network ⋆

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Abstract: In this paper, a novel robust adaptive controller based on wavelet neural network is proposed for voltage-type magnetic levitation system with unknown control direction. First, coordinate transformation is used to simplify the model of the voltage-type magnetic levitation system. Second, the mean value theorem and Nussbaum function are used to deal with the problem of the implicit control input of nonlinear function and unknown control direction, the approximation to the unknown nonlinear function is realized by wavelet neural network. Finally, all the signals of the closed loop system are bounded according to Lyapunov function method.Brdys:99

Keywords: Magnetic levitation system (MLS), wavelet neural network, backstepping, Nussbaum function, robust adaptive control, Lyapunov function.

1. INTRODUCTION

The electromechanical dynamics of magnetic levitation device is generally represented by a nonlinear model composed of position, velocity and coil current signal. Because the system has open-loop instability and non-linearity in electromechanical system, a series of magnetic levitation controllers are proposed. In Rote (2002), in order to improve the robustness of feedback linearization control design, a sliding-mode controller was introduced. However, under the condition of parameter uncertainty, the stability and control performance of the system were not clearly analyzed. In Hajjaji (2001), a robust feedback linearization controller suitable for magnetic levitation system was proposed. Although the stability of the control system was guaranteed in theory, there seemed to be large overshoots and transient oscillation in the experimental results.

Recent years, people have developed great interest in using RBF neural network Yang (2001) and RNN neural network Lin (2007) to deal with magnetic levitation system. However, the second-order system of magnetic levitation system was generally considered, which has limitations in practical application and low approximation accuracy. As is known to all, fuzzy neural networks can approximate any continuous function and are widely used in electrical systems Cao (2006), Brdys (1999). However, FNN is a static mapping and cannot represent a dynamic mapping without staggered delay.

In this study, a nonlinear dynamic model is first used to represent the magnetic levitation device, and the model is simplified by coordinate transformation. The velocity signal can be obtained in the form of pseudo differential, which can make the state of the whole magnetic levitation system available. Backstepping design method is applied to divide the system into three subsystems: position error signal subsystem, velocity error subsystem and acceleration error subsystem. Then the mean value theorem and Nussbaum function are used to deal with the problem of the implicit control input of nonlinear function and unknown control direction. The approximation to the unknown nonlinear function and the design of the adaptive controller are realized by wavelet neural network and Lyapunov function respectively. Finally, the effectiveness of the proposed robust adaptive controller based on wavelet neural network is verified by simulation.

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2. MODEL OF THE MAGNETIC LEVITATION SYSTEM

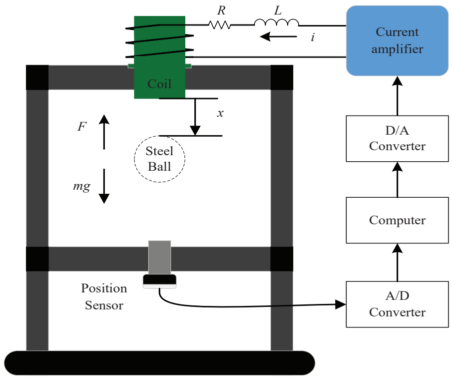

Fig. 1. Schematic diagram of magnetic levitation system

Considering the magnetic levitation system in Fig.1, the dynamic equation of MLS is Cho (1993), Wong (1986)

$$\begin{aligned}\frac{dx}{dt} &= v\\ \frac{d(L(x)i)}{dt} &= -Ri + e\\ m\frac{dv}{dt} &= mg - \frac{Qi^2}{2x^2}\end{aligned} \tag{1}$$

where x is the position of the steel ball, v is the velocity of the steel ball, i is the coil current, e is the applied voltage, R is the coil resistance, $L(x)$ is the coil inductance, g is gravity acceleration, m is the mass of the steel ball, Q is the positive constant determined by the characteristics of the magnetic core. In (1), it is shown that the inductance $L(x)$ is a nonlinear function about the position x of the steel ball Wong (1986), where

$$L(x) = L_1 + \frac{L_0 x_0}{x} \tag{2}$$

L_1 is the static inductance when the steel ball is not in the electromagnetic field, L_0 is the increased inductance in the coil when the steel ball is in the electromagnetic field (that is, the increased inductance when the air gap is zero), and x_0 is the arbitrary reference position relative to the inductance coil. Substituting (2) into the second equation of (1), we have

$$e = Ri + L\frac{di}{dt} - (\frac{L_0 x_0}{x^2} i)\frac{dx}{dt} \tag{3}$$

Thus, (3) is rewritten as

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{L_0 x_0}{L}\frac{vi}{x^2} + \frac{1}{L}e \tag{4}$$

The conservation of energy argument shows that $Q = L_0 x_0$. Combining with $\boldsymbol{x} = [x_1, x_2, x_3]^{\mathrm{T}} = [x, v, i]^{\mathrm{T}}$, as the system state variables and $u = e$, as the control input, the state equations of MLS in state-space form is

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= g - \frac{Q}{2m}\frac{x_3^2}{x_1^2}\\ \dot{x}_3 &= -\frac{R}{L}x_3 + \frac{Q}{L}\frac{x_3}{x_1^2}x_2 + \frac{1}{L}u\end{aligned} \tag{5}$$

In order to convert the original nonlinear system into a simpler system, the following nonlinear coordinate transformation is adopted in the sense of more direct controller synthesis Yang (2008), Yang (2007).

$$\boldsymbol{\xi} = [\xi_1, \xi_2, \xi_3]^{\mathrm{T}} = [x_1, x_2, \dot{x}_2]^{\mathrm{T}} \tag{6}$$

Note that $\boldsymbol{\xi}$ is only in the feasible region $\Omega_x = \{\boldsymbol{x} | 0 \le x_1 \le x_{1\mathrm{M}}\}$, regardless of the control strategy. The result of (6) is the generation of the new state variables ξ_1, ξ_2 and ξ_3 which are the position, velocity and acceleration of the steel ball, respectively. Because

$$\dot{\xi}_3 = -\frac{Q}{m}(-\frac{R}{L}\frac{x_3^2}{x_1^2} + \frac{Q}{L}\frac{x_3^2 x_2}{x_1^4} - \frac{x_3^2 x_2}{x_1^3}) - \frac{Q}{mL}\frac{x_3}{x_1^2}u \tag{7}$$

Thus, system model (5) is converted to

$$\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= \xi_3\\ \dot{\xi}_3 &= \varphi(\boldsymbol{x}, u)\end{aligned} \tag{8}$$

where

$$\varphi(\boldsymbol{x}, u) = -\frac{Q}{m}(-\frac{R}{L}\frac{x_3^2}{x_1^2} + \frac{Q}{L}\frac{x_3^2 x_2}{x_1^4} - \frac{x_3^2 x_2}{x_1^3}) - \frac{Q}{mL}\frac{x_3}{x_1^2}u \tag{9}$$

Assumption 1: The reference trajectory y_r is designed to be smooth enough, $y_r^{(i)} \in R$, $\forall i = 0, 1, \cdots, 3$ exists and is bounded, and is within the effective induction range of the laser sensor.

3. THE CONTROLLER DESIGN

The control object is to design an effective and reliable wavelet neural network controller for voltage-type magnetic levitation system with external interference and model mismatch, so that the system output position signal ξ_1 can effectively track the reference trajectory y_r, and ensure that all signals of the closed-loop system are bounded. The detailed design process is given below.

Step1: Design S_1 subsystem.

Define position error signal and velocity error signal respectively, and introduce backstepping design

$$z_1 = \xi_1 - y_r \tag{10}$$

$$z_2 = \xi_2 - \alpha_1 \tag{11}$$

where α_1 is the virtual control to be designed to stabilize z_1 and then subsystem S_1 is written as

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{y}_r \tag{12}$$

The virtual control input α_1 is designed as

$$\alpha_1 = -k_1 z_1 + \dot{y}_r \tag{13}$$

where $k_1 > 0$. Notice that

$$\dot{\alpha}_1 = -k_1(\xi_2 - \dot{y}_r) + \ddot{y}_r \tag{14}$$

Applying α_1 to (12), we get

$$\dot{z}_1 = z_2 - k_1 z_1 \tag{15}$$

Define the Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 \tag{16}$$

The derivative of V_1 along (12) is

$$\dot{V}_1 = -k_1 z_1^2 + z_1 z_2 \tag{17}$$

If $z_1 z_2$ is eliminated in the next design, subsystem S_1 will be stable.

Step2: Design S_2 subsystem.

Define acceleration error

$$z_3 = \xi_3 - \alpha_2 \tag{18}$$

where α_2 is the virtual control input to stabilize z_2. And then we get subsystem S_2:

$$\dot{z}_2 = z_3 + \alpha_2 - \dot{\alpha}_1 \tag{19}$$

The virtual control input α_2 is taken as

$$\alpha_2 = -k_2 z_2 - z_1 + \dot{\alpha}_1 \tag{20}$$

where $k_2 > 0$. Notice that

$$\begin{aligned}\dot{\alpha}_2 = &-(k_1k_2+1)\xi_2 - (k_1+k_2)\xi_3 + (k_1k_2+1)\dot{y}_r \\ &+(k_1+k_2)\ddot{y}_r + \dddot{y}_r\end{aligned} \tag{21}$$

Applying α_2 to (19), we have

$$\dot{z}_2 = z_3 - k_2 z_2 - z_1 \tag{22}$$

Define the Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{23}$$

The derivative of V_2 along (19) is

$$\dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 + z_2 z_3 \tag{24}$$

If $z_2 z_3$ is eliminated in the next design, subsystem S_2 will be stable.

Step3: Obtain the actual control law u.

The dynamic equation of acceleration subsystem S_3 is

$$\dot{z}_3 = \varphi(\boldsymbol{x}, u) - \dot{\alpha}_2 \tag{25}$$

Assuming the sign of the current is unknown, according to the mean value theorem in Ge (2003), there exists a $u^* \in (0, u)$ such that

$$\varphi(\boldsymbol{x}, u) = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x}, u^*)u \tag{26}$$

where $\alpha(\boldsymbol{x}) = \varphi(\boldsymbol{x}, 0)$, $\beta(\boldsymbol{x}, u^*) = \frac{\partial \varphi(\boldsymbol{x},u)}{\partial u}|_{u=u^*}$. In actual control, $\alpha(\boldsymbol{x})$ and $\beta(\boldsymbol{x}, u^*)$ are unknown functions, and the sign of $\beta(\boldsymbol{x}, u^*)$ is unknown. Then

$$\dot{z}_3 = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x}, u^*)u - \dot{\alpha}_2 \tag{27}$$

Assumption 2: $\beta(\boldsymbol{x}, u^*)$ is a smooth bounded function whose sign is unknown, and there exists unknown constants $\beta_{\min}$ and $\beta_{\max}$, so that

$$0 < \beta_{\min} \le |\beta(\boldsymbol{x}, u^*)| \le \beta_{\max} \tag{28}$$

The unknown nonlinear function $\alpha(\boldsymbol{x})$ is approximated by wavelet neural network as

$$\alpha(\boldsymbol{x}) = W_F^{*\mathrm{T}} \phi(\boldsymbol{x}, d^*, c^*) + \varepsilon_F \tag{29}$$

where W_F^*, d^* and c^* are the ideal weight vector, ideal extension parameter vector and ideal translation parameter vector of the wavelet neural network, respectively. ε_F is the approximate error of wavelet neural network, which satisfies $\|\varepsilon_F\| \le \|\varepsilon_{FM}\|$. Since W_F^*, d^* and c^* are hard to get, then we use $\hat{W}_F$, $\hat{d}$ and $\hat{c}$ to estimate, respectively. Define estimation error $\tilde{W}_F = W_F^* - \hat{W}_F$, $\tilde{d} = d^* - \hat{d}$, $\tilde{c} = c^* - \hat{c}$, $\tilde{\phi} = \phi(\boldsymbol{x}, d^*, c^*) - \phi(\boldsymbol{x}, \hat{d}, \hat{c})$, abbreviated as $\tilde{\phi} = \phi^* - \hat{\phi}$. Then the actual output of wavelet neural network is

$$\hat{\alpha}(\boldsymbol{x}) = \hat{W}_F^{\mathrm{T}} \phi(\boldsymbol{x}, \hat{d}, \hat{c}) \tag{30}$$

Assumption 3: The ideal weight vector W_F^*, ideal extension parameter d^* and ideal translation parameter vector c^* of wavelet neural network are bounded, that is, $\|W_F^*\| \le W_{FM}$, $\|d^*\| \le d_M$ and $\|c^*\| \le c_M$, where W_{FM}, d_M and c_M are positive constants.

The error of approximate nonlinear function $\alpha(\boldsymbol{x})$ by wavelet neural network is

$$\begin{aligned}\tilde{\alpha}(\boldsymbol{x}) &= \alpha(\boldsymbol{x}) - \hat{\alpha}(\boldsymbol{x}) \\ &= W_F^{*\mathrm{T}} \phi(\boldsymbol{x}, d^*, c^*) + \varepsilon_F - \hat{W}_F^{\mathrm{T}} \phi(\boldsymbol{x}, \hat{d}, \hat{c}) \\ &= (\tilde{W}_F + \hat{W}_F)^{\mathrm{T}}(\tilde{\phi} + \hat{\phi}) - \hat{W}_F^{\mathrm{T}}\hat{\phi} + \varepsilon_F \\ &= \tilde{W}_F^{\mathrm{T}}\tilde{\phi} + \tilde{W}_F^{\mathrm{T}}\hat{\phi} + \hat{W}_F^{\mathrm{T}}\tilde{\phi} + \hat{W}_F^{\mathrm{T}}\hat{\phi} - \hat{W}_F^{\mathrm{T}}\hat{\phi} + \varepsilon_F \\ &= \tilde{W}_F^{\mathrm{T}}\tilde{\phi} + \tilde{W}_F^{\mathrm{T}}\hat{\phi} + \hat{W}_F^{\mathrm{T}}\tilde{\phi} + \varepsilon_F\end{aligned} \tag{31}$$

According to Taylor linearization expansion method, $\tilde{\phi}$ is linearly expanded as

$$\tilde{\phi} = I^{\mathrm{T}}\tilde{d} + J^{\mathrm{T}}\tilde{c} + \mu \tag{32}$$

where μ is a higher-order term,

$$\begin{aligned}I &= [\partial\phi_1/\partial d, \cdots, \partial\phi_n/\partial d]|_{d=\hat{d}} \\ J &= [\partial\phi_1/\partial c, \cdots, \partial\phi_n/\partial c]|_{c=\hat{c}}\end{aligned}$$

We have the following assumptions.

Assumption 4: Higher-order term μ, matrices I and J are bounded, that is, $\|\mu\| \le \mu_M$, $\|I\| \le I_M$ and $\|J\| \le J_M$, where μ_M, I_M and J_M are positive constants.

Substituting (32) into (31), we obtain

$$\tilde{\alpha}(\boldsymbol{x}) = \tilde{W}_F^{\mathrm{T}}(\hat{\phi} - I^{\mathrm{T}}\hat{d} - J^{\mathrm{T}}\hat{c}) + \tilde{d}^{\mathrm{T}} I \hat{W}_F + \tilde{c}^{\mathrm{T}} J \hat{W}_F + \triangle \tag{33}$$

where

$$\begin{aligned}\triangle &= \tilde{W}_F^{\mathrm{T}} I^{\mathrm{T}} d^* + \tilde{W}_F^{\mathrm{T}} J^{\mathrm{T}} c^* + W_F^{*\mathrm{T}}\mu + \varepsilon_F \\ &= (W_F^{*\mathrm{T}} - \hat{W}_F^{\mathrm{T}}) I^{\mathrm{T}} d^* + (W_F^{*\mathrm{T}} - \hat{W}_F^{\mathrm{T}}) J^{\mathrm{T}} c^* \\ &\quad + W_F^{*\mathrm{T}}\mu + \varepsilon_F \\ &= W_F^{*\mathrm{T}}(I^{\mathrm{T}} d^* + J^{\mathrm{T}} c^* + \mu) - \hat{W}_F^{\mathrm{T}}(I^{\mathrm{T}} d^* + J^{\mathrm{T}} c^*) \\ &\quad + \varepsilon_F \\ &\le W_F^{*\mathrm{T}}(I^{\mathrm{T}} d^* + J^{\mathrm{T}} c^* + \mu) + (I^{\mathrm{T}} d^* + J^{\mathrm{T}} c^*) + \varepsilon_F \\ &\le (1 + \|\hat{W}_F^{\mathrm{T}}\|)[W_F^{*\mathrm{T}}(I^{\mathrm{T}} d^* + J^{\mathrm{T}} c^* + \mu) + (I^{\mathrm{T}} d^* \\ &\quad + J^{\mathrm{T}} c^*) + \varepsilon_F] \\ &\le \zeta\sigma\end{aligned} \tag{34}$$

where

$$\begin{aligned}\zeta &= 1 + \|\hat{W}_F^{\mathrm{T}}\| \\ \sigma &= \max\{W_{FM}^{\mathrm{T}}(I_M^{\mathrm{T}} d_M + J_M^{\mathrm{T}} c_M + \mu_M) + \varepsilon_{FM}, \\ &\quad I_M^{\mathrm{T}} d_M + J_M^{\mathrm{T}} c_M\}\end{aligned}$$

For the nonlinear function $\beta(\boldsymbol{x}, u^*)$ with unknown sign, Nussbaum function is introduced, and the controller is designed as

$$u = N_b(\psi)(\hat{\alpha}(\boldsymbol{x}) - \dot{\alpha}_2 + z_2 + k_3 z_3 + u_r) \tag{35}$$

$$\dot{\psi} = z_3(\hat{\alpha}(\boldsymbol{x}) - \dot{\alpha}_2 + z_2 + k_3 z_3 + u_r) \tag{36}$$

where $k_3 > 0$, $N_b(\psi) = \psi^2\cos(\psi)$ is the Nussbaum function. $u_r = \sigma\zeta\tanh(z_3\zeta/\iota)$ is the robust term, where $\iota > 0$. Then

$$\dot{z}_3 = \alpha(\boldsymbol{x}) + \beta(\boldsymbol{x}, u^*)N_b(\psi)(\dot{\hat{\alpha}}(\boldsymbol{x}) - \dot{\alpha}_2 + z_2 + k_3 z_3 + u_r) - \dot{\alpha}_2 \quad (37)$$

Define the Lyapunov function

$$V_3 = V_2 + \frac{1}{2}z_3^2 + \frac{1}{2\eta}\tilde{W}_F^{\mathrm{T}}\tilde{W}_F + \frac{1}{2\eta}\tilde{d}^{\mathrm{T}}\tilde{d} + \frac{1}{2\eta}\tilde{c}^{\mathrm{T}}\tilde{c} \quad (38)$$

where $\eta > 0$. The derivative of (38) is

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + z_3\dot{z}_3 - \frac{1}{\eta}\tilde{W}_F^{\mathrm{T}}\dot{\hat{W}}_F - \frac{1}{\eta}\tilde{d}^{\mathrm{T}}\dot{\hat{d}} - \frac{1}{\eta}\tilde{c}^{\mathrm{T}}\dot{\hat{c}} \\ &= -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 + z_3(\alpha(\boldsymbol{x}) - \hat{\alpha}(\boldsymbol{x})) \\ &\quad +[\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi} - z_3u_r - \frac{1}{\eta}\tilde{W}_F^{\mathrm{T}}\dot{\hat{W}}_F \\ &\quad -\frac{1}{\eta}\tilde{d}^{\mathrm{T}}\dot{\hat{d}} - \frac{1}{\eta}\tilde{c}^{\mathrm{T}}\dot{\hat{c}} \end{aligned} \quad (39)$$

Substituting (33) into (39), we have

$$\begin{aligned}\dot{V}_3 &= -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 + [\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi} \\ &\quad +\frac{1}{\eta}\tilde{W}_F^{\mathrm{T}}[\eta(\hat{\phi} - I^{\mathrm{T}}\hat{d} - J^{\mathrm{T}}\hat{c})z_3 - \dot{\hat{W}}_F] \\ &\quad +\frac{1}{\eta}\tilde{d}^{\mathrm{T}}(\eta I\hat{W}_F z_3 - \dot{\hat{d}}) + \frac{1}{\eta}\tilde{c}^{\mathrm{T}}(\eta J\hat{W}_F z_3 - \dot{\hat{c}}) \\ &\quad +z_3\triangle - z_3u_r \end{aligned} \quad (40)$$

The adaptive laws of weight $\hat{W}_F$, learning expansion parameter $\hat{d}$ and translation parameter $\hat{c}$ of the wavelet neural network are respectively taken as

$$\dot{\hat{W}}_F = \eta[z_3(\hat{\phi} - I^{\mathrm{T}}\hat{d} - J^{\mathrm{T}}\hat{c}) - k_F\hat{W}_F] \quad (41)$$

$$\dot{\hat{d}} = \eta(z_3 I\hat{W}_F - k_D\hat{d}) \quad (42)$$

$$\dot{\hat{c}} = \eta(z_3 J\hat{W}_F - k_C\hat{c}) \quad (43)$$

where k_F, k_D and $k_C > 0$, substituting the adaptive law (41)-(43) into (40), we get

$$\begin{aligned}\dot{V}_3 &\le -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 + [\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi} \\ &\quad +k_F\tilde{W}_F^{\mathrm{T}}\hat{W}_F + k_D\tilde{d}^{\mathrm{T}}\hat{d} + k_C\tilde{c}^{\mathrm{T}}\hat{c} + \sigma|z_3\zeta| \\ &\quad -\sigma z_3\zeta\tanh(z_3\zeta/\iota) \end{aligned} \quad (44)$$

According to the inequality $0 \le |\zeta z_3| - \zeta z_3\tanh(\zeta z_3/\iota) \le \iota$, we get

$$\begin{aligned}\dot{V}_3 &\le -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 + [\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi} \\ &\quad +k_F\tilde{W}_F^{\mathrm{T}}\hat{W}_F + k_D\tilde{d}^{\mathrm{T}}\hat{d} + k_C\tilde{c}^{\mathrm{T}}\hat{c} + \sigma\iota \end{aligned} \quad (45)$$

Using inequality

$$\begin{aligned}\tilde{W}_F^{\mathrm{T}}\hat{W}_F &= \frac{1}{2}\{\tilde{W}_F^{\mathrm{T}}(W_F^* - \tilde{W}_F) + (W_F^* - \hat{W}_F)^{\mathrm{T}}\hat{W}_F\} \\ &= \frac{1}{2}\{\tilde{W}_F^{\mathrm{T}}W_F^* - \tilde{W}_F^{\mathrm{T}}\tilde{W}_F + W_F^{*\mathrm{T}}\hat{W}_F - \hat{W}_F^{\mathrm{T}}\hat{W}_F\} \\ &= \frac{1}{2}\{-\|\tilde{W}_F\|^2 + (W_F^* - \hat{W}_F)^{\mathrm{T}}W_F^* - \|\hat{W}_F\|^2 \\ &\quad +W_F^{*\mathrm{T}}\hat{W}_F\} \\ &= \frac{1}{2}\{-\|\tilde{W}_F\|^2 - \|\hat{W}_F\|^2 + \|W_F^*\|^2\} \\ &\le \frac{1}{2}\{-\|\tilde{W}_F\|^2 + \|W_F^*\|^2\} \end{aligned} \quad (46)$$

Similarly

$$\tilde{d}^{\mathrm{T}}\hat{d} \le \frac{1}{2}\{-\|\tilde{d}\|^2 + \|d^*\|^2\} \quad (47)$$

$$\tilde{c}^{\mathrm{T}}\hat{c} \le \frac{1}{2}\{-\|\tilde{c}\|^2 + \|c^*\|^2\} \quad (48)$$

Then

$$\begin{aligned}\dot{V}_3 &\le -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 + [\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi} \\ &\quad -\frac{k_F}{2}\|\tilde{W}_F\|^2 - \frac{k_D}{2}\|\tilde{d}\|^2 - \frac{k_C}{2}\|\tilde{c}\|^2 + \chi_2 \end{aligned} \quad (49)$$

where

$$\chi_2 = \frac{k_F}{2}\|W_F^*\|^2 + \frac{k_D}{2}\|d^*\|^2 + \frac{k_C}{2}\|c^*\|^2 + \sigma\iota \quad (50)$$

Define $\chi_1 = \min\{2k_1, 2k_2, 2k_3, \eta k_F, \eta k_D, \eta k_C\}$, then

$$\dot{V}_3 \le -\chi_1V_3 + [\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi} + \chi_2 \quad (51)$$

Multiplying both sides of (51) by $e^{\chi_1 t}$, we get

$$\frac{d(V_3e^{\chi_1 t})}{dt} \le e^{\chi_1 t}\{[\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi} + \chi_2\} \quad (52)$$

Integrating (52) over $[0, t]$, we have

$$\begin{aligned}V_3(t) &\le (V_3(0) - \frac{\chi_2}{\chi_1})e^{-\chi_1 t} + \frac{\chi_2}{\chi_1} \\ &\quad + \int_0^t e^{-\chi_1(t-\tau)}\{[\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi}\}d\tau \end{aligned} \quad (53)$$

Because $0 < e^{-\chi_1 t} < 1$ and $\frac{\chi_2}{\chi_1}e^{-\chi_1 t} > 0$, so

$$(V_3(0) - \frac{\chi_2}{\chi_1})e^{-\chi_1 t} < V_3(0) \quad (54)$$

If substituting (54) into (53), we get

$$\begin{aligned}V_3(t) &\le \chi_3 \\ &\quad + \int_0^t e^{-\chi_1(t-\tau)}\{[\beta(\boldsymbol{x}, u^*)N_b(\psi) + 1]\dot{\psi}\}d\tau \end{aligned} \quad (55)$$

where $\chi_3 = V_3(0) + \frac{\chi_2}{\chi_1}$.

4. STABILITY ANALYSIS

Due to the continuity of $y_r(t), \cdots, y_r^{(3)}(t)$ and the smooth non-linearity of the system (8), the solution of the closed-loop adaptive system exists and is unique. Define an alternative item of Lyapunov function:

$$V = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2) + \frac{1}{2\eta}\tilde{W}_F^{\mathrm{T}}\tilde{W}_F + \frac{1}{2\eta}\tilde{d}^{\mathrm{T}}\tilde{d} + \frac{1}{2\eta}\tilde{c}^{\mathrm{T}}\tilde{c} \quad (56)$$

The time derivative of V is

$$\dot{V} = z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 - \frac{1}{\eta}\tilde{W}_F^{\mathrm{T}}\dot{\hat{W}}_F - \frac{1}{\eta}\tilde{d}^{\mathrm{T}}\dot{\hat{d}} - \frac{1}{\eta}\tilde{c}^{\mathrm{T}}\dot{\hat{c}} \quad (57)$$

Substituting (15), (22), (33)-(37) and (41)-(43) into (57), we get

$$\dot{V} \le -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 + \sigma|z_3\zeta| - \sigma z_3\zeta\tanh(z_3\zeta/\iota)$$

$$+[\beta(\boldsymbol{x},u^*)N_b+1]\dot{\psi}+k_F\tilde{W}_F^{\mathrm{T}}\hat{W}_F+k_D\tilde{d}^{\mathrm{T}}\hat{d}$$
$$+k_C\tilde{c}^{\mathrm{T}}\hat{c}$$

Using inequality (46)-(48) and $0 \leq |\zeta z_3| - \zeta z_3 \tanh(\zeta z_3/\iota) \leq \iota$, we get

$$\dot{V} \leq -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 + [\beta(\boldsymbol{x},u^*)N_b(\psi)+1]\dot{\psi} - \frac{k_F}{2}\|\tilde{W}_F\|^2 - \frac{k_D}{2}\|\tilde{d}\|^2 - \frac{k_C}{2}\|\tilde{c}\|^2 + \chi_2 \quad (58)$$

where

$$\chi_2 = \frac{k_F}{2}\|W_F^*\|^2 + \frac{k_D}{2}\|d^*\|^2 + \frac{k_C}{2}\|c^*\|^2 + \sigma\iota \quad (59)$$

Define $\chi_1 = \min\{2k_1, 2k_2, 2k_3, \eta k_F, \eta k_D, \eta k_C\}$, then

$$\dot{V} \leq -\chi_1 V + [\beta(\boldsymbol{x},u^*)N_b(\psi)+1]\dot{\psi} + \chi_2 \quad (60)$$

Multiplying both sides of (60) by $e^{\chi_1 t}$, we get

$$\frac{d(Ve^{\chi_1 t})}{dt} \leq e^{\chi_1 t}\{[\beta(\boldsymbol{x},u^*)N_b(\psi)+1]\dot{\psi} + \chi_2\} \quad (61)$$

Integrating (61) over $[0,t]$

$$V(t) \leq (V(0) - \frac{\chi_2}{\chi_1})e^{-\chi_1 t} + \frac{\chi_2}{\chi_1} + \int_0^t e^{-\chi_1(t-\tau)}\{[\beta(\boldsymbol{x},u^*)N_b(\psi)+1]\dot{\psi}\}d\tau \quad (62)$$

Because $0 < e^{-\chi_1 t} < 1$ and $\frac{\chi_2}{\chi_1}e^{-\chi_1 t} > 0$, so

$$(V(0) - \frac{\chi_2}{\chi_1})e^{-\chi_1 t} < V(0) \quad (63)$$

If substituting (63) into (62), we get

$$V(t) \leq \chi_3 + \int_0^t e^{-\chi_1(t-\tau)}\{[\beta(\boldsymbol{x},u^*)N_b(\psi)+1]\dot{\psi}\}d\tau \quad (64)$$

where $\chi_3 = V(0) + \frac{\chi_2}{\chi_1}$.

According to lemma 1 in Hong (2009), we can get that V, ψ and $\int_0^t \beta(\boldsymbol{x},u^*)N_b(\psi)\dot{\psi}d\tau$ are bounded on $[0,t_f]$. According to lemma 2 in Hong (2009), the above conclusion is also true when $t_f = \infty$. we know that $\tilde{W}_F$, $\tilde{d}$, $\tilde{c}$ and thus $z_i, i = 1,2,3$ are bounded. z_1 and y_r are bounded, so x_1 is bounded on $[0,\infty]$.

5. NUMERICAL SIMULATION

In order to demonstrate the performance of the proposed control design, a digital simulation is presented. MATLAB/Simulink is used to simulate the above MLS control design and verify its performance. The velocity ξ_2 is measured by pseudo-differentiation of the measured position ξ_1 as $s\xi_1/(0.004s+1)$ Yang (2001). The parameters of MLS in Yang (2007).

$$m = 0.54\text{kg} \quad g = 9.8\text{m/s}^2 \quad Q = 0.001624\text{Hm}$$
$$L = 0.8052\text{H} \quad R = 11.88\Omega$$

It is worth noting that the proposed controllers do not need too precise dynamic parameters or their optimal region

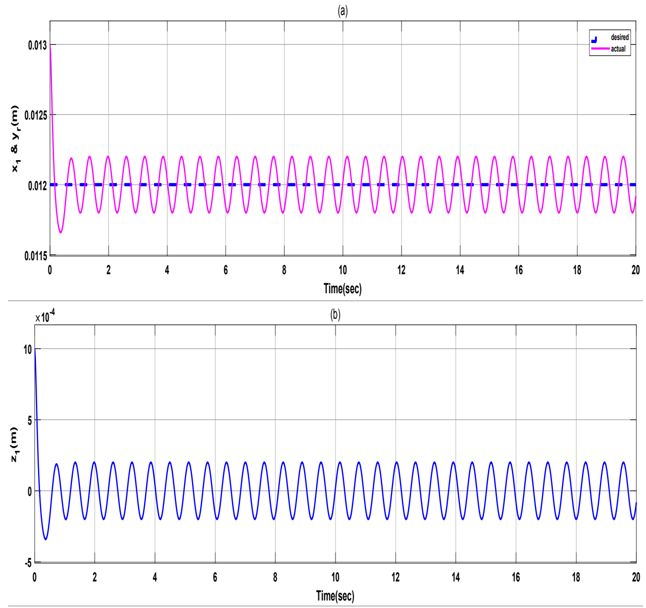

Fig. 2. The output and tracking error waveform after coordinate reconstruction.

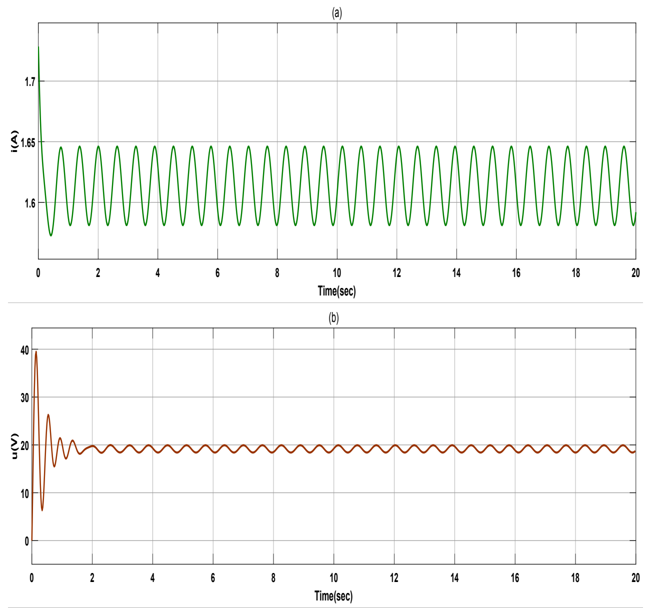

Fig. 3. The input current i and Controller u.

of interest. To illustrate the transient performance of the proposed control design, an active reference trajectory is used. In order to satisfy assumption 1, the third-order derivative of the reference trajectory is continuous. Here, we design the reference trajectory as $y_r = 0.012$m. In the study of magnetic levitation system, ξ_1 is the distance between the electromagnetic coil and a point on the surface of the small suspending steel ball. In practice, the distance between the electromagnetic coil and the centroid of the small suspending steel ball needs to be considered, so the centroid distance compensation needs to be carried out, and the compensating distance is 0.007987m. The initial position of the steel ball is set as 0.03m, and the initial velocity is set as 0.01m/s. Wavelet neural network is used to approximate function $\alpha(\boldsymbol{x})$, control input are x_1, x_2 and x_3, the number of product layers is $l = 10$, and the initial value of parameters $\hat{W}_F$, $\hat{d}$ and $\hat{c}$ are set as 0.25, respectively. The control gains of the proposed controller are chosen as below to obtain the best tracking performance

$$k_1 = 40 \quad k_2 = 40 \quad k_3 = 20 \quad \iota = 0.3$$
$$\eta = 100 \quad k_F = 0.3 \quad k_D = 0.3 \quad k_C = 0.3$$

When y_r is a fixed constant, the simulation results of the proposed controller are shown in Fig.2 and Fig.3. Fig.2 shows the actual trajectory (solid line) ξ_1, desired trajectory (dashed line) y_r, and tracking error z_1 under the control of the proposed control design. Fig.3 shows the waveform of input current i and controller u. Except the initial time, the current value i and voltage value u are uniformly adjusted in the range of 1.58A-1.65A and 18V-20V, respectively. The position ξ_1 of the steel ball fluctuates around the reference track y_r=0.012m, the tracking error z_1 approaches zero.

6. CONCLUSION

Aiming at the voltage-type magnetic levitation system with unknown control direction, this paper proposes a robust adaptive controller based on wavelet neural network by using coordinate transformation, backstepping, mean value theorem, Nussbaum function and adaptive boundary technology. Small changes in coil inductance will not be ignored since there is a functional relationship between coil inductance L and position state variable ξ_1. The unmeasurable velocity state ξ_2 is obtained by means of pseudo differentiation, which does not require prior knowledge of the control direction and can obtain good tracking performance on the premise that all signals of the closed-loop system are bounded.

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