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CONTROLLER DESIGN FOR LARGE-SCALE NEUTRAL TIME-DELAY SYSTEMS USING OVERLAPPING DECOMPOSITIONS *

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Abstract: Decentralized controller design for large-scale linear time-invariant (LTI) neutral time-delay systems, using the approach of overlapping decompositions, is presented. The approach is based on the principle of *restriction*, which is a special case of *inclusion*. The considered controllers have the most general form of LTI neutral time-delay controllers. However, they include LTI retarded time-delay, as well as finite-dimensional, controllers as special cases.

Keywords: Large-scale systems; time-delay systems; neutral systems; inclusion principle; overlapping decompositions; controller design; decentralized control; time-delay controllers.

1. INTRODUCTION

Many reasons such as communication or transportation delays may cause time-delays to appear in physical systems. Sometimes such time-delays may be ignored or approximated by finite-dimensional dynamics. However, when they are large, compared to the time-constant of the system, they must be considered explicitly (Loiseau et al. (2009)). Time-delay systems are in the class of infinite-dimensional systems, since their state can not be represented by finitely many state variables (Curtain and Zwart (1995)). For this reason, analysis of and controller design for time-delay systems are more difficult than for finite-dimensional systems. There are, in general, two types of time-delay systems: *retarded* and *neutral* (Niculescu (2001)). Although retarded systems have only finitely many modes in any given right-half complex-plane, neutral systems may have infinite chains of modes extending to infinity along vertical asymptotes (Michiels and Niculescu (2007)). For this reason, it is, in general, more difficult to deal with neutral systems, compared to retarded systems.

A common way to attack the problem of analysis of and/or controller design for a large-scale system is to first decompose such a system into smaller subsystems (Šiljak (1978)). However, many typical large-scale systems may have an *overlapping part* through which subsystems are interconnected (Šiljak (1991)). For such a system a disjoint decomposition may not be useful. The *overlapping decompositions* approach has first been proposed by Ikeda and Šiljak (1980) to deal with such systems. This approach, since then, has been used successfully to analyze and/or design controllers for such finite-dimensional systems (e.g., Ikeda et al. (1981); Ikeda and Šiljak (1986); Hodžić and Šiljak (1986); İftar and Özgüner (1987, 1990, 1998); Özgüner et al. (1997); Ataşlar and İftar (1999); Stanković et al. (2000); Aybar and İftar (2002)). Overlapping decompositions approach is based on the principle of *inclusion*. A special case of inclusion, which is especially useful in controller design is *restriction* (Ikeda et al. (1984)).

Both the inclusion principle and the overlapping decompositions approach have been widely considered for finite-dimensional systems (see the references in the previous paragraph). However, they have been extended to time-delay systems only recently (e.g., see Bakule et al. (2005a,b); Bakule and Rossell (2008); İftar (2008); Momeni and Aghdam (2009); İftar (2014, 2016, 2017, 2018)). Furthermore, most of these works (with the exception of İftar (2016, 2018), which deal with distributed time-delay) have been restricted to retarded time-delay systems. The inclusion principle and the approach of overlapping decompositions have been recently presented for the case of neutral time-delay systems with pointwise time-delays in İftar (2019). Specifically, the inclusion principle and two special cases of it, restriction and aggregation, were defined and how to obtain an expansion of a given overlappingly decoposed system such that the original system is either a restriction or an aggregation of the expanded system were presented in İftar (2019). However, as long as controller design using the overlapping decompositions approach is concerned, only static state vector feedback controllers were considered in İftar (2019).

In the present work, using the results of İftar (2019) (which are summarized in Section 2 below), we present the decentralized controller design approach using overlapping decompositions for linear time-invariant (LTI) neutral time-delay systems. The controllers we consider have the most general form of LTI neutral time-delay controllers. However, they also include LTI retarded time-delay, as well as finite-dimensional, controllers as special cases.

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Throughout the paper, for positive integers k and l, $\mathbf{R}^k$ and $\mathbf{R}^{k\times l}$ denote the spaces of, respectively, k-dimensional real vectors and $k\times l$-dimensional real matrices. I_k denotes the $k \times k$-dimensional identity matrix. 0 may denote either the scalar zero or the zero matrix of appropriate dimensions. rank$(\cdot)$ denotes the rank of $\cdot$. For a vector function x, $\dot{x}$ is the derivative of x.

2. BACKGROUND AND PRELIMINARIES

2.1 Inclusion and Restriction

Consider two LTI neutral time-delay systems, Σ:

$$E_0\dot{x}(t) + \sum_{\tau\in\mathcal{T}} E_\tau \dot{x}(t-\tau) = A_0 x(t) + B_0 u(t) + \sum_{\tau\in\mathcal{T}} \left(A_\tau x(t-\tau) + B_\tau u(t-\tau)\right) \tag{1}$$

$$y(t) = C_0 x(t) + \sum_{\tau\in\mathcal{T}} C_\tau x(t-\tau) \tag{2}$$

and $\hat{\Sigma}$:

$$\hat{E}_0\dot{\hat{x}}(t) + \sum_{\tau\in\hat{\mathcal{T}}} \hat{E}_\tau \dot{\hat{x}}(t-\tau) = \hat{A}_0 \hat{x}(t) + \hat{B}_0 \hat{u}(t) + \sum_{\tau\in\hat{\mathcal{T}}} \left(\hat{A}_\tau \hat{x}(t-\tau) + \hat{B}_\tau \hat{u}(t-\tau)\right) \tag{3}$$

$$\hat{y}(t) = \hat{C}_0 \hat{x}(t) + \sum_{\tau\in\hat{\mathcal{T}}} \hat{C}_\tau \hat{x}(t-\tau) \tag{4}$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^p$, and $y \in \mathbf{R}^q$ are, respectively, the state, the input, and the output vectors of Σ, and $\hat{x} \in \mathbf{R}^{\hat{n}}$, $\hat{u} \in \mathbf{R}^p$, and $\hat{y} \in \mathbf{R}^q$ are, respectively, the state, the input, and the output vectors of $\hat{\Sigma}$. It is assumed that Σ and $\hat{\Sigma}$ have the same number of inputs (p) and the same number of outputs (q); however, the state vector for $\hat{\Sigma}$ is larger dimensional than the state vector of Σ, i.e., $\hat{n} > n$. The sets $\mathcal{T}$ and $\hat{\mathcal{T}}$ include the time-delays, which are positive real numbers, of Σ and $\hat{\Sigma}$, respectively. The matrices E_τ, A_τ, B_τ, C_τ, for $\tau \in \{0\} \cup \mathcal{T}$, and $\hat{E}_\tau$, $\hat{A}_\tau$, $\hat{B}_\tau$, $\hat{C}_\tau$, for $\tau \in \{0\} \cup \hat{\mathcal{T}}$, are appropriately dimensioned constant real matrices. For non-triviality, it is assumed that, for any $\tau \in \mathcal{T}$, at least one of E_τ, A_τ, B_τ, or C_τ is non-zero, and, for any $\tau \in \hat{\mathcal{T}}$, at least one of $\hat{E}_\tau$, $\hat{A}_\tau$, $\hat{B}_\tau$, or $\hat{C}_\tau$ is non-zero. When rank$(E_0) = n$ (respectively, rank$(\hat{E}_0) = \hat{n}$) and $E_\tau = 0$, $\forall \tau \in \mathcal{T}$ (respectively, $\hat{E}_\tau = 0$, $\forall \tau \in \hat{\mathcal{T}}$), Σ (respectively, $\hat{\Sigma}$) reduces to a retarded system. Although in the sequel we assume that Σ and $\hat{\Sigma}$ are neutral systems, in general, our results continue to hold even when any one of these systems reduces to a retarded system. It is known that (Hale and Verduyn-Lunel (1993)) the existence and uniqueness (with given appropriate initial conditions) of solutions to (1) (respectively, to (3)) are guaranteed when rank$(E_0) = n$ (respectively, rank$(\hat{E}_0) = \hat{n}$). When rank$(E_0) < n$ (respectively, rank$(\hat{E}_0) < \hat{n}$), the system Σ (respectively, $\hat{\Sigma}$) is sometimes called a *descriptor-type* neutral system and some additional conditions (e.g., see (Erol and İftar (2016))) are required to guarantee the existence and uniqueness of solutions (which we assume to hold throughout the present paper). For some differentiable functions $\phi : [-\bar{\tau}, 0] \to \mathbf{R}^n$ and $\hat{\phi} : [-\hat{\tau}, 0] \to \mathbf{R}^{\hat{n}}$, where

$$\bar{\tau} := \max_{\tau\in\mathcal{T}}(\tau) \quad \text{and} \quad \hat{\tau} := \max_{\tau\in\hat{\mathcal{T}}}(\tau) , \tag{5}$$

the initial conditions for Σ and $\hat{\Sigma}$ are assumed to be given as:

$$x(\theta) = \phi(\theta) , \quad \theta \in [-\bar{\tau}, 0] \tag{6}$$

and

$$\hat{x}(\theta) = \hat{\phi}(\theta) , \quad \theta \in [-\hat{\tau}, 0] \tag{7}$$

respectively.

Inclusion can now be defined as in İftar (2019):

Definition 1: $\hat{\Sigma}$ is said to *include* Σ if there exists a full column-rank matrix $V \in \mathbf{R}^{\hat{n}\times n}$ and a full row-rank matrix $U \in \mathbf{R}^{n\times\hat{n}}$, satisfying $UV = I_n$, such that for all $\phi(\cdot)$ and for all $u(\cdot)$, the choice

$$\hat{\phi}(\theta) = V\phi(\theta) , \quad \theta \in [-\tilde{\tau}, 0] , \tag{8}$$

where $\tilde{\tau} := \min\{\bar{\tau}, \hat{\tau}\}$, and

$$\hat{u}(t) = u(t) , \quad t \geq -\tilde{\tau} \tag{9}$$

implies

$$x(t) = U\hat{x}(t) , \quad t \geq -\tilde{\tau} \tag{10}$$

and

$$y(t) = \hat{y}(t) , \quad t \geq 0 . \tag{11}$$

It has been shown by İftar (2019) that when $\hat{\Sigma}$ includes Σ,

(i) Σ and $\hat{\Sigma}$ have the same input-output map;
(ii) Σ is bounded-input bounded-output (BIBO) stable if and only if $\hat{\Sigma}$ is BIBO stable; and
(iii) if $\hat{\Sigma}$ is internally (Lyapunov, asymptotic, and/or exponential) stable, then Σ is internally stable.

Although internal stability of Σ does not necessarily imply the internal stability of $\hat{\Sigma}$, the important direction is the one given in (iii), since stabilizing controllers are to be first designed for the expanded system and then contracted for implementation on the original system (as to be presented below).

An important special case of inclusion is restriction.

Definition 2: Σ is said to be a *restriction* of $\hat{\Sigma}$ if there exists a full column-rank matrix $V \in \mathbf{R}^{\hat{n}\times n}$ such that for all $\phi(\cdot)$ and for all $u(\cdot)$, the choice (8) and (9) implies

$$\hat{x}(t) = Vx(t) , \quad t \geq -\tilde{\tau} \tag{12}$$

and (11).

It has been shown by İftar (2019) that if Σ is a restriction of $\hat{\Sigma}$ then $\hat{\Sigma}$ includes Σ. Furthermore, the necessary and sufficient conditions for restriction have also been derived by İftar (2019):

Theorem 1: Σ is a restriction of $\hat{\Sigma}$ if and only if there exist a full column-rank matrix $V \in \mathbf{R}^{\hat{n}\times n}$ such that

i) $\mathcal{T} \setminus \hat{\mathcal{T}} = \emptyset$,
ii) for any $\tau \in \hat{\mathcal{T}} \setminus \mathcal{T}$, $\hat{E}_\tau V = 0$, $\hat{A}_\tau V = 0$, $\hat{B}_\tau = 0$, and $\hat{C}_\tau V = 0$,

and

iii) for any $\tau \in \{0\} \cup (\mathcal{T} \cap \hat{\mathcal{T}})$, $\hat{E}_\tau V = V E_\tau$, $\hat{A}_\tau V = V A_\tau$, $\hat{B}_\tau = V B_\tau$, and $\hat{C}_\tau V = C_\tau$.

To facilitate the definition of the matrices of the expanded system in the overlapping decompositions approach, without any loss of generality, the matrices of Σ and $\hat{\Sigma}$ can be related as follows:

$$\hat{E}_\tau = V E_\tau U + R_\tau \qquad \hat{A}_\tau = V A_\tau U + M_\tau \tag{13}$$

$$\hat{B}_\tau = V B_\tau + N_\tau \qquad \hat{C}_\tau = C_\tau U + L_\tau \tag{14}$$

for all $\tau \in \{0\} \cup \mathcal{T} \cup \hat{\mathcal{T}}$, where V and U are as in Definition 1 and R_τ, M_τ, N_τ, and L_τ are appropriately dimensioned real matrices (commonly called as *complementary matrices*). For $\tau \notin \{0\} \cup \mathcal{T}$, E_τ, A_τ, B_τ, and C_τ in (13)–(14) are defined as appropriately dimensioned zero matrices (thus, in this case, $R_\tau = \hat{E}_\tau$, etc) and, for $\tau \notin \{0\} \cup \hat{\mathcal{T}}$, $\hat{E}_\tau$, $\hat{A}_\tau$, $\hat{B}_\tau$, and $\hat{C}_\tau$ in (13)–(14) are defined as appropriately dimensioned zero matrices (thus, in this case, $R_\tau = -V E_\tau U$, etc). The necessary and sufficient conditions for restriction can now be stated in terms of these complementary matrices (see İftar (2019) for the proof):

Corollary 1: Σ is a restriction of $\hat{\Sigma}$ if and only if $\mathcal{T} \subset \hat{\mathcal{T}}$ and there exist a full column-rank matrix $V \in \mathbf{R}^{\hat{n} \times n}$ and a full row-rank matrix $U \in \mathbf{R}^{n \times \hat{n}}$, satisfying $UV = I_n$, such that, for any $\tau \in \{0\} \cup \hat{\mathcal{T}}$, (13)–(14) are satisfied with complementary matrices satisfying $R_\tau V = 0$, $M_\tau V = 0$, $N_\tau = 0$, and $L_\tau V = 0$.

2.2 Overlapping Decompositions and Expansions

Large-scale systems may be composed of subsystems which may overlap in many different ways (see İftar (1993)). One such case is when two subsystems have a common dynamic part. Suppose that such a system is described by (1)–(2) and is denoted by Σ. The state, the input, and the output vectors of this system can then be decomposed as

$$x = \begin{bmatrix} x_1 \\ x_c \\ x_2 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \tag{15}$$

where $x_i \in \mathbf{R}^{n_i}$, $u_i \in \mathbf{R}^{p_i}$, and $y_i \in \mathbf{R}^{q_i}$ are, respectively, the state, the input, and the output vectors of the i^{th} subsystem, for $i = 1, 2$, and $x_c \in \mathbf{R}^{n_c}$ is the state vector of the overlapping dynamics. Let us partition the matrices of this system as follows:

$$E_\tau = \begin{bmatrix} E_\tau^{11} & E_\tau^{1c} & E_\tau^{12} \\ E_\tau^{c1} & E_\tau^{cc} & E_\tau^{c2} \\ E_\tau^{21} & E_\tau^{2c} & E_\tau^{22} \end{bmatrix},$$

$$A_\tau = \begin{bmatrix} A_\tau^{11} & A_\tau^{1c} & A_\tau^{12} \\ A_\tau^{c1} & A_\tau^{cc} & A_\tau^{c2} \\ A_\tau^{21} & A_\tau^{2c} & A_\tau^{22} \end{bmatrix}, \; B_\tau = \begin{bmatrix} B_\tau^{11} & B_\tau^{12} \\ B_\tau^{c1} & B_\tau^{c2} \\ B_\tau^{21} & B_\tau^{22} \end{bmatrix},$$

and

$$C_\tau = \begin{bmatrix} C_\tau^{11} & C_\tau^{1c} & C_\tau^{12} \\ C_\tau^{21} & C_\tau^{2c} & C_\tau^{22} \end{bmatrix}.$$

for all $\tau \in \{0\} \cup \mathcal{T}$, where the partitioning is compatible with that in (15).

Following İftar (2019), an *expansion* of this system, which is to be described by (3)–(4) and to be denoted by $\hat{\Sigma}$, can be obtained by choosing

$$V = \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & I_{n_c} & 0 \\ 0 & I_{n_c} & 0 \\ 0 & 0 & I_{n_2} \end{bmatrix}, \; U = \begin{bmatrix} I_{n_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2} I_{n_c} & \frac{1}{2} I_{n_c} & 0 \\ 0 & 0 & 0 & I_{n_2} \end{bmatrix}, \tag{16}$$

$\hat{\mathcal{T}} = \mathcal{T}$, and the matrices in (3)–(4) as in (13)–(14), with

$$R_\tau = \begin{bmatrix} 0 & \frac{1}{2}E_\tau^{1c} & -\frac{1}{2}E_\tau^{1c} & 0 \\ 0 & \frac{1}{2}E_\tau^{cc} & -\frac{1}{2}E_\tau^{cc} & 0 \\ 0 & -\frac{1}{2}E_\tau^{cc} & \frac{1}{2}E_\tau^{cc} & 0 \\ 0 & -\frac{1}{2}E_\tau^{2c} & \frac{1}{2}E_\tau^{2c} & 0 \end{bmatrix},$$

$$M_\tau = \begin{bmatrix} 0 & \frac{1}{2}A_\tau^{1c} & -\frac{1}{2}A_\tau^{1c} & 0 \\ 0 & \frac{1}{2}A_\tau^{cc} & -\frac{1}{2}A_\tau^{cc} & 0 \\ 0 & -\frac{1}{2}A_\tau^{cc} & \frac{1}{2}A_\tau^{cc} & 0 \\ 0 & -\frac{1}{2}A_\tau^{2c} & \frac{1}{2}A_\tau^{2c} & 0 \end{bmatrix}, \; N_\tau = 0$$

and

$$L_\tau = \begin{bmatrix} 0 & \frac{1}{2}C_\tau^{1c} & -\frac{1}{2}C_\tau^{1c} & 0 \\ 0 & -\frac{1}{2}C_\tau^{2c} & \frac{1}{2}C_\tau^{2c} & 0 \end{bmatrix},$$

for all $\tau \in \{0\} \cup \mathcal{T}$. These complementary matrices are chosen to facilitate the decomposition of $\hat{\Sigma}$ into two decoupled subsystems with minimal interaction between them (see İftar (2019)). We note that the above choices satisfy the conditions of Corollary 1; hence, the *original system* Σ is a restriction of the *expanded system* $\hat{\Sigma}$.

Once the expanded system $\hat{\Sigma}$ is obtained as above, as in İftar (2019), it can be decomposed into two disjoint subsystems: $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$, each of which is described as:

$$\hat{E}_0^i \dot{\hat{x}}_i(t) + \sum_{\tau \in \mathcal{T}} \hat{E}_\tau^i \dot{\hat{x}}_i(t-\tau) = \hat{A}_0^i \hat{x}_i(t) + \hat{B}_0^i \hat{u}_i(t) + \sum_{\tau \in \mathcal{T}} \Big(\hat{A}_\tau^i \hat{x}_i(t-\tau) + \hat{B}_\tau^i \hat{u}_i(t-\tau) \Big) \tag{17}$$

$$\hat{y}_i(t) = \hat{C}_0^i \hat{x}_i(t) + \sum_{\tau \in \mathcal{T}} \hat{C}_\tau^i \hat{x}_i(t-\tau) \tag{18}$$

for $i = 1, 2$, where, $\hat{x}_i \in \mathbf{R}^{\hat{n}_i}$, where $\hat{n}_i := n_i + n_c$, $\hat{u}_i \in \mathbf{R}^{p_i}$, and $\hat{y}_i \in \mathbf{R}^{q_i}$ are, respectively, the state, the input, and the output vectors of $\hat{\Sigma}_i$ and, for $\tau \in \{0\} \cup \mathcal{T}$,

$$\hat{E}_\tau^1 = \begin{bmatrix} E_\tau^{11} & E_\tau^{1c} \\ E_\tau^{c1} & E_\tau^{cc} \end{bmatrix}, \; \hat{A}_\tau^1 = \begin{bmatrix} A_\tau^{11} & A_\tau^{1c} \\ A_\tau^{c1} & A_\tau^{cc} \end{bmatrix}, \; \hat{B}_\tau^1 = \begin{bmatrix} B_\tau^{11} \\ B_\tau^{c1} \end{bmatrix},$$

$$\hat{E}_\tau^2 = \begin{bmatrix} E_\tau^{cc} & E_\tau^{c2} \\ E_\tau^{2c} & E_\tau^{22} \end{bmatrix}, \; \hat{A}_\tau^2 = \begin{bmatrix} A_\tau^{cc} & A_\tau^{c2} \\ A_\tau^{2c} & A_\tau^{22} \end{bmatrix}, \; \hat{B}_\tau^2 = \begin{bmatrix} B_\tau^{c2} \\ B_\tau^{22} \end{bmatrix},$$

$$\hat{C}_\tau^1 = \begin{bmatrix} C_\tau^{11} & C_\tau^{1c} \end{bmatrix}, \text{ and } \hat{C}_\tau^2 = \begin{bmatrix} C_\tau^{2c} & C_\tau^{22} \end{bmatrix}.$$

Assuming that the only strong interconnections between the original subsystems are through the overlapping part, due to the choice of the complementary matrices above, these two disjoint subsystems will have only a weak interaction (which is not shown in the above equations) among them (see İftar (2019)).

3. CONTRACTIBILITY OF CONTROLLERS

Now, let us consider a controller Γ of the form

$$J_0 \dot{z}(t) + \sum_{\tau \in \mathcal{T}_c} J_\tau \dot{z}(t-\tau) = F_0 z(t) + G_0 w(t) + \sum_{\tau \in \mathcal{T}_c} (F_\tau z(t-\tau) + G_\tau w(t-\tau)) \tag{19}$$

$$v(t) = H_0 z(t) + K_0 w(t) + \sum_{\tau \in \mathcal{T}_c} (H_\tau z(t-\tau) + K_\tau w(t-\tau)) \quad (20)$$

for Σ and a controller $\hat{\Gamma}$ of the form

$$\hat{J}_0 \dot{\hat{z}}(t) + \sum_{\tau \in \hat{\mathcal{T}}_c} \hat{J}_\tau \dot{\hat{z}}(t-\tau) = \hat{F}_0 \hat{z}(t) + \hat{G}_0 \hat{w}(t) + \sum_{\tau \in \hat{\mathcal{T}}_c} \left(\hat{F}_\tau \hat{z}(t-\tau) + \hat{G}_\tau \hat{w}(t-\tau) \right) \quad (21)$$

$$\hat{v}(t) = \hat{H}_0 \hat{z}(t) + \hat{K}_0 \hat{w}(t) + \sum_{\tau \in \hat{\mathcal{T}}_c} \left(\hat{H}_\tau \hat{z}(t-\tau) + \hat{K}_\tau \hat{w}(t-\tau) \right) \quad (22)$$

for $\hat{\Sigma}$. Here, $z \in \mathbf{R}^m$, $w \in \mathbf{R}^q$, and $v \in \mathbf{R}^p$ are, respectively, the state, the input, and the output vectors of Γ and $\hat{z} \in \mathbf{R}^{\hat{m}}$, $\hat{w} \in \mathbf{R}^q$, and $\hat{v} \in \mathbf{R}^p$ are, respectively, the state, the input, and the output vectors of $\hat{\Gamma}$. The sets $\mathcal{T}_c$ and $\hat{\mathcal{T}}_c$ include the time-delays, which are positive real numbers, of Γ and $\hat{\Gamma}$, respectively. The matrices J_τ, F_τ, G_τ, and H_τ, for $\tau \in \{0\} \cup \mathcal{T}_c$, and $\hat{J}_\tau$, $\hat{F}_\tau$, $\hat{G}_\tau$, and $\hat{H}_\tau$, for $\tau \in \{0\} \cup \hat{\mathcal{T}}_c$, are appropriately dimensioned constant real matrices. For some differentiable functions $\zeta : [-\bar{\tau}_c, 0] \to \mathbf{R}^m$ and $\hat{\zeta} : [-\hat{\tau}_c, 0] \to \mathbf{R}^{\hat{m}}$, where

$$\bar{\tau}_c := \max_{\tau \in \mathcal{T}_c}(\tau) \quad \text{and} \quad \hat{\tau}_c := \max_{\tau \in \hat{\mathcal{T}}_c}(\tau) , \quad (23)$$

the initial conditions for Γ and $\hat{\Gamma}$ are assumed to be given as:

$$z(\theta) = \zeta(\theta) , \quad \theta \in [-\bar{\tau}_c, 0] \quad (24)$$

and

$$\hat{z}(\theta) = \hat{\zeta}(\theta) , \quad \theta \in [-\hat{\tau}_c, 0] \quad (25)$$

respectively.

As shown by Erol and İftar (2016), the form (19)–(20) (likewise, the form (21)–(22)) describe the most general form of LTI neutral time-delay controllers (including descriptor-type when $\text{rank}(J_0) < m$). Such a controller reduces to a retarded time-delay controller when $\text{rank}(J_0) = m$ and $J_\tau = 0$, $\forall \tau \in \mathcal{T}_c$. It reduces to a finite-dimensional dynamic controller when $\text{rank}(J_0) = m$ and $\mathcal{T}_c = \emptyset$. It reduces to a time-delay controller of the form $v(t) = K_0 w(t) + \sum_{\tau \in \mathcal{T}_c} K_\tau w(t-\tau)$ when $m = 0$ and further reduces to a static output feedback controller of the form $v(t) = K_0 w(t)$ when $m = 0$ and $\mathcal{T}_c = \emptyset$.

Here, Γ is to be applied to Σ and $\hat{\Gamma}$ is to be applied to $\hat{\Sigma}$ by letting

$$w(t) = y(t) - r(t) \quad \text{and} \quad \hat{w}(t) = \hat{y}(t) - \hat{r}(t) \quad (26)$$

and

$$u(t) = v(t) + e(t) \quad \text{and} \quad \hat{u}(t) = \hat{v}(t) + \hat{e}(t) \quad (27)$$

for $t \geq 0$, where $r \in \mathbf{R}^q$, $\hat{r} \in \mathbf{R}^q$, $e \in \mathbf{R}^p$, and $\hat{e} \in \mathbf{R}^p$ are some *external inputs*. Here it is assumed that, the controllers Γ and $\hat{\Gamma}$ are such that the closed-loop systems obtained by making the connections (26) and (27) are well-defined and well-posed. To satisfy condition (9) following the application of the controllers, the following property must be satisfied.

Definition 3: Suppose that the connection in (26) is made but the connection in (27) is not made. The controller $\hat{\Gamma}$ for $\hat{\Sigma}$ is said to be *contractible* to the controller Γ for Σ if there exist a full column-rank matrix $V \in \mathbf{R}^{\hat{n} \times n}$ and a full row-rank matrix $P \in \mathbf{R}^{m \times \hat{m}}$, such that for all $\phi(\cdot)$, for all $u(\cdot)$, for all $r(\cdot)$, and for all $\hat{\zeta}(\cdot)$, the choice (8), (9),

$$\zeta(\theta) = P\hat{\zeta}(\theta) , \quad \theta \in [-\tilde{\tau}_c, 0] \quad (28)$$

where $\tilde{\tau}_c := \min\{\bar{\tau}_c, \hat{\tau}_c\}$, and

$$\hat{r}(t) = r(t) , \quad t \geq -\tilde{\tau}_c \quad (29)$$

implies

$$z(t) = P\hat{z}(t) , \quad t \geq -\tilde{\tau}_c \quad (30)$$

and

$$\hat{v}(t) = v(t) , \quad t \geq 0 . \quad (31)$$

We note that condition (31) need to be satisfied only for $t \geq 0$, since the controllers are assumed to be applied starting at time $t = 0$. We also note that, the existence of a full row-rank matrix $P \in \mathbf{R}^{m \times \hat{m}}$, in particular implies $\hat{m} \geq m$. This, however, as indicated elsewhere (e.g., see Šiljak (1991)), is natural, since Σ, in general, forms a part of $\hat{\Sigma}$, and hence, should not require a controller with a larger dimensional state vector.

Now, let Σ be a restriction of $\hat{\Sigma}$. Then, the conditions for $\hat{\Gamma}$ to be contractible to Γ are given by the following theorem.

Theorem 2: Suppose that Σ is a restriction of $\hat{\Sigma}$. Then, the controller $\hat{\Gamma}$ for $\hat{\Sigma}$ is contractible to the controller Γ for Σ if there exists a full row-rank matrix $P \in \mathbf{R}^{m \times \hat{m}}$ such that

i) $\mathcal{T}_c \setminus \hat{\mathcal{T}}_c = \emptyset$,
ii) for any $\tau \in \hat{\mathcal{T}}_c \setminus \mathcal{T}_c$, $P\hat{J}_\tau = 0$, $P\hat{F}_\tau = 0$, $P\hat{G}_\tau = 0$, $\hat{H}_\tau = 0$, and $\hat{K}_\tau = 0$,

and

iii) for any $\tau \in \{0\} \cup (\mathcal{T}_c \cap \hat{\mathcal{T}}_c)$, $P\hat{J}_\tau = J_\tau P$, $P\hat{F}_\tau = F_\tau P$, $P\hat{G}_\tau = G_\tau$, $\hat{H}_\tau = H_\tau P$, and $\hat{K}_\tau = K_\tau$.

Proof: Since the connection (26) is made at time $t = 0$, (11) and (29) implies

$$\hat{w}(t) = w(t) , \quad t \geq -\tilde{\tau}_c . \quad (32)$$

Note that (30) for $t \in [-\tilde{\tau}_c, 0]$ is implied by (28). To establish (30) for $t \geq 0$, premultiply both sides of (21) by P, use (32) and the conditions (i)–(iii) above and compare with (19). Together with (28), by the uniqueness of solutions, this establishes (30) for $t \geq 0$. Then, to establish (31), use (30), (32), and the conditions (i)–(iii) above in (20) and compare with (22). This concludes the proof. □

Now, suppose that Σ is a restriction of $\hat{\Sigma}$ and $\hat{\Gamma}$ is contractible to Γ. Suppose that Γ is applied to Σ and $\hat{\Gamma}$ is applied to $\hat{\Sigma}$ by making the connections (26) and (27) for $t \geq 0$. Let us denote the closed-loop systems obtained in this way by Σ_c and $\hat{\Sigma}_c$, respectively. Then we obtain the following important result.

Theorem 3: Suppose that Σ is a restriction of $\hat{\Sigma}$ and $\hat{\Gamma}$ is contractible to Γ. Then, $\hat{\Sigma}_c$ includes Σ_c.

Proof: Let the maximum time-delay in Σ_c and $\hat{\Sigma}_c$ be denoted by $\bar{\sigma}$ and $\hat{\sigma}$, respectively (there exists such $\bar{\sigma} \geq$

$\max\{\bar{\tau}, \bar{\tau}_c\}$ and $\hat{\sigma} \geq \max\{\hat{\tau}, \hat{\tau}_c\}$, since Σ_c and $\hat{\Sigma}_c$ are assumed to be well-posed). Let $\xi(t) := \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$ and $\hat{\xi}(t) := \begin{bmatrix} \hat{x}(t) \\ \hat{z}(t) \end{bmatrix}$ denote the state vectors of Σ_c and $\hat{\Sigma}_c$, respectively. Then, since the connections (26) and (27) are made at $t = 0$, the initial conditions of Σ_c and $\hat{\Sigma}_c$ are respectively given as

$$\xi(\theta) = \psi(\theta) , \qquad \theta \in [-\bar{\sigma}, 0] \tag{33}$$

and

$$\hat{\xi}(\theta) = \hat{\psi}(\theta) , \qquad \theta \in [-\hat{\sigma}, 0] , \tag{34}$$

where $\psi(\theta) = \begin{bmatrix} \psi_1(\theta) \\ \psi_2(\theta) \end{bmatrix}$, $\theta \in [-\bar{\sigma}, 0]$, and $\hat{\psi}(\theta) = \begin{bmatrix} \hat{\psi}_1(\theta) \\ \hat{\psi}_2(\theta) \end{bmatrix}$, $\theta \in [-\hat{\sigma}, 0]$, where

$$\psi_1(\theta) := \begin{cases} \phi(\theta) , & \theta \in [-\bar{\tau}, 0] \\ 0 , & \theta \in [-\bar{\sigma}, -\bar{\tau}) \end{cases} ,$$

$$\psi_2(\theta) := \begin{cases} \zeta(\theta) , & \theta \in [-\bar{\tau}_c, 0] \\ 0 , & \theta \in [-\bar{\sigma}, -\bar{\tau}_c) \end{cases} ,$$

$$\hat{\psi}_1(\theta) := \begin{cases} \hat{\phi}(\theta) , & \theta \in [-\hat{\tau}, 0] \\ 0 , & \theta \in [-\hat{\sigma}, -\hat{\tau}) \end{cases} ,$$

and

$$\hat{\psi}_2(\theta) := \begin{cases} \hat{\zeta}(\theta) , & \theta \in [-\hat{\tau}_c, 0] \\ 0 , & \theta \in [-\hat{\sigma}, -\hat{\tau}_c) \end{cases} .$$

Let $Q \in \mathbf{R}^{\hat{m} \times m}$ be a full column-rank matrix satisfying $PQ = I_m$ and $U \in \mathbf{R}^{n \times \hat{n}}$ be a full row-rank matrix satisfying $UV = I_n$, where V and P are as in Definitions 2 and 3, respectively. Let $R := \begin{bmatrix} V & 0 \\ 0 & Q \end{bmatrix}$ and $S := \begin{bmatrix} U & 0 \\ 0 & P \end{bmatrix}$. Note that $SR = I_{n+m}$. Let

$$\hat{\psi}(\theta) = R\psi(\theta) , \qquad \theta \in [-\tilde{\sigma}, 0] , \tag{35}$$

where $\tilde{\sigma} := \min\{\bar{\sigma}, \hat{\sigma}\}$. This then implies (8) and (28). Also let

$$\hat{r}(t) = r(t) \quad \text{and} \quad \hat{e}(t) = e(t) , \qquad t \geq -\tilde{\sigma} , \tag{36}$$

which, in particular, implies (29) and (together with (31)) (9). Then, (12) and (30) hold. Thus,

$$\xi(t) = S\hat{\xi}(t) , \qquad t \geq -\tilde{\sigma} , \tag{37}$$

which, together with (11) and (31) implies that $\hat{\Sigma}_c$ includes Σ_c. □

The above result in particular implies that $\hat{\Sigma}_c$ and Σ_c have the same input-output map (respectively from $\begin{bmatrix} \hat{r} \\ \hat{e} \end{bmatrix}$ to $\begin{bmatrix} \hat{y} \\ \hat{v} \end{bmatrix}$ and from $\begin{bmatrix} r \\ e \end{bmatrix}$ to $\begin{bmatrix} y \\ v \end{bmatrix}$) and that if $\hat{\Gamma}$ stabilizes $\hat{\Sigma}$, then Γ stabilizes Σ.

4. DECENTRALIZED CONTROLLER DESIGN

For each of the decoupled subsystems $\hat{\Sigma}_i$ $(i = 1, 2)$, described by (17)–(18), let us consider a controller, to be denoted by Γ_i, of the form:

$$J_0^i \dot{z}_i(t) + \sum_{\tau \in \mathcal{T}_i} J_\tau^i \dot{z}_i(t - \tau) = F_0^i z_i(t) + G_0^i w_i(t) + \sum_{\tau \in \mathcal{T}_i} \left(F_\tau^i z_i(t - \tau) + G_\tau^i w_i(t - \tau) \right) \tag{38}$$

$$v_i(t) = H_0^i z_i(t) + K_0^i w_i(t) + \sum_{\tau \in \mathcal{T}_i} \left(H_\tau^i z_i(t - \tau) + K_\tau^i w_i(t - \tau) \right) \tag{39}$$

where $z_i \in \mathbf{R}^{m_i}$, $w_i \in \mathbf{R}^{q_i}$, and $v_i \in \mathbf{R}^{p_i}$ are, respectively, the state, the input, and the output vectors of Γ_i and the set $\mathcal{T}_i$ includes the time-delays, which are positive real numbers, of Γ_i $(i = 1, 2)$. The matrices J_τ^i, F_τ^i, G_τ^i, and H_τ^i, for $\tau \in \{0\} \cup \mathcal{T}_i$, are appropriately dimensioned constant real matrices.

Now, let

$$\hat{z} := \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} , \quad \hat{w} := \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} , \quad \text{and} \quad \hat{v} := \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} . \tag{40}$$

Also let $\hat{\mathcal{T}} := \mathcal{T}_1 \cup \mathcal{T}_2$ and, for $\tau \in \{0\} \cup \hat{\mathcal{T}}$, define

$$\hat{J}_\tau := \begin{bmatrix} J_\tau^1 & 0 \\ 0 & J_\tau^2 \end{bmatrix} , \quad \hat{F}_\tau := \begin{bmatrix} F_\tau^1 & 0 \\ 0 & F_\tau^2 \end{bmatrix} , \quad \hat{G}_\tau := \begin{bmatrix} G_\tau^1 & 0 \\ 0 & G_\tau^2 \end{bmatrix} ,$$

$$\hat{H}_\tau := \begin{bmatrix} H_\tau^1 & 0 \\ 0 & H_\tau^2 \end{bmatrix} , \quad \text{and} \quad \hat{K}_\tau := \begin{bmatrix} K_\tau^1 & 0 \\ 0 & K_\tau^2 \end{bmatrix} ,$$

where, for $i \in \{1, 2\}$ and $\tau \notin \{0\} \cup \mathcal{T}_i$, J_τ^i, F_τ^i, G_τ^i, H_τ^i, and K_τ^i are appropriately dimensioned zero matrices. Let $\hat{\Gamma}$ be defined by (21)–(22) and be applied to the expanded system $\hat{\Sigma}$. Suppose that the resulting closed-loop system $\hat{\Sigma}_c$ is well-defined and well-posed. Then, by Theorem 3, this expanded closed-loop system includes the actual closed-loop system, obtained by applying the local controllers (38)–(39) to the original system Σ by letting

$$w_i(t) = y_i(t) - r_i(t) \quad \text{and} \quad u_i(t) = v_i(t) + e_i(t) \tag{41}$$

for $t \geq 0$, where $r_i \in \mathbf{R}^{q_i}$ and $e_i \in \mathbf{R}^{p_i}$ $(i = 1, 2)$ are some external inputs. Thus, assuming that $\hat{\Gamma}$ stabilizes $\hat{\Sigma}$, the local controllers (38)–(39) stabilizes the original system Σ. Furthermore, the original closed-loop system has the same input-output map as the expanded closed-loop system.

5. CONCLUSIONS

A decentralized controller design approach for large-scale LTI neutral time-delay systems has been presented. The approach is based on the principle of restriction, which is a special case of inclusion. In this approach, an overlappingly decomposed neutral time-delay system is first expanded such that the original system is a restriction of the expanded system. The expanded system then appears as an interconnection of disjoint subsystems, which have only weak interactions between them. Then it is possible to design decentralized controllers for these decoupled subsystems, e.g., by the approach of Erol and İftar (2017). Since the original system is a restriction of the expanded system, these controllers can be contracted for implementation on the original system.

In the present work, it has been shown that, the expanded closed-loop system, obtained by applying the designed decentralized controllers to the expanded system, includes the original closed-loop system, obtained by applying the contracted controllers to the original system. This then implies that, if the decentralized controllers are designed to stabilize the expanded system, then the contracted controllers stabilize the original system. Furthermore, the two closed-loop systems will have the same input-output map. This property then guarantees that, if the designed decentralized controllers satisfy certain input-output performance criteria for the expanded system, then the contracted controllers satisfy the same performance criteria for the original system.

Although only the case of two subsystems has been presented for brevity, the approach can also be applied to the case of more subsystems overlappingly decomposed in different ways. Furthermore, although only the overlapping decompositions of the state-space has been considered, the approach can be extended to the case of overlapping decompositions of the input and the output spaces as well.

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