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Control of Heterogeneous Platoons Using a Delay-Based Spacing Policy

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Abstract: A control design approach for heterogeneous platoons is derived that achieves tracking of the desired delay-based spacing policy. The delay-based spacing policy induces an identical spatially varying velocity reference for all vehicles of the platoon. Thus, the control law is derived in the spatial domain. For a heterogeneous platoon, with individual dynamics of each vehicle, the first step of the control design is an individual exact linearization of each vehicle dynamics that transforms the heterogeneous platoon into a homogeneous platoon, where all vehicles have identical dynamics with respect to the new input. Then, a well-known control design for homogeneous platoons may be applied. The results are illustrated by simulations.

Keywords: Vehicle platoons, heterogeneous platoons, delay-based spacing policy, decentralized control, cascaded systems.

1. INTRODUCTION

Assisted and automated driving applications have gained more and more interest over the course of the last decades. Both the private sector and academic researchers have invested immensely to promote this progress. Even the public sector has established institutions to develop solutions for present and future mobility challenges. It is predictable that the demand for logistic capabilities will increase faster than the expansion of the present transportation infrastructure. Therefore, different approaches have been proposed to utilize the given capacity more efficiently. One concern is to improve traffic flow and to optimize utilization of transportation networks to increase throughput. Another approach is to reduce inter-vehicular spacing to put more vessels on driveways. Their combination further enhances the overall performance.

Platooning is therefore one of the most prominent tasks for automated and assisted driving. However, this application raises an array of peculiarities. Usually, performance shall not only be optimized for an individual member of a platoon, but with respect to the platoon as a whole. Most approaches seek to optimize said performance via the criterion of fuel consumption. Effective ways to minimize this are smooth trajectories to reduce actuation effort and establishing small inter-vehicular distances to exploit the resulting slipstream. However, these means raise issues of safety and stability. Safety here means guaranteed collision avoidance and stability is understood in the context of string stability, which is usually denoted as the capacity for attenuating propagated disturbances in the upstream direction of the platoon, after Shaw and Hedrick (2007) and Ploeg et al. (2014). These aspects are further exacerbated by the typical target platforms for platooning, as heavy duty vessels usually offer only very limited dynamics. For example, a low limit for acceleration leads to a delayed response to sudden velocity changes and therefore infringes stability. Limited deceleration on the other hand implies long braking distances and compromises safety.

From the control engineering view of the context presented, many different approaches have been developed. Adaptive Cruise Controllers (ACC) and Cooperative Adaptive Cruise Controllers (CACC) may already be considered as established and traditional approaches; their design is compliant to standard methods of controller design for linear time-invariant (LTI) systems. Due to their simple nature, their properties are easy to evaluate using LTI system analysis. A handful of CACC implementations shall be pointed out: the original PATH-CACC as described in Rajamani (2012), developed at Berkeley University, California, the CACC by J. Ploeg et al. (2011), and the Flatbed-CACC by Ali et al. (2015). The named CACC are well established approaches within the community and various experiments have proven their capability and fit for purpose. Moreover, most of these implementations rely on a minimal set of peripheral hardware requirements which are usually already part of current vehicle models. Cruise Control is already a common feature; an extension to ACC merely requires a sufficiently reliable distance measurement unit. In turn extending the ACC to CACC calls for communication capabilities, which might as well be implemented with minor effort. The computational requirements for the named vehicle controllers are considered to be very moderate and may even be implemented alongside with already existing embedded systems. The group of consensus controllers are an extension of the

CACC approach. Foremost, they differ in implementing different communication topologies, whereas traditional CACC are usually subject to a simple, rigid, and limited set of communication links. An example of a consensus controller may be found in Santini et al. (2017).

Closely related to the controller design is the choice of a sufficient spacing policy. The constant spacing and the time headway spacing policy are arguably the most widespread. However, these approaches suffer not only from issues with safety and stability but also are not necessarily optimal in terms of fuel consumption. Considering a platoon route over hilly terrain illustrates that the demand for instantaneous tracking of the velocity of a leading vehicle yields no benefit. In Besselink and Johansson (2017) a controller is proposed alongside a novel spatial spacing policy. The delay-based spacing policy refers to the tracking of a reference velocity profile in the spatial domain, meaning that the reference is a now function of space, rather than time, as implied by other spacing policies. The delay-based spacing policy therefore yields many advantages: Turri et al. (2017) showed that slipstream performance and fuel consumption can be optimized. In fact, features that determine an optimal speed profile such as slopes, curves, speed limits, traffic lights, intersections, freeway ramps etc. are inherently spatial and fixed in space. Sufficient information on a route may allow *a priori* and optimized planning of the platoon's trajectory.

Based on Remark 6 in Besselink and Johansson (2017) ("[...] even though it is assumed that all vehicles have identical dynamics [...], the results in this paper have the potential to be extended to heterogeneous vehicle platoons."), an extension of the the control design approach in Besselink and Johansson (2017) to a heterogeneous vehicle platoon is derived in this paper.

The remainder of this paper is organized as follows. In Section 2, the control problem is formulated, presenting the delay-based spacing policy and the dynamic model of a heterogeneous platoon. The controller design for heterogeneous platoons that achieves tracking of the desired delay-based spacing policy is derived in Section 3. Section 4 presents simulation results for an academic example. Future work is stated in Section 5.

2. PROBLEM FORMULATION

2.1 Delay-Based Spacing Policy

The delay-based spacing policy introduced in Besselink and Johansson (2017) is defined as

$$s_{\mathrm{ref},i}(t) = s_{i-1}(t - \Delta t) \quad , \tag{1}$$

where $s_{\mathrm{ref},i}(t)$ denotes the reference position of the vehicle i and $s_{i-1}(t)$ the position of the preceding vehicle $i-1$. Thus, each follower vehicle tracks a time-delayed version of the trajectory of the preceding vehicle, with time gap $\Delta t > 0$. Assuming perfect tracking, this policy achieves identical velocity profiles in space for all vehicles,

$$v_i(s) = v_{\mathrm{ref}}(s) \tag{2}$$

for $i \in \mathcal{T}_N^0 = \{0, 1, \ldots, N\}$, where N denotes the number of follower vehicles. On the other hand, perfect tracking of the reference velocity yields

$$s_i(t) = s_{i-1}(t - \Delta t) \quad . \tag{3}$$

See Besselink and Johansson (2017) for the proof.

Note that the reference velocity has to be bounded by $v_{\min}$ with

$$v_{\min}\,\Delta t > L_{\max} \quad , \tag{4}$$

where $L_{\max}$ denotes the maximum vehicle length, such that subsequent vehicles do not collide when they perfectly track the reference velocity.

2.2 Platoon Modeling

Consider a *heterogeneous* platoon of $N+1$ vehicles, in which each vehicle satisfies the *individual* longitudinal dynamics in the time domain

$$\dot{s}_i(t) = \tilde{h}_i\left(\underline{\chi}_i(t)\right) \quad , \tag{5}$$

$$\underline{\dot{\chi}}_i(t) = \underline{\tilde{f}}_i\left(\underline{\chi}_i(t)\right) + \underline{\tilde{g}}_i\left(\underline{\chi}_i(t)\right)\tilde{u}_i(t) \tag{6}$$

for $i \in \mathcal{T}_N^0$. Here, $s_i(t)$ denotes the position of vehicle i, such that (5) represents the kinematic relation with velocity $\tilde{v}_i(t) = \tilde{h}_i\left(\underline{\chi}_i(t)\right)$. Equation (6) with state $\underline{\chi}_i(t)$ describes the remaining dynamics, e.g. engine and drivetrain dynamics. It is assumed that all vehicle models have the same order n, i.e. $\dim \underline{\chi}_i = n-1$ for $i \in \mathcal{T}_N^0$, and that the functions $\underline{\tilde{f}}_i$, $\underline{\tilde{g}}_i$, and $\tilde{h}_i$ are sufficiently smooth. The controller synthesis is done in the spatial domain, considering s as independent variable. Thus, the platoon dynamics (5,6) is now written in the spatial domain.

Let us first consider a single vehicle, neglecting the index i. The kinematic equation

$$\frac{\mathrm{d}s(t)}{\mathrm{d}t} = \tilde{v}(t) \tag{7}$$

yields

$$\frac{\mathrm{d}t(s)}{\mathrm{d}s} = \frac{1}{\tilde{v}(t(s))} \quad , \tag{8}$$

where $t(s)$ denotes the time instance where the vehicle passes s. Substituting the definition

$$v(s) = \tilde{v}(t(s)) \tag{9}$$

yields

$$t'(s) = \frac{1}{v(s)} \quad , \tag{10}$$

where $'$ denotes the spatial derivative, i.e.

$$t'(s) = \frac{\mathrm{d}t(s)}{\mathrm{d}s} \quad . \tag{11}$$

The equation for the remaining dynamics,

$$\underline{\dot{\chi}}(t) = \underline{\tilde{f}}\left(\underline{\chi}(t)\right) + \underline{\tilde{g}}\left(\underline{\chi}(t)\right)\tilde{u}(t) \quad , \tag{12}$$

may be written as

$$\frac{\mathrm{d}\underline{\chi}(t(s))}{\mathrm{d}s}\frac{\mathrm{d}s(t)}{\mathrm{d}t} = \underline{\tilde{f}}\left(\underline{\chi}(t(s))\right) + \underline{\tilde{g}}\left(\underline{\chi}(t(s))\right)\tilde{u}(t(s)) \quad , \tag{13}$$

and substituting (7) yields

$$\frac{\mathrm{d}\underline{\chi}(t(s))}{\mathrm{d}s} = \frac{\underline{\tilde{f}}\left(\underline{\chi}_i(t(s))\right)}{\tilde{v}(t(s))} + \frac{\underline{\tilde{g}}\left(\underline{\chi}_i(t(s))\right)}{\tilde{v}(t(s))}\tilde{u}_i(t(s)) \quad . \tag{14}$$

Substituting the definitions

$$\underline{\xi}(s) = \underline{\chi}(t(s)) \quad , \tag{15}$$

$$\underline{f}\left(\underline{\xi}(s)\right) = \frac{\underline{\tilde{f}}\left(\underline{\chi}(t(s))\right)}{\tilde{v}(t(s))} = \frac{\underline{\tilde{f}}\left(\underline{\chi}(t(s))\right)}{\tilde{h}\left(\underline{\chi}(t(s))\right)} \quad , \tag{16}$$

$$\underline{g}\left(\underline{\xi}(s)\right) = \frac{\underline{\tilde{g}}\left(\underline{\chi}(t(s))\right)}{\tilde{v}(t(s))} = \frac{\underline{\tilde{g}}\left(\underline{\chi}(t(s))\right)}{\tilde{h}\left(\underline{\chi}(t(s))\right)} \quad , \tag{17}$$

$$u(s) = \tilde{u}(t(s)) \tag{18}$$

yields

$$\underline{\xi}'(s) = \underline{f}\left(\underline{\xi}(s)\right) + \underline{g}\left(\underline{\xi}(s)\right) u(s) \quad . \tag{19}$$

Substituting the definition

$$h\left(\underline{\xi}(s)\right) = \frac{1}{v(s)} \tag{20}$$

into (10) yields

$$t'(s) = h\left(\underline{\xi}(s)\right) \quad . \tag{21}$$

Thus, the longitudinal dynamics of a platoon of $N+1$ vehicles according to (5,6) in the time domain may be written in the spatial domain as

$$t_i'(s) = h_i\left(\underline{\xi}_i(s)\right) \quad , \tag{22}$$

$$\underline{\xi}_i'(s) = \underline{f}_i\left(\underline{\xi}_i(s)\right) + \underline{g}_i\left(\underline{\xi}_i(s)\right) u_i(s) \tag{23}$$

for $i \in \mathcal{T}_N^0$, where $t_i(s)$ denotes the time instance where the vehicle i passes s, and $\underline{\xi}_i(s)$, $\underline{f}_i\left(\underline{\xi}_i(s)\right)$, $\underline{g}_i\left(\underline{\xi}_i(s)\right)$, $h_i\left(\underline{\xi}_i(s)\right)$, $u_i(s)$ are defined similar to (15,16,17,20,18) by augmenting the index i.

Now, a controller has to be designed such that each vehicle i tracks the reference velocity $v_{\mathrm{ref}}(s)$, which is assumed to be at least $n-2$ times continuously differentiable.

3. CONTROLLER DESIGN

Consider a platoon of $N+1$ vehicles, with the longitudinal dynamics in the spatial domain according to (22,23) for $i \in \mathcal{T}_N^0$. For tracking of the reference velocity $v_{\mathrm{ref}}(s)$, the reference for $h_i\left(\underline{\xi}_i(s)\right)$ is defined as

$$h_i^{\mathrm{ref}}\left(\underline{\xi}_i(s)\right) = \frac{1}{v_{\mathrm{ref}}(s)} \tag{24}$$

and the tracking error of the inverse velocity as

$$e_{1,i}(s) = h_i\left(\underline{\xi}_i(s)\right) - \frac{1}{v_{\mathrm{ref}}(s)} \quad . \tag{25}$$

We assume that the relative degree of

$$h_i\left(\underline{\xi}_i(s)\right) = \frac{1}{v_i(s)} \tag{26}$$

equals $n-1$. Thus,

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}s} h_i\left(\underline{\xi}_i(s)\right) \\
&\quad = \left(\frac{\partial h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \underline{\xi}_i'(s) \\
&\quad = \left(\frac{\partial h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \left(\underline{f}_i\left(\underline{\xi}_i(s)\right) + \underline{g}_i\left(\underline{\xi}_i(s)\right) u_i(s)\right) \\
&\quad = L_{\underline{f}_i} h_i + L_{\underline{g}_i} h_i \, u_i(s) \\
&\quad = L_{\underline{f}_i} h_i \quad ,
\end{aligned} \tag{27}$$

$$\begin{aligned}
&\frac{\mathrm{d}^2}{\mathrm{d}s^2} h_i\left(\underline{\xi}_i(s)\right) \\
&\quad = \left(\frac{\partial L_{\underline{f}_i} h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \underline{\xi}_i'(s) \\
&\quad = \left(\frac{\partial L_{\underline{f}_i} h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \left(\underline{f}_i\left(\underline{\xi}_i(s)\right) + \underline{g}_i\left(\underline{\xi}_i(s)\right) u_i(s)\right) \\
&\quad = L_{\underline{f}_i}^2 h_i + L_{\underline{g}_i} L_{\underline{f}_i} h_i \, u_i(s) \\
&\quad = L_{\underline{f}_i}^2 h_i \quad ,
\end{aligned} \tag{28}$$

$$\vdots$$

$$\begin{aligned}
&\frac{\mathrm{d}^{n-2}}{\mathrm{d}s^{n-2}} h_i\left(\underline{\xi}_i(s)\right) \\
&\quad = \left(\frac{\partial L_{\underline{f}_i}^{n-3} h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \underline{\xi}_i'(s) \\
&\quad = \left(\frac{\partial L_{\underline{f}_i}^{n-3} h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \left(\underline{f}_i\left(\underline{\xi}_i(s)\right) + \underline{g}_i\left(\underline{\xi}_i(s)\right) u_i(s)\right) \\
&\quad = L_{\underline{f}_i}^{n-2} h_i + L_{\underline{g}_i} L_{\underline{f}_i}^{n-3} h_i \, u_i(s) \\
&\quad = L_{\underline{f}_i}^{n-2} h_i \quad ,
\end{aligned} \tag{29}$$

$$\begin{aligned}
&\frac{\mathrm{d}^{n-1}}{\mathrm{d}s^{n-1}} h_i\left(\underline{\xi}_i(s)\right) \\
&\quad = \left(\frac{\partial L_{\underline{f}_i}^{n-2} h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \underline{\xi}_i' \\
&\quad = \left(\frac{\partial L_{\underline{f}_i}^{n-2} h_i}{\partial \underline{\xi}_i}\right)^{\mathrm{T}} \left(\underline{f}_i\left(\underline{\xi}_i(s)\right) + \underline{g}_i\left(\underline{\xi}_i(s)\right) u_i(s)\right) \\
&\quad = L_{\underline{f}_i}^{n-1} h_i + L_{\underline{g}_i} L_{\underline{f}_i}^{n-2} h_i \, u_i(s) \quad ,
\end{aligned} \tag{30}$$

where $L_{\underline{f}_i} h_i$ denotes the Lie derivative of h_i along $\underline{f}_i$, see Isidori (1995).

We define the desired transfer equation in the frequency domain as

$$E_{1,i}(p) = \frac{1}{p^{n-1}} \bar{U}_i(p) \quad , \tag{31}$$

where p denotes the complex frequency. This yields

$$p^{n-1} E_{1,i}(p) = \bar{U}_i(p) \tag{32}$$

or

$$\overset{(n-1)}{e}_{1,i}(s) = \bar{u}_i(s) \tag{33}$$

with

$$\overset{(n-1)}{e}_{1,i}(s) = \frac{\mathrm{d}^{n-1}}{\mathrm{d}s^{n-1}}\left(h_i\left(\underline{\xi}_i(s)\right) - \frac{1}{v_{\text{ref}}(s)}\right) = \frac{\mathrm{d}^{n-1}}{\mathrm{d}s^{n-1}}h_i\left(\underline{\xi}_i(s)\right) - \frac{\mathrm{d}^{n-1}}{\mathrm{d}s^{n-1}}\left(\frac{1}{v_{\text{ref}}(s)}\right) \quad . \tag{34}$$

Substituting (30) yields

$$\overset{(n-1)}{e}_{1,i}(s) = L_{\underline{f}_i}^{n-1}h_i + L_{\underline{g}_i}L_{\underline{f}_i}^{n-2}h_i\,u_i(s) - \frac{\mathrm{d}^{n-1}}{\mathrm{d}s^{n-1}}\left(\frac{1}{v_{\text{ref}}(s)}\right) \quad . \tag{35}$$

Substituting into (33) yields

$$L_{\underline{f}_i}^{n-1}h_i + L_{\underline{g}_i}L_{\underline{f}_i}^{n-2}h_i\,u_i(s) - \frac{\mathrm{d}^{n-1}}{\mathrm{d}s^{n-1}}\left(\frac{1}{v_{\text{ref}}(s)}\right) = \bar{u}_i(s) \tag{36}$$

or

$$u_i(s) = \frac{1}{L_{\underline{g}_i}L_{\underline{f}_i}^{n-2}h_i}\left(\bar{u}_i(s) - L_{\underline{f}_i}^{n-1}h_i + \frac{\mathrm{d}^{n-1}}{\mathrm{d}s^{n-1}}\left(\frac{1}{v_{\text{ref}}(s)}\right)\right) \quad . \tag{37}$$

Using the state variables

$$e_{1,i}(s) = h_i\left(\underline{\xi}_i(s)\right) - \frac{1}{v_{\text{ref}}(s)} \quad , \tag{38}$$

$$e_{k,i}(s) = \frac{\mathrm{d}^{k-1}}{\mathrm{d}s^{k-1}}e_{1,i}(s) \quad \text{for } k = 2,\dots,n-1 \tag{39}$$

yields

$$e'_{1,i}(s) = e_{2,i}(s) \quad , \tag{40}$$

$$\vdots$$

$$e'_{n-2,i}(s) = e_{n-1,i}(s) \tag{41}$$

or

$$e'_{k,i}(s) = e_{k+1,i}(s) \quad \text{for } k = 1,\dots,n-2 \quad . \tag{42}$$

Equation (33) may be written as

$$\frac{\mathrm{d}}{\mathrm{d}s}\overset{(n-2)}{e}_{1,i}(s) = \bar{u}_i(s) \quad . \tag{43}$$

Substituting (39) for $k = n-1$ yields

$$\frac{\mathrm{d}}{\mathrm{d}s}e_{n-1,i}(s) = \bar{u}_i(s) \quad . \tag{44}$$

Using the state vector

$$\underline{e}_i = \left[\, e_{1,i} \;\cdots\; e_{n-1,i} \,\right]^{\mathrm{T}} \tag{45}$$

yields the linear state equation

$$\underline{e}'_i = \underline{A}\,\underline{e}_i + \underline{b}\,\bar{u}_i(s) \tag{46}$$

with

$$\underline{A} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & 0 & 1 \\ 0 & & & 0 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad . \tag{47}$$

Denoting the deviation from the nominal time gap to the preceding vehicle as $\Delta_i(s)$ and to the leading vehicle as $\Delta_i^0(s)$,

$$\Delta_i(s) = t_i(s) - t_{i-1}(s) - \Delta t \quad , \tag{48}$$

$$\Delta_i^0(s) = t_i(s) - t_0(s) - i\,\Delta t \quad , \tag{49}$$

the time gap tracking error is defined as

$$\delta_{1,i}(s) = (1-\kappa_0)\,\Delta_i(s) + \kappa_0\,\Delta_i^0(s) + \kappa\,e_{1,i}(s) \tag{50}$$

with $0 \le \kappa_0 < 1$ and the tracking error of the inverse velocity $e_{1,i}(s)$ according to (38).

Note that

$$\Delta_i^0(s) = \Delta_i(s) + \Delta_{i-1}^0(s) \tag{51}$$

and

$$\Delta'_i(s) = t'_i(s) - t'_{i-1}(s) = h_i\left(\underline{\xi}_i(s)\right) - h_{i-1}\left(\underline{\xi}_{i-1}(s)\right) \tag{52}$$

with

$$h_i\left(\underline{\xi}_i(s)\right) = e_{1,i}(s) + \frac{1}{v_{\text{ref}}(s)} \quad , \tag{53}$$

$$h_{i-1}\left(\underline{\xi}_{i-1}(s)\right) = e_{1,i-1}(s) + \frac{1}{v_{\text{ref}}(s)} \quad , \tag{54}$$

i.e.

$$\Delta'_i(s) = e_{1,i}(s) - e_{1,i-1}(s) \tag{55}$$

or

$$\kappa\,\Delta'_i(s) = \kappa\,e_{1,i}(s) - \kappa\,e_{1,i-1}(s) \quad . \tag{56}$$

Solving the definition (50) of the time gap tracking error for $\kappa\,e_{1,i}(s)$ and substituting into (56) yields

$$\begin{aligned} &\kappa\,\Delta'_i(s) \\ &= \delta_{1,i}(s) - (1-\kappa_0)\,\Delta_i(s) - \kappa_0\,\Delta_i^0(s) - \kappa\,e_{1,i-1}(s) \\ &= -\Delta_i(s) + \delta_{1,i}(s) + \kappa_0\left(\Delta_i(s) - \Delta_i^0(s)\right) - \kappa\,e_{1,i-1}(s) \quad . \end{aligned} \tag{57}$$

Substituting (51) yields

$$\begin{aligned} &\kappa\,\Delta'_i(s) \\ &= -\Delta_i(s) + \delta_{1,i}(s) + \kappa_0\left(\Delta_i(s) - \left(\Delta_i(s) - \Delta_{i-1}^0(s)\right)\right) \\ &\quad - \kappa\,e_{1,i-1}(s) \\ &= -\Delta_i(s) + \delta_{1,i}(s) - \kappa_0\,\Delta_{i-1}^0(s) - \kappa\,e_{1,i-1}(s) \quad . \end{aligned} \tag{58}$$

This dynamics is induced by the definition of the time gap tracking error according to (50).

The terms of the preceding vehicle are summarized according to

$$y_{i-1}(s) = -\kappa_0\,\Delta_{i-1}^0(s) - \kappa\,e_{1,i-1}(s) \quad , \tag{59}$$

which yields

$$\kappa\,\Delta'_i(s) = -\Delta_i(s) + \delta_{1,i}(s) + y_{i-1}(s) \quad . \tag{60}$$

Additional time gap tracking error coordinates are defined as

$$\delta_{k,i}(s) = (1-\kappa_0)\,(e_{k-1,i} - e_{k-1,i-1}) + \kappa_0\,(e_{k-1,i} - e_{k-1,0}) + \kappa\,e_{k,i} \tag{61}$$

for $k = 2,\dots,n-1$ and $i \in \mathcal{T}_N$. The dynamics of the platoon is now represented using the timing error coordinates

$$\underline{x}_i = \left[\,\Delta_i \;\; \underline{\delta}_i^{\mathrm{T}}\,\right]^{\mathrm{T}} \tag{62}$$

with

$$\underline{\delta}_i^{\mathrm{T}} = \left[\,\delta_{1,i} \;\cdots\; \delta_{n-1,i}\,\right] \quad . \tag{63}$$

Equation (60) may be written as

$$\Delta_i'(s) = \frac{1}{\kappa}\left(-\Delta_i(s) + \delta_{1,i}(s) + y_{i-1}(s)\right) \quad . \tag{64}$$

Equation (50) yields

$$\delta_{1,i}'(s) = (1-\kappa_0)\,\Delta_i'(s) + \kappa_0 \frac{\mathrm{d}}{\mathrm{d}s}\Delta_i^0(s) + \kappa\, e_{1,i}'(s) \quad . \tag{65}$$

Note that

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}s}\Delta_i^0(s) &= t_i'(s) - t_0'(s) \\ &= h_i\left(\underline{\xi}_i(s)\right) - h_0\left(\underline{\xi}_0(s)\right) \\ &= e_{1,i}(s) + \frac{1}{v_{\mathrm{ref}}(s)} - \left(e_{1,0}(s) + \frac{1}{v_{\mathrm{ref}}(s)}\right) \\ &= e_{1,i}(s) - e_{1,0}(s) \end{aligned} \tag{66}$$

holds. Substituting (55), (66), and (40) into (65) yields

$$\begin{aligned} \delta_{1,i}'(s) = &(1-\kappa_0)\,\left(e_{1,i}(s) - e_{1,i-1}(s)\right) \\ &+ \kappa_0\,\left(e_{1,i}(s) - e_{1,0}(s)\right) + \kappa\, e_{2,i}(s) \quad , \end{aligned} \tag{67}$$

and substituting (61) for $k=2$ yields

$$\delta_{1,i}'(s) = \delta_{2,i}(s) \quad . \tag{68}$$

For $k = 2, \ldots, n-2$, differentiating (61) yields

$$\begin{aligned} \delta_{k,i}'(s) = &(1-\kappa_0)\,\left(e_{k-1,i}'(s) - e_{k-1,i-1}'(s)\right) \\ &+ \kappa_0\,\left(e_{k-1,i}'(s) - e_{k-1,0}'(s)\right) + \kappa\, e_{k,i}'(s) \quad , \end{aligned} \tag{69}$$

and substituting (42) yields

$$\begin{aligned} \delta_{k,i}'(s) = &(1-\kappa_0)\,\left(e_{k,i}(s) - e_{k,i-1}(s)\right) \\ &+ \kappa_0\,\left(e_{k,i}(s) - e_{k,0}(s)\right) + \kappa\, e_{k+1,i}(s) \quad . \end{aligned} \tag{70}$$

Substituting (61) yields

$$\delta_{k,i}'(s) = \delta_{k+1,i}(s) \quad . \tag{71}$$

For $k = n-1$, differentiating (61) yields

$$\begin{aligned} \delta_{n-1,i}'(s) = &(1-\kappa_0)\,\left(e_{n-2,i}'(s) - e_{n-2,i-1}'(s)\right) \\ &+ \kappa_0\,\left(e_{n-2,i}'(s) - e_{n-2,0}'(s)\right) + \kappa\, e_{n-1,i}'(s) \quad . \end{aligned} \tag{72}$$

Substituting (41) and (44) yields

$$\begin{aligned} \delta_{n-1,i}'(s) = &(1-\kappa_0)\,\left(e_{n-1,i}(s) - e_{n-1,i-1}(s)\right) \\ &+ \kappa_0\,\left(e_{n-1,i}(s) - e_{n-1,0}(s)\right) + \kappa\, \bar{u}_i(s) \quad . \end{aligned} \tag{73}$$

Introducing $\tilde{u}_i(s)$ according to

$$\begin{aligned} \bar{u}_i(s) = &-\kappa^{-1}\,(1-\kappa_0)\,\left(e_{n-1,i}(s) - e_{n-1,i-1}(s)\right) \\ &-\kappa^{-1}\,\kappa_0\,\left(e_{n-1,i}(s) - e_{n-1,0}(s)\right) + \tilde{u}_i(s) \quad , \end{aligned} \tag{74}$$

(73) may be written as

$$\begin{aligned} \delta_{n-1,i}'(s) = &(1-\kappa_0)\,\left(e_{n-1,i}(s) - e_{n-1,i-1}(s)\right) \\ &+ \kappa_0\,\left(e_{n-1,i}(s) - e_{n-1,0}(s)\right) \\ &- (1-\kappa_0)\,\left(e_{n-1,i}(s) - e_{n-1,i-1}(s)\right) \\ &- \kappa_0\,\left(e_{n-1,i}(s) - e_{n-1,0}(s)\right) + \kappa\, \tilde{u}_i(s) \\ = &\,\kappa\, \tilde{u}_i(s) \quad . \end{aligned} \tag{75}$$

Using (64), (71), and (75), the state equation of the follower vehicle $i \in \mathcal{T}_N$ in timing error coordinates may be written as

$$\begin{aligned} \underline{x}_i'(s) = &\begin{bmatrix} -\frac{1}{\kappa} & \begin{bmatrix} \frac{1}{\kappa} & 0 & \cdots & 0 \end{bmatrix} \\ \underline{0} & \underline{A} \end{bmatrix} \underline{x}_i(s) + \begin{bmatrix} 0 \\ \kappa\, \underline{b} \end{bmatrix} \tilde{u}_i(s) \\ &+ \begin{bmatrix} \frac{1}{\kappa} \\ \underline{0} \end{bmatrix} y_{i-1}(s) \end{aligned} \tag{76}$$

with $\underline{A}$ and $\underline{b}$ according to (47).

Equation (59) may be written as

$$y_i(s) = -\kappa_0\,\Delta_i^0(s) - \kappa\, e_{1,i}(s) \quad , \tag{77}$$

and (50) as

$$-\kappa_0\,\Delta_i^0(s) - \kappa\, e_{1,i}(s) = (1-\kappa_0)\,\Delta_i(s) - \delta_{1,i}(s) \quad . \tag{78}$$

Substituting (78) into (77) yields

$$y_i(s) = (1-\kappa_0)\,\Delta_i(s) - \delta_{1,i}(s) \tag{79}$$

or

$$y_i(s) = \begin{bmatrix} (1-\kappa_0) & -1 & 0 & \cdots & 0 \end{bmatrix} \underline{x}_i(s) \quad , \tag{80}$$

the output equation of the follower vehicle $i \in \mathcal{T}_N$ in timing error coordinates.

Using the decentralized controller

$$\tilde{u}_i(s) = \begin{bmatrix} 0 & \underline{k}_i^{\mathrm{T}} \end{bmatrix} \underline{x}_i(s) \quad , \tag{81}$$

the closed-loop dynamics of the follower vehicle $i \in \mathcal{T}_N$ in timing error coordinates is given by

$$\begin{aligned} \underline{x}_i'(s) = &\begin{bmatrix} -\frac{1}{\kappa} & \begin{bmatrix} \frac{1}{\kappa} & 0 & \cdots & 0 \end{bmatrix} \\ \underline{0} & \underline{A} \end{bmatrix} \underline{x}_i(s) + \begin{bmatrix} 0 \\ \kappa\, \underline{b} \end{bmatrix} \begin{bmatrix} 0 & \underline{k}_i^{\mathrm{T}} \end{bmatrix} \underline{x}_i(s) \\ &+ \begin{bmatrix} \frac{1}{\kappa} \\ \underline{0} \end{bmatrix} y_{i-1}(s) \\ = &\begin{bmatrix} -\frac{1}{\kappa} & \begin{bmatrix} \frac{1}{\kappa} & 0 & \cdots & 0 \end{bmatrix} \\ \underline{0} & \underline{A} \end{bmatrix} \underline{x}_i(s) + \begin{bmatrix} 0 & \underline{0}^{\mathrm{T}} \\ \underline{0} & \kappa\, \underline{b}\, \underline{k}_i^{\mathrm{T}} \end{bmatrix} \underline{x}_i(s) \\ &+ \begin{bmatrix} \frac{1}{\kappa} \\ \underline{0} \end{bmatrix} y_{i-1}(s) \\ = &\begin{bmatrix} -\frac{1}{\kappa} & \begin{bmatrix} \frac{1}{\kappa} & 0 & \cdots & 0 \end{bmatrix} \\ 0 & \underline{A} + \kappa\, \underline{b}\, \underline{k}_i^{\mathrm{T}} \end{bmatrix} \underline{x}_i(s) + \begin{bmatrix} \frac{1}{\kappa} \\ \underline{0} \end{bmatrix} y_{i-1}(s) \quad . \end{aligned} \tag{82}$$

Choosing $\kappa > 0$ and $\underline{k}_i^{\mathrm{T}}$ such that $\underline{A} + \kappa\, \underline{b}\, \underline{k}_i^{\mathrm{T}}$ is stable yields a stable dynamics of the controlled follower vehicle $i \in \mathcal{T}_N$.

For the leader vehicle $i = 0$, the deviation from the nominal time gap is defined as

$$\Delta_0(s) = t_0(s) - \int_{\sigma=0}^{s} \frac{1}{v_{\mathrm{ref}}(\sigma)} \mathrm{d}\sigma \quad , \tag{83}$$

$$\Delta_0^0(s) = \Delta_0(s) \quad , \tag{84}$$

and the additional time gap tracking error coordinates as

$$\delta_{k,0}(s) = e_{k-1,0}(s) + \kappa\, e_{k,0}(s) \tag{85}$$

for $k = 2, \ldots, n-1$. This yields the state equation of the leader vehicle $i = 0$ in timing error coordinates

$$\underline{x}_0'(s) = \begin{bmatrix} -\frac{1}{\kappa} & \begin{bmatrix} \frac{1}{\kappa} & 0 & \cdots & 0 \end{bmatrix} \\ \underline{0} & \underline{A} \end{bmatrix} \underline{x}_0(s) + \begin{bmatrix} 0 \\ \kappa\, \underline{b} \end{bmatrix} \tilde{u}_0(s) \tag{86}$$

with $\underline{A}$ and $\underline{b}$ according to (47). The output equation is given by

$$y_0(s) = \left[\,(1-\kappa_0)\ -1\ 0\ \cdots\ 0\,\right]\underline{x}_0(s) \quad . \tag{87}$$

Using the decentralized controller

$$\tilde{u}_0(s) = \left[\,0\ \underline{k}_0^{\mathrm{T}}\,\right]\underline{x}_0(s) \quad , \tag{88}$$

the closed-loop dynamics of the leader vehicle $i = 0$ in timing error coordinates is given by

$$\underline{x}_0'(s) = \begin{bmatrix} -\frac{1}{\kappa} & \begin{bmatrix} \frac{1}{\kappa} & 0 & \cdots & 0 \end{bmatrix} \\ \underline{0} & \underline{A} + \kappa\,\underline{b}\,\underline{k}_0^{\mathrm{T}} \end{bmatrix}\underline{x}_0(s) \quad . \tag{89}$$

4. SIMULATION EXAMPLE

Consider the vehicle dynamics in the time domain

$$\frac{\mathrm{d}}{\mathrm{d}t}s_i(t) = \tilde{v}_i(t) \quad , \tag{90}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{v}_i(t) = \frac{1}{m_i}\tilde{F}_i(t) \quad , \tag{91}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{F}_i(t) = \frac{1}{\tau_i}\left(-\tilde{F}_i(t) + \tilde{u}_i(t)\right) \quad . \tag{92}$$

The vehicle dynamics in the spatial domain is then given by

$$t_i'(s) = \frac{1}{v_i(s)} \quad , \tag{93}$$

$$v_i'(s) = \frac{F_i(s)}{m_i\,v_i(s)} \quad , \tag{94}$$

$$F_i'(s) = -\frac{F_i(s)}{\tau_i\,v_i(s)} + \frac{1}{\tau_i\,v_i(s)}\,u_i(s) \quad . \tag{95}$$

The Lie derivatives in (37) are given by

$$L_{\underline{f}_i}^2 h_i = \frac{1}{m_i}\left(\frac{3}{m_i}\frac{F_i^2(s)}{v_i^5(s)} + \frac{1}{\tau_i}\frac{F_i(s)}{v_i^4(s)}\right) \quad , \tag{96}$$

$$L_{\underline{g}_i}L_{\underline{f}_i}h_i = -\frac{1}{m_i\tau_i}\frac{1}{v_i^4(s)} \quad . \tag{97}$$

Thus, the feedback linearizing control (37) may be written as

$$\begin{aligned} u_i(s) = {} & F_i(s) + 3\,\frac{\tau_i}{m_i}\frac{F_i^2(s)}{v_i(s)} \\ & - m_i\,\tau_i\,v_i^4(s)\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2}\left(\frac{1}{v_{\mathrm{ref}}(s)}\right) + \bar{u}_i(s)\right) \end{aligned} \tag{98}$$

with

$$\begin{aligned} \bar{u}_i(s) = {} & \frac{1}{\kappa}\frac{1}{m_i}\frac{F_i(s)}{v_i^3(s)} - \frac{1-\kappa_0}{\kappa}\frac{1}{m_{i-1}}\frac{F_{i-1}(s)}{v_{i-1}^3(s)} \\ & - \frac{\kappa_0}{\kappa}\frac{1}{m_0}\frac{F_0(s)}{v_0^3(s)} + k_{1,i}\,\delta_{1,i}(s) + k_{2,i}\,\delta_{2,i}(s) \end{aligned} \tag{99}$$

for $i \in \mathcal{T}_N$. The reference velocity is defined according to

$$v_{\mathrm{ref}}(s) = \begin{cases} 20 - 2\left(1 - \cos\left(10^{-2}\pi\,(s-300)\right)\right) & \text{for } 300 \le s \le 500 \\ 20 & \text{otherwise} \end{cases} \tag{100}$$

as in Besselink and Johansson (2017). Thus,

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{1}{v_{\mathrm{ref}}(s)}\right) = \begin{cases} \dfrac{0.02\,\pi\sin\left(10^{-2}\pi\,(s-300)\right)}{v_{\mathrm{ref}}^2(s)} & \text{for } 300 \le s \le 500 \\ 0 & \text{otherwise} \end{cases} \tag{101}$$

and

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\left(\frac{1}{v_{\mathrm{ref}}(s)}\right) = \begin{cases} \dfrac{0.0002\,\pi^2\cos\left(10^{-2}\pi\,(s-300)\right)}{v_{\mathrm{ref}}^2(s)} \\ \quad + \dfrac{0.0008\,\pi^2\sin^2\left(10^{-2}\pi\,(s-300)\right)}{v_{\mathrm{ref}}^3(s)} & \text{for } 300 \le s \le 500 \\ 0 & \text{otherwise} \end{cases} \quad . \tag{102}$$

The system parameter values used in the simulations are given in Table 1. Note that the dynamics of each vehicle is defined using an individual mass m_i and an individual time constant τ_i. The control parameters according to Table 2 are chosen as in Besselink and Johansson (2017) with the corrections given in Besselink and Johansson (2018).

Table 1. System parameter values

Parameter	Value	Parameter	Value
m_i	$i+1$	τ_i	$i+1$

Table 2. Control parameter values

Parameter	Value	Parameter	Value
Δt	1	ω_0	0.05
κ_0	0.1	ζ_0	0.9
κ	2	$k_{1,i}$	$-\frac{\omega_0^2}{\kappa}$
		$k_{2,i}$	$-\frac{2\zeta_0\omega_0}{\kappa}$

The simulation results for $N = 5, 20, 40$ are shown in Fig. 1, Fig. 2, and Fig. 3, respectively. For $N = 5$, the initial velocities are chosen as

$$\left[\,v_0(0)\ \cdots\ v_5(0)\,\right]^{\mathrm{T}} = \left[\,19\ 22\ 18\ 21\ 18.5\ 23\,\right]^{\mathrm{T}} \quad . \tag{103}$$

For $N = 20$ and $N = 40$, the initial velocities are randomly chosen in the interval $[18, 22]$. The initial time is chosen in compliance with the delay-based spacing policy as $t_i(0) = i\,\Delta t$ and the initial force as $F_i(0) = 0$ for $i \in \mathcal{T}_N^0$. The simulations were executed in Matlab using `ode45` for solving the differential equations.

In these simulations, the leader vehicle $i = 0$ is automatically controlled in order to track the predefined desired velocity profile. In another scenario, the leader vehicle may be driven by a human driver, thus defining the desired velocity profile for the follower vehicles online.

5. CONCLUSION

The control design for homogeneous platoons presented in Besselink and Johansson (2017) that achieves tracking of the desired delay-based spacing policy was extended in this paper to the case of heterogeneous platoons. The results were illustrated by an academic example. Next, we will implement the control in the simulation environment Plexe, see Segata et al. (2015), and compare the results for realistic vehicle models with other control approaches, see Seeland et al. (2020).

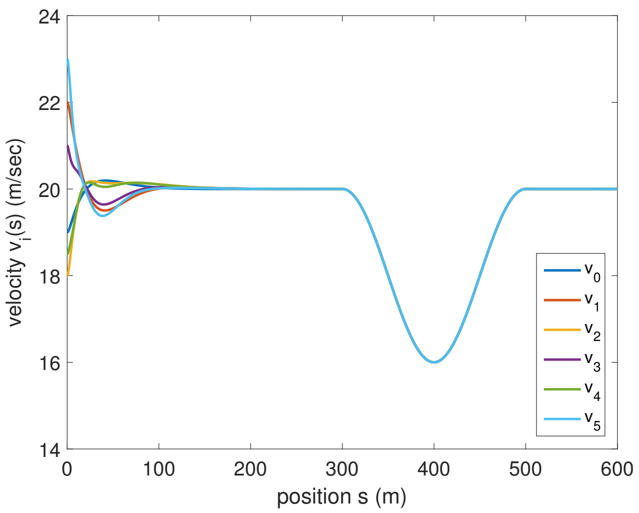

Fig. 1. Simulation 1: Velocities $v_i(s)$ for $i \in \mathcal{T}_5^0 = \{0, 1, 2, 3, 4, 5\}$

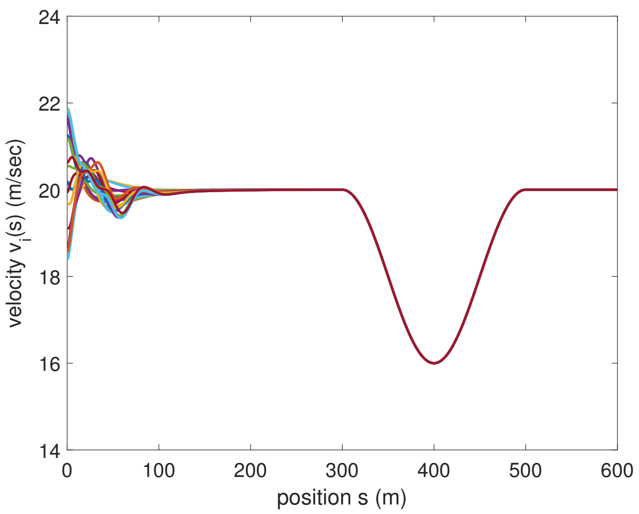

Fig. 2. Simulation 2: Velocities $v_i(s)$ for $i \in \mathcal{T}_{20}^0 = \{0, 1, \ldots, 20\}$

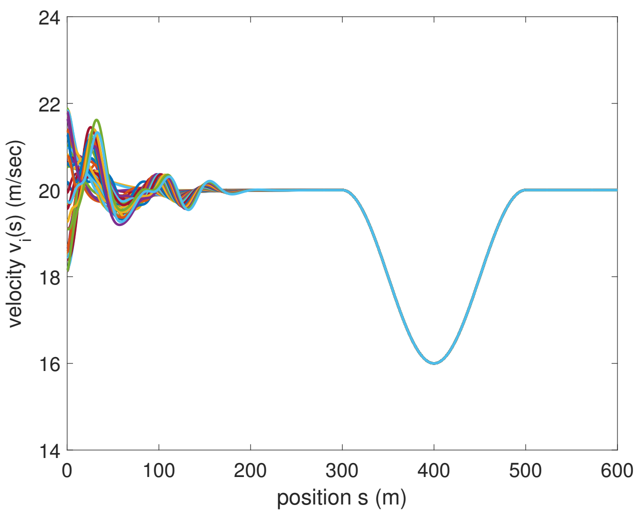

Fig. 3. Simulation 3: Velocities $v_i(s)$ for $i \in \mathcal{T}_{40}^0 = \{0, 1, \ldots, 40\}$

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