Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

PID Control of Nonlinear Stochastic Systems with Structural Uncertainties ⋆

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Abstract: It is widely known that the classical PID (proportional-integral-derivative) controller still plays a dominating role in engineering control systems, and that most of the theoretical studies on PID control focus on linear deterministic systems. In this paper, we will extend the authors recent theoretical investigation by considering additional uncertainties in the input channel, and try to establish a theoretical foundation on the PID control for a class of high-dimensional nonlinear stochastic systems with structural uncertainties consisting of dynamics uncertainty, diffusion uncertainty and input channel uncertainty. We will construct a three dimensional parameter set based on the available information, so that under the classical PID control, the closed-loop control system can be globally stabilized with regulation error tending to zero in the mean square sense, as long as the three PID parameters are chosen from this set. We will further show that global stabilization and asymptotic regulation of a class of multi-agent uncertain nonlinear stochastic systems can also be achieved by uncoupled PID controllers of the agents.

Keywords: PID control, nonlinear systems, random noises

1. INTRODUCTION

With the rapid development in information technology, various advanced control techniques have been proposed and investigated over the past 60 years. However, the classical linear proportional-integral-derivative (PID) controller is still found to be a dominating feedback law in engineering control systems (see, e.g. Åström and Hägglund (1995), Åström et al. (2006), Samad (2017)).

The PID controller is constructed based on the present (P-term), past (I-term) and future (D-term) control errors and it does not rely on the precise mathematical models of the controlled system. The simple and easy-to-use linear structure of the PID control has shown its supremacy in real world control applications, however, most of the practical PID control systems are not well tuned (see, e.g. O'Dwyer (2006)). In fact, almost all of the existing tuning methods including the well-known Ziegler-Nichols rules heavily rely on experience of the operators, which makes it rather complicated to achieve the desired control performance. In view of this, extensive research attention has been paid to the theoretical investigation on PID control. However, except for a few related investigation on nonlinear systems (see, e.g. Killingsworth and Krstic (2006), Xue and Huang (2018)), mostly are focus on linear deterministic systems (see, e.g. Åström et al. (2006), Silva et al. (2004)). Furthermore, practical control systems are bound to contain various uncertainties including random noise and hence the random cases also deserve investigation (see, Cong and Guo (2017), Zhang and Guo (2019a)).

These motivate our investigation on the fundamental theory of PID control(see, e.g. Zhao and Guo (2017a), Zhao and Guo (2017b), Krstic (2017), Zhang and Guo (2019b), Cong and Guo (2017),Zhang and Guo (2019a)). For instance, it has been shown in Zhao and Guo (2017b) that a three dimensional parameter set can be constructed explicitly within which the PID parameters can be chosen arbitrarily to achieve global stabilization for a class of one dimensional uncertain nonlinear systems, and such result can be extended to stochastic system, see, e.g.Cong and Guo (2017). In Zhang and Guo (2019b), we proved that similar results can be obtained for high-dimensional nonlinear system and the regulation error vanishes exponentially. Moreover, it has also been shown in Zhang and Guo (2019a) that such method can be extended to stochastic system without input channel uncertainty.

The main purpose of the current paper is to extend our previous results on stochastic nonlinear uncertain systems to include uncertainty in the input channel. To be specific, we will construct a three dimensional PID parameters' set on the basis of the upper bounds of partial derivatives of both the drift and diffusion functions, and will show that the PID controlled closed-loop can be globally stabilized with exponential convergence rate in the mean square sense. Moreover, we will show that a similar approach can be used to deal with a class of multi-agent uncertain stochastic systems, and will demonstrate that uncoupled PID controllers have the expected capability in dealing with coupled nonlinear systems.

⋆ This work was supported by the National Natural Science Foundation of China under Grants 11688101. (Corresponding author: Lei Guo.)

The remainder of this paper is organized as follows. In Section 2, we will introduce some denotations and formulate the mathematical problem based on Newton's second law. Main results of this paper will be provided in Section 3, with their proofs given in Section 4.

2. PROBLEM FORMULATION

First, we will give some notations to be used in the rest of this paper:

Denote $\mathbb{R}^{3+} = (0,\infty)\times(0,\infty)\times(0,\infty)$. The Euclidean norm of a vector or matrix is $\|\cdot\|$. Let $\boldsymbol{\Phi}:\mathbb{R}^n\to\mathbb{R}^m$ be a matrix function defined by

$$\frac{\partial\boldsymbol{\Phi}(\boldsymbol{x})}{\partial\boldsymbol{x}} = \begin{bmatrix} \frac{\partial\Phi_1}{\partial x_1} & \cdots & \frac{\partial\phi_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial\phi_m}{\partial x_1} & \cdots & \frac{\partial\phi_m}{\partial x_n}\end{bmatrix}, \quad \forall\boldsymbol{x}\in\mathbb{R}^n.$$

Now, consider a moving body in $\mathbb{R}^n$, with its position, velocity and acceleration denoted by $\boldsymbol{p}(t),\boldsymbol{v}(t)$, and $\boldsymbol{a}(t)$ respectively. Assume that the moving body is affected by three kind of external forces: uncertain system force $\boldsymbol{f}(\boldsymbol{p},\boldsymbol{v})$, an unknown disturbance force $\boldsymbol{\sigma}$"white noise", and the control force $B\boldsymbol{u}$, where $B\in\mathbb{R}^{n\times n}$ is an unknown positive definite parameter matrix in the input channel with known lower and upper bounds, i.e., $\bar{b}I_n \ge B \ge \underline{b}I_n > 0$.

By the well-known Newton's second law, we have the following kinetic equation:

$$m\boldsymbol{a} = \boldsymbol{f}(\boldsymbol{p},\boldsymbol{v}) + B\boldsymbol{u} + \boldsymbol{\sigma}(\boldsymbol{p},\boldsymbol{v})\text{“}\textit{white noise}\text{”}, \qquad (1)$$

where m is the mass of the body.

We assume that the control force is produced by a classical PID controller:

$$\boldsymbol{u}(t) = k_p\boldsymbol{e}(t) + k_i\int_0^t \boldsymbol{e}(s)\,ds + k_d\dot{\boldsymbol{e}}(t), \qquad (2)$$

where k_p,k_i,k_d are the three PID parameters to be designed in the paper, and where $\boldsymbol{e}$ is the control error, defined by

$$\boldsymbol{e}(t) = \boldsymbol{r^*} - \boldsymbol{p}(t),$$

with $\boldsymbol{r^*}$ being a given vector-valued setpoint.

The control objective is to design a PID controller to guarantee that the moving body can be induced to any setpoint $\boldsymbol{r^*}$ from any initial position with arbitrary initial velocity.

In the sequel, we assume that the body has the unit mass $m=1$ for simplicity. Mathematically, the "white noise" in the continuous-time case can be roughly regarded as the "derivative" of a standard Brownian motion $\{\omega(t)\}_{t\ge0}$ defined on a complete probability space $(\Omega,\mathscr{F},P)$. Denote $\boldsymbol{x}_1=\boldsymbol{p}$ and $\boldsymbol{x}_2=\dot{\boldsymbol{p}}$, then we have the following corresponding state space equation with PID control:

$$\begin{cases} d\boldsymbol{x_1} = \boldsymbol{x_2}dt \\ d\boldsymbol{x_2} = \boldsymbol{f}(\boldsymbol{x_1},\boldsymbol{x_2},t)dt + B\boldsymbol{u}(t)dt + \boldsymbol{\sigma}(\boldsymbol{x_1},\boldsymbol{x_2},t)d\omega(t) \\ \boldsymbol{u}(t) = k_p\boldsymbol{e}(t) + k_i\int_0^t \boldsymbol{e}(s)\,ds + k_d\dot{\boldsymbol{e}}(t) \end{cases} \qquad (3)$$

where $\boldsymbol{x_1}(0),\boldsymbol{x_2}(0)\in\mathbb{R}^n$ and $\boldsymbol{e}(t) = \boldsymbol{r}^* - \boldsymbol{x_1}(t)$.

We will show that under the control law (2) with three PID controller parameters k_p,k_i,k_d chosen from a constructed set, the position of the body can track a given vector-valued setpoint $\boldsymbol{r^*}$ with the regulation error vanishes exponentially fast in the mean square sense for any initial position and velocity, as long as both $\boldsymbol{f}=\boldsymbol{f}(\boldsymbol{x_1},\boldsymbol{x_2},t)$ and $\boldsymbol{\sigma}(\boldsymbol{x_1},\boldsymbol{x_2},t)$ are continuously differentiable functions with known upper bounds for their partial derivatives.

3. MAIN RESULTS

Motivated by the investigation of maximum feedback capability in Xie and Guo (2000), we first define two function spaces to describe the uncertainty of systems mathematically as follows:

$$\mathcal{F}_{M_1,M_2} = \left\{\boldsymbol{f}\in C^1(\mathbb{R}^{2n}\times\mathbb{R}^+,\mathbb{R}^n)\,\middle|\, \left\|\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x_1}}\right\|\le M_1, \left\|\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x_2}}\right\|\le M_2,\ \forall\boldsymbol{x_1},\boldsymbol{x_2}\in\mathbb{R}^n,\ \forall t\in\mathbb{R}^+,\right\}$$

$$\mathcal{P}_{N_1,N_2} = \left\{\boldsymbol{\sigma}\in C^1(\mathbb{R}^{2n}\times\mathbb{R}^+,\mathbb{R}^n)\,\middle|\, \left\|\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{x_1}}\right\|\le N_1, \left\|\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{x_2}}\right\|\le N_2,\ \forall\boldsymbol{x_1},\boldsymbol{x_2}\in\mathbb{R}^n,\ \forall t\in\mathbb{R}^+,\right\}$$

where M_1, M_2, N_1 and N_2 are known positive constants, and $C^1(\mathbb{R}^{2n}\times\mathbb{R}^+,\mathbb{R}^n)$ denotes the space of all functions from $\mathbb{R}^{2n}\times\mathbb{R}^+$ to $\mathbb{R}^n$ which are piecewise continuous in t and with continuous partial derivatives with respect to $(\boldsymbol{x_1},\boldsymbol{x_2})$. We further assume that for all $t\in\mathbb{R}^+$ and $\boldsymbol{r}\in\mathbb{R}^n$, $\boldsymbol{f}(\boldsymbol{r},0,t)=\boldsymbol{f}(\boldsymbol{r},0,0)$, $\boldsymbol{\sigma}(\boldsymbol{r},0,t)=\boldsymbol{\sigma}(\boldsymbol{r},0,0)$ and $\boldsymbol{\sigma}(\boldsymbol{r^*},0,t)=0$.

Next, we introduce the following three-dimensional parameter set:

$$\Omega_{pid} = \left\{(\bar{k}_p,k_i,\bar{k}_d)\in\mathbb{R}^{3+}\,\middle|\,\underline{b}^2\bar{k}_p\bar{k}_d - \bar{b}k_i - \frac{1}{2}N_1^2 > \{(\underline{b}\bar{k}_pM_1 + \underline{b}\bar{k}_pN_1N_2 + \bar{b}k_iM_2 + \frac{1}{2}M_2N_1^2)(M_1 + \underline{b}\bar{k}_dM_2 + N_1N_2)\}^{\frac{1}{2}}\right\}$$

where $\bar{k}_p = k_p - \frac{1}{\underline{b}}M_1$, $\bar{k}_d = k_d - \frac{1}{\underline{b}}(M_2 + \frac{1}{2}N_2^2)$.

Below is the main result of this paper:

Theorem 1. Consider the PID controlled uncertain system (3) with unknown nonlinear function $\boldsymbol{f}\in\mathcal{F}_{M_1,M_2}$ and $\boldsymbol{\sigma}\in\mathcal{P}_{N_1,N_2}$. Then for any $M_1,M_2,N_1,N_2>0$, whenever the controller parameters (k_p,k_i,k_d) are taken such that $(\bar{k}_p,k_i,\bar{k}_d)\in\Omega_{pid}$, the state of the closed-loop control system (3) will satisfy

$$\lim_{t\to\infty}E\|\boldsymbol{x_1}(t)-\boldsymbol{r^*}\|^2=0,\ \lim_{t\to\infty}E\|\boldsymbol{x_2}(t)\|^2=0,$$

with exponential convergence rate, for any initial value $\boldsymbol{x_1}(0),\boldsymbol{x_2}(0)\in\mathbb{R}^n$ and any vector-valued setpoint $\boldsymbol{r}^*\in\mathbb{R}^n$.

Remark 1. When $\sigma=0$, i.e. $N_1=0,N_2=0$, and $\bar{b}=\underline{b}=1$, Theorem 1 will be degenerated to the special case considered in Zhang and Guo (2019b).

Remark 2. It is quite obvious that the set Ω_{pid} defined in Theorem 1 is an open unbounded set, which gives some flexibility for the choice of PID parameters. Besides, it is easy to convince oneself that the feedback gains in the PID controller are not necessary large.

Remark 3. The upper bounds of the derivatives for both the drift and diffusion terms may be obtained based on the physical mechanism or some prior information. Also, when the upper bounds are not constants, some semi-global results may be derived (see, e.g.Zhao and Guo (2019)).

Furthermore, we point out that similar results may be got for a class of coupled multi-agent stochastic systems (see, Yuan et al. (2018)), where each agent can be described as:

$$\begin{cases} dx_{1j} = dx_{2j}dt, \\ dx_{2j} = f_j(\boldsymbol{x_1}, \boldsymbol{x_2}, t)dt + B_j u_j(t)dt + \sigma_j(\boldsymbol{x_1}, \boldsymbol{x_2}, t)d\omega(t), \\ u_j(t) = k_p^j e_j(t) + k_i^j \displaystyle\int_0^t e_j(s)\, ds + k_d^j \dot{e}_j(t), \end{cases} \tag{4}$$

where $x_{1j}, x_{2j} \in \mathbb{R}^m$ and $\bar{b}I_m \geq B_j \geq \underline{b}I_m > 0$, $j = 1, 2, \cdots, n$.

Denote

$$\boldsymbol{x_1} = (x_{11}^{\boldsymbol{\tau}}, x_{12}^{\boldsymbol{\tau}}, \cdots, x_{1n}^{\boldsymbol{\tau}})^{\boldsymbol{\tau}}, \quad \boldsymbol{x_2} = (x_{21}^{\boldsymbol{\tau}}, x_{22}^{\boldsymbol{\tau}}, \cdots, x_{2n}^{\boldsymbol{\tau}})^{\boldsymbol{\tau}},$$
$$\boldsymbol{f} = (f_1^{\boldsymbol{\tau}}, f_2^{\boldsymbol{\tau}}, \cdots, f_n^{\boldsymbol{\tau}})^{\boldsymbol{\tau}}, \qquad \boldsymbol{\sigma} = (\sigma_1^{\boldsymbol{\tau}}, \sigma_2^{\boldsymbol{\tau}}, \cdots, \sigma_n^{\boldsymbol{\tau}})^{\boldsymbol{\tau}}.$$

To deal with this problem, we extend the PID parameters and the input channel to positive definite matrices as follows:

$$\begin{aligned} \boldsymbol{k_p} &= diag(k_p^1 I_m, k_p^2 I_m, \cdots, k_p^n I_m), \\ \boldsymbol{k_i} &= diag(k_i^1 I_m, k_i^2 I_m, \cdots, k_i^n I_m), \\ \boldsymbol{k_d} &= diag(k_d^1 I_m, k_d^2 I_m, \cdots, k_d^n I_m), \\ B &= diag(B_1, B_2, \cdots, B_n), \end{aligned}$$

where $k_p^j, k_i^j, k_d^j > 0$, $j = 1, 2, \cdots, n$.

Then $\boldsymbol{k_p}, \boldsymbol{k_i}, \boldsymbol{k_d}, B \in \mathbb{R}^{n^* \times n^*}$, where $n^* = mn$.

Next, we introduce the following parameter set:

$$\Omega_{pid}^* = \Bigg\{ (\bar{\boldsymbol{k}}_{\boldsymbol{p}}, \|\boldsymbol{k_i}\|, \bar{\boldsymbol{k}}_{\boldsymbol{d}}) \in \mathbb{R}^{3+} \Bigg| \underline{b}^2 \bar{\boldsymbol{k}}_{\boldsymbol{p}} \bar{\boldsymbol{k}}_{\boldsymbol{d}} - \bar{b}\|\boldsymbol{k_i}\| - \frac{1}{2}N_1^2 > \big\{ (\underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{p}} M_1 + \underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{p}} N_1 N_2 + \bar{b}\|\boldsymbol{k_i}\| M_2 + \frac{1}{2} M_2 N_1^2)(M_1 + \underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{d}} M_2 + N_1 N_2) \big\}^{\frac{1}{2}} \Bigg\}$$

where $\bar{\boldsymbol{k}}_{\boldsymbol{p}} = \min\limits_j k_p^j - \frac{1}{\underline{b}} M_1$, $\bar{\boldsymbol{k}}_{\boldsymbol{d}} = \min\limits_j k_d^j - \frac{1}{\underline{b}}(M_2 + \frac{1}{2}N_2^2)$ and $\|\boldsymbol{k_i}\| = \max\limits_j k_i^j$.

The following Theorem shows that the uncoupled PID controllers have the expected capability in dealing with coupled nonlinear systems.

Theorem 2. Consider the PID controlled stochastic systems (4) with uncertain functions $\boldsymbol{f} \in \mathcal{F}_{M_1,M_2}$ and $\boldsymbol{\sigma} \in \mathcal{P}_{N_1,N_2}$. Then for any $M_1, M_2, N_1, N_2 > 0$, whenever the controller parameter matrix $(\boldsymbol{k_p}, \boldsymbol{k_i}, \boldsymbol{k_d})$ are taken such that $(\bar{\boldsymbol{k}}_{\boldsymbol{p}}, \|\boldsymbol{k_i}\|, \bar{\boldsymbol{k}}_{\boldsymbol{d}}) \in \Omega_{pid}^*$, the state of the closed-loop control system (3) will satisfy

$$\lim_{t\to\infty} E\|\boldsymbol{x_1}(t) - \boldsymbol{r^*}\|^2 = 0, \ \lim_{t\to\infty} E\|\boldsymbol{x_2}(t)\|^2 = 0,$$

with exponential convergence rate, for any initial value $\boldsymbol{x_1}(0), \boldsymbol{x_2}(0) \in \mathbb{R}^n$ and any vector-valued setpoint $\boldsymbol{r}^* \in \mathbb{R}^n$.

4. PROOFS OF THE THEOREMS

4.1 *Proof of Theorem 1*

Proof. Denote $\boldsymbol{e_i}(t) = \int_0^t \boldsymbol{e}(s)\, ds + (Bk_i)^{-1}\boldsymbol{f}(\boldsymbol{r}^*, 0, 0)$, $\boldsymbol{e}(t) = \boldsymbol{e}(t), \boldsymbol{e_d}(t) = \dot{\boldsymbol{e}}(t), \boldsymbol{g_1}(\boldsymbol{e}, \boldsymbol{e_d}, t) = -\boldsymbol{f}(\boldsymbol{r}^* - \boldsymbol{e}, -\boldsymbol{e_d}, t) + \boldsymbol{f}(\boldsymbol{r}^*, 0, t)$ and $\boldsymbol{g_2}(\boldsymbol{e}, \boldsymbol{e_d}, t) = -\boldsymbol{\sigma}(\boldsymbol{r}^* - \boldsymbol{e}, -\boldsymbol{e_d}, t)$, then (3) can be rewritten as

$$\begin{cases} d\boldsymbol{e_i} = \boldsymbol{e}dt \\ d\boldsymbol{e} = \boldsymbol{e_d}dt \\ d\boldsymbol{e_d} = [\boldsymbol{g_1}(\boldsymbol{e}, \boldsymbol{e_d}, t) - Bk_i\boldsymbol{e_i} - Bk_p\boldsymbol{e} - Bk_d\boldsymbol{e_d}]dt \\ \qquad + \boldsymbol{g_2}(\boldsymbol{e}, \boldsymbol{e_d}, t)d\omega(t) \end{cases} \tag{5}$$

From $f \in \mathcal{F}_{M_1,M_2}$, we have $\boldsymbol{g_1} \in \mathcal{F}_{M_1,M_2}$ and $\boldsymbol{g_1}(0, 0, t) = 0$, $\forall t > 0$. Similarly, it is easy to see that $\boldsymbol{g_2} \in \mathcal{P}_{M_1,M_2}$ and $\boldsymbol{g_2}(0, 0, t) = 0$, $\forall t > 0$. Hence (0,0,0) is an equilibrium of (5).

Note that by utilizing the mean value theorem of integral type (see, e.g. Huang and Liu (2016)), $\boldsymbol{g_1}(\boldsymbol{e}, \boldsymbol{e_d}, t)$ can be decomposed as :

$$\begin{aligned} &\boldsymbol{g_1}(\boldsymbol{e}, \boldsymbol{e_d}, t) \\ =&[\boldsymbol{g_1}(\boldsymbol{e}, 0, t) - \boldsymbol{g_1}(0, 0, t)] + [\boldsymbol{g_1}(\boldsymbol{e}, \boldsymbol{e_d}, t) - \boldsymbol{g_1}(\boldsymbol{e}, 0, t)] \\ =&\left\{\int_0^1 \frac{\partial \boldsymbol{g_1}(\bar{\boldsymbol{e}}, 0, t)}{\partial \bar{\boldsymbol{e}}}\, d\lambda\right\}\boldsymbol{e} + \left\{\int_0^1 \frac{\partial \boldsymbol{g_1}(\boldsymbol{e}, \bar{\boldsymbol{e}}_{\boldsymbol{d}}, t)}{\partial \bar{\boldsymbol{e}}_{\boldsymbol{d}}}\, d\lambda\right\}\boldsymbol{e_d} \\ \triangleq&\beta_1(\boldsymbol{e}, t)\boldsymbol{e} + \alpha_1(\boldsymbol{e}, \boldsymbol{e_d}, t)\boldsymbol{e_d} \end{aligned}$$

where $\bar{\boldsymbol{e}} = \lambda\boldsymbol{e}$ and $\bar{\boldsymbol{e}}_{\boldsymbol{d}} = \lambda\boldsymbol{e_d}$.

Notice the fact that $\boldsymbol{g_1} \in \mathcal{F}_{M_1,M_2}$, we can deduce the upper bounds of $\|\beta_1(\boldsymbol{e}, t)\|$ and $\|\alpha_1(\boldsymbol{e}, \boldsymbol{e_d}, t)\|$ by using Schwarz inequality:

$$\begin{aligned} \|\beta_1(\boldsymbol{e}, t)\| &= \left\|\int_0^1 \frac{\partial \boldsymbol{g_1}(\bar{\boldsymbol{e}}, 0, t)}{\partial \bar{\boldsymbol{e}}}\, d\lambda\right\| \\ &\leq \sqrt{\int_0^1 \left\|\frac{\partial \boldsymbol{g_1}(\bar{\boldsymbol{e}}, 0, t)}{\partial \bar{\boldsymbol{e}}}\right\|^2 d\lambda} \leq M_1, \end{aligned}$$

Similarly, we can deduce $\|\alpha_1(\boldsymbol{e}, \boldsymbol{e_d}, t)\| \leq M_2$.

Through the same approach, $\boldsymbol{g_2}(\boldsymbol{e}, \boldsymbol{e_d}, t)$ can be expressed as

$$\boldsymbol{g_2}(\boldsymbol{e}, \boldsymbol{e_d}, t) \triangleq \beta_2(\boldsymbol{e}, t)\boldsymbol{e} + \alpha_2(\boldsymbol{e}, \boldsymbol{e_d}, t)\boldsymbol{e_d},$$

where $\|\beta_2(\boldsymbol{e}, \boldsymbol{e_d}, t)\| \leq N_1$ and $\|\alpha_2(\boldsymbol{e}, \boldsymbol{e_d}, t)\| \leq N_2$.

Hence,the closed-loop equation (5) goes over into:

$$d\boldsymbol{x} = F(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t)dt + G(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t)d\omega(t) \tag{6}$$

where

$$\boldsymbol{x}^{\boldsymbol{\tau}} = [\boldsymbol{e_i^{\tau}}, \boldsymbol{e^{\tau}}, \boldsymbol{e_d^{\tau}}]$$
$$F(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) = \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{e_d} \\ \boldsymbol{\Phi}(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) \end{bmatrix}$$
$$G(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{g_2}(\boldsymbol{e}, \boldsymbol{e_d}, t) \end{bmatrix}$$

where

$$\begin{aligned} \boldsymbol{\Phi}(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) =& - Bk_i\boldsymbol{e_i} + \big(- Bk_p + \beta_1(\boldsymbol{e}, t)\big)\boldsymbol{e} \\ &+ \big(- Bk_d + \alpha_1(\boldsymbol{e}, \boldsymbol{e_d}, t)\big)\boldsymbol{e_d}. \end{aligned}$$

Similar to Reissig et al. (1974) ,we now proceed to show that the following quadratic form is indeed a Lyapunov function,

$$V(\boldsymbol{x}) = \boldsymbol{x}^{\boldsymbol{\tau}} P \boldsymbol{x}$$

where the constant matrix P is

$$P = \frac{1}{2}\begin{bmatrix} \mu B k_i & B k_i & \varepsilon I_n \\ B k_i & B k_p + \mu B k_d & \mu I_n \\ \varepsilon I_n & \mu I_n & I_n \end{bmatrix}, \tag{7}$$

μ is a constant defined by

$$\mu = \frac{2(\underline{b}^2 \bar{k}_d \bar{k}_p + \bar{b} k_i) + N_1^2 - M_1 M_2 - N_1 N_2 M_2}{4\underline{b}\bar{k}_p + M_2^2},$$

and ε is an arbitrarily small positive constant.

It is quite obvious that the matrix P is positive definite as long as P_0 is positive definite, where P_0 is a symmetric matrix defined by

$$P_0 = \begin{bmatrix} \mu B k_i & B k_i & 0 \\ B k_i & (B k_p + \mu B k_d) & \mu I_n \\ 0 & \mu I_n & I_n \end{bmatrix}.$$

To show P_0 is positive definite, we now prove the following inequalities hold,

$$\mu > 0 \tag{8}$$

$$\mu < \underline{b}\bar{k}_d \tag{9}$$

$$\begin{aligned} & 4(-\bar{b}k_i + \mu\underline{b}\bar{k}_p - \frac{1}{2}N_1^2)(-\mu + \underline{b}\bar{k}_d) \\ & > (\mu M_2 + M_1 + N_1 N_2)^2 \end{aligned} \tag{10}$$

$$-\bar{b}k_i + \mu\underline{b}\bar{k}_p - \frac{1}{2}N_1^2 > 0 \tag{11}$$

From the definition of Ω_{pid}, it is obvious to see that

$$\underline{b}^2 \bar{k}_p \bar{k}_d - \bar{b}k_i - \frac{1}{2}N_1^2 > 0,$$

and that

$$\begin{aligned} & (\underline{b}^2 \bar{k}_p \bar{k}_d - \bar{b}k_i - \frac{1}{2}N_1^2)^2 > (\underline{b}\bar{k}_p M_1 + \underline{b}\bar{k}_p N_1 N_2 + \bar{b}k_i M_2 \\ & + \frac{1}{2}M_2 N_1^2)(M_1 + \underline{b}\bar{k}_d M_2 + N_1 N_2) \end{aligned} \tag{12}$$

Consequently, the inequality (9) can be verified since

$$\begin{aligned} \mu - \underline{b}\bar{k}_d = & \frac{1}{(4\underline{b}\bar{k}_p + M_2^2)}[-2(\underline{b}^2 \bar{k}_p \bar{k}_d - \bar{b}k_i - \frac{1}{2}N_1^2) \\ & - N_1 N_2 M_2 - M_1 M_2 - M_2^2 \underline{b}\bar{k}_d] < 0. \end{aligned}$$

Furthermore, by using (12), we have

$$\begin{aligned} & 4(-\bar{b}k_i + \mu\underline{b}\bar{k}_p - \frac{1}{2}N_1^2)(-\mu + \underline{b}\bar{k}_d) - (\mu M_2 + M_1 \\ & + N_1 N_2)^2 \\ = & -(4\underline{b}\bar{k}_p + M_2^2)\mu^2 + [4(\underline{b}^2 \bar{k}_p \bar{k}_d + \bar{b}k_i) + 2N_1^2 \\ & - 2(M_1 + N_1 N_2)M_2]\mu - (4\bar{b}k_i + 2N_1^2)\underline{b}\bar{k}_d \\ & - (M_1 + N_1 N_2)^2 \\ = & \frac{4}{4\underline{b}\bar{k}_p + M_2^2}\big[(\underline{b}^2 \bar{k}_p \bar{k}_d - \bar{b}k_i - \frac{1}{2}N_1^2)^2 - (\underline{b}\bar{k}_p M_1 + \bar{b}k_i M_2 \\ & + \underline{b}\bar{k}_p N_1 N_2 + \frac{1}{2}M_2 N_1^2)(M_1 + \underline{b}\bar{k}_d M_2 + N_1 N_2)\big] \\ > & 0. \end{aligned}$$

Thus, we can deduce (10), and consequently (11) follows from (9) and (10). Additionally, from (11) and the fact $k_i, \bar{k}_d > 0$, we will see (8) is valid.

Let $H_1 = \begin{bmatrix} I_n & 0 & 0 \\ -\frac{1}{\mu}I_n & I_n & -\mu I_n \\ 0 & 0 & I_n \end{bmatrix}$, then we have

$$H_1 P_0 H_1^{\boldsymbol{\tau}} = \begin{bmatrix} \mu B k_i & & \\ & P' & \\ & & I_n \end{bmatrix},$$

where $P' = Bk_p + \mu B k_d - \frac{1}{\mu}Bk_i - \mu^2 I_n$.

Since H_1 is invertible, we only to show both the matrices $\mu B k_i$ and P' are positive definite. Noted that B is positive definite and $\mu k_i > 0$, thus we only need to verify P' is positive definite.

In fact, from (9) and (11), $\forall y \neq 0$, we have

$$\begin{aligned} & y^{\boldsymbol{\tau}} P' y \\ = & y^{\boldsymbol{\tau}}(Bk_p + \mu B k_d - \frac{1}{\mu}Bk_i - \mu^2 I_n)y \\ \geq & \frac{1}{\mu}(\mu\underline{b}k_p + \mu^2 \underline{b}k_d - \bar{b}k_i - \mu^3)\|y\|^2 > 0, \end{aligned}$$

which means P' is positive definite. As a result, we have both the matrices P_0 and P are positive defintie.

Let $\bar{\gamma}$ and $\underline{\gamma}$ be the maximum and minimum eigenvalues of P respectively. Then $\bar{\gamma} \geq \underline{\gamma} > 0$. Hence, we have

$$\underline{\gamma}\boldsymbol{x}^{\boldsymbol{\tau}}\boldsymbol{x} \leq V(\boldsymbol{x}) \leq \bar{\gamma}\boldsymbol{x}^{\boldsymbol{\tau}}\boldsymbol{x}, \tag{13}$$

and consequently, $V(\boldsymbol{x})$ is a positive definite function which is radically unbounded in $\boldsymbol{x}$.

Next, we define the differential operator L acts on the function $V(\boldsymbol{x})$ associated with the equation (6) as follows:

$$LV(\boldsymbol{x}) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}}F + \frac{1}{2}trace[G^{\boldsymbol{\tau}}\frac{\partial^2 V}{\partial \boldsymbol{x}^2}G].$$

Then, by simple calculation, we have:

$$\begin{aligned} & LV(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}) \\ = & \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}}F + \frac{1}{2}G^{\boldsymbol{\tau}}\frac{\partial^2 V}{\partial \boldsymbol{x}^2}G \\ = & \varepsilon[-k_i \boldsymbol{e_i}^{\tau} B \boldsymbol{e_i} + \boldsymbol{e_i}^{\tau}(-Bk_p + \beta_1)\boldsymbol{e} + \boldsymbol{e_i}^{\tau}(-Bk_d + \alpha_1)\boldsymbol{e_d}] \\ & - \boldsymbol{e}^{\boldsymbol{\tau}}\left[-Bk_i + \mu(Bk_p - \frac{\beta_1 + \beta_1^{\boldsymbol{\tau}}}{2}) - \frac{\beta_2^{\boldsymbol{\tau}}\beta_2}{2}\right]\boldsymbol{e} \\ & + \boldsymbol{e}^{\boldsymbol{\tau}}(\mu\alpha_1 + \beta_1^{\boldsymbol{\tau}} + \varepsilon I_n + \beta_2^{\boldsymbol{\tau}}\alpha_2)\boldsymbol{e_d} \\ & - \boldsymbol{e_d}^{\boldsymbol{\tau}}\left(-\mu I_n + Bk_d - \frac{\alpha_1 + \alpha_1^{\boldsymbol{\tau}} + \alpha_2^{\boldsymbol{\tau}}\alpha_2}{2}\right)\boldsymbol{e_d} \\ \leq & -\underline{b}k_i\varepsilon\|\boldsymbol{e_i}\|^2 + \varepsilon(\bar{b}k_p + M_1)\|\boldsymbol{e_i}\|\|\boldsymbol{e}\| \\ & + \varepsilon(\bar{b}k_d + M_2)\|\boldsymbol{e_i}\|\|\boldsymbol{e_d}\| - [-\bar{b}k_i + \mu(\underline{b}k_p - M_1) \\ & - \frac{1}{2}N_1^2]\|\boldsymbol{e}\|^2 + (\mu M_2 + M_1 + \varepsilon + N_1 N_2)\|\boldsymbol{e}\|\|\boldsymbol{e_d}\| \\ & - (-\mu + \underline{b}k_d - M_2 - \frac{1}{2}N_2^2)\|\boldsymbol{e_d}\|^2 \\ = & -[\|\boldsymbol{e_i}\|, \|\boldsymbol{e}\|, \|\boldsymbol{e_d}\|]Q[\|\boldsymbol{e_i}\|, \|\boldsymbol{e}\|, \|\boldsymbol{e_d}\|]^{\boldsymbol{\tau}}, \end{aligned}$$

where Q is a symmetric matrix, expressed by

$$Q = \begin{bmatrix} \underline{b}k_i\varepsilon & * & * \\ \frac{-(\bar{b}k_p + M_1)\varepsilon}{2} & -\bar{b}k_i + \mu\underline{b}\bar{k}_p - \frac{1}{2}N_1^2 & * \\ \frac{-(\bar{b}k_d + M_2)\varepsilon}{2} & -\frac{\mu M_2 + M_1 + N_1N_2 + \varepsilon}{2} & -\mu + \underline{b}\bar{k}_d \end{bmatrix} \tag{14}$$

Next, we will show that the matrix Q is also positive definite.

Let $H = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\bar{b}k_p + M_1}{2\underline{b}k_i} & 1 & 0 \\ \frac{\bar{b}k_d + M_2}{2\underline{b}k_i} & 0 & 1 \end{bmatrix}$, then we have

$$HQH^{\tau} = \begin{bmatrix} \underline{b}k_i\varepsilon & \\ & Q' - \varepsilon C \end{bmatrix},$$

where Q' and C are both symmetric matrices, defined by

$$Q' = \begin{bmatrix} -\bar{b}k_i + \mu\underline{b}\bar{k}_p - \frac{1}{2}N_1^2 & * \\ \frac{\mu M_2 + M_1 + N_1N_2}{2} & -\mu + \underline{b}\bar{k}_d \end{bmatrix},$$

and

$$C = \begin{bmatrix} \frac{(\bar{b}k_p + M_1)^2}{4\underline{b}k_i} & * \\ \frac{(\bar{b}k_p + M_1)(\bar{b}k_d + M_2) + 2\underline{b}k_i}{4\underline{b}k_i} & \frac{(\bar{b}k_d + M_2)^2}{4\underline{b}k_i} \end{bmatrix}.$$

Noted that H is invertible, thus we only prove $Q' - \varepsilon C$ is positive definite. By using (10) and (11), we have

$$\lim_{\varepsilon\to 0^+} -\bar{b}k_i + \mu\underline{b}\bar{k}_p - \frac{1}{2}N_1^2 - \frac{(\bar{b}k_p + M_1)^2}{4\underline{b}k_i}\varepsilon > 0,$$
$$\lim_{\varepsilon\to 0^+} det(Q' - \varepsilon C) > 0,$$

which means $Q' - \varepsilon C$ is positive definite as long as ε is small enough. Consequently, the positive definiteness of Q is valid.

Let λ present the minimum eigenvalue of Q. Thus, we have $\lambda > 0$, since Q is positive definite. Then, it is obvious to see

$$LV(\boldsymbol{x}) \le -\lambda(\boldsymbol{x}^{\tau}\boldsymbol{x}) \tag{15}$$

Next, by using the Itô formula, we have

$$dV(\boldsymbol{x}(t)) = LV(\boldsymbol{x}(t))dt + \Phi(\boldsymbol{x}(t), t)d\omega(t),$$

where $\Phi(\boldsymbol{x}, t)$ is expressed as:

$$\Phi(\boldsymbol{x}, t) = (\varepsilon\boldsymbol{e_i^{\tau}} + \mu\boldsymbol{e^{\tau}} + \boldsymbol{e_d^{\tau}})(\beta_2(\boldsymbol{e}, t)\boldsymbol{e} + \alpha_2(\boldsymbol{e}, \boldsymbol{e_d}, t)\boldsymbol{e_d}).$$

Then, for any $T > 0$, we have

$$\begin{aligned} &V(\boldsymbol{x}(T)) \\ =&V(\boldsymbol{x}(0)) + \int_0^T LV(\boldsymbol{x}(t))dt + \int_0^T \Phi(\boldsymbol{x}(t), t)d\omega(t). \end{aligned} \tag{16}$$

Noted that, by the boundedness of $\alpha_2(\boldsymbol{e}, \boldsymbol{e_d}, t)$ and $\beta_2(\boldsymbol{e}, t)$, we can get

$$|\Phi(\boldsymbol{x}(t), t)|^2 = O(\|\boldsymbol{x}(t)\|^4).$$

Hence, by using Theorem 2.4.1 in Mao (2007), it is obvious to see that

$$E\int_0^T |\Phi(\boldsymbol{x}(t), t)|^2 dt < \infty,$$

and consequently, we can get

$$E\int_0^T \Phi(\boldsymbol{x}(t), t)d\omega(t) = 0.$$

By taking mathematical expectation on both sides of (16) and taking derivative with respect to T, we have

$$\big(EV(\dot{\boldsymbol{x}}(T))\big) = ELV(\boldsymbol{x}(T)),$$

thus, by using (13) and (15) we can deduce the following inequality:

$$EV(\boldsymbol{x}(t)) \le EV(\boldsymbol{x}(0))e^{-\frac{\lambda}{\bar{\gamma}}t},$$

hence, we can get

$$E\|\boldsymbol{x}(t)\|^2 \le \frac{1}{\underline{\gamma}}EV(\boldsymbol{x}(0))e^{-\frac{\lambda}{\bar{\gamma}}t}, \tag{17}$$

which means

$$\lim_{t\to\infty} E\|\boldsymbol{x_1}(t) - \boldsymbol{r}^*\|^2 = 0,$$
$$\lim_{t\to\infty} E\|\boldsymbol{x_2}(t)\|^2 = 0,$$

exponentially fast.

4.2 Proof of Theorem 2

Proof. Similar to the proof of Theorem 1, the system (4) can be transformed into the following form:

$$d\boldsymbol{x} = F(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t)dt + G(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t)d\omega(t) \tag{18}$$

where

$$F(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) = \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{e_d} \\ \boldsymbol{\Phi}(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) \end{bmatrix}$$

$$G(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{g_2}(\boldsymbol{e}, \boldsymbol{e_d}, t) \end{bmatrix}$$

and

$$\begin{aligned} \boldsymbol{\Phi}(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}, t) =& - B\boldsymbol{k_i}\boldsymbol{e_i} + \big(- B\boldsymbol{k_p} + \beta_1(\boldsymbol{e}, t)\big)\boldsymbol{e} \\ &+ \big(- B\boldsymbol{k_d} + \alpha_1(\boldsymbol{e}, \boldsymbol{e_d}, t)\big)\boldsymbol{e_d}. \end{aligned}$$

Similar to the previous proof, we consider the following quadratic form

$$V(\boldsymbol{x}) = \boldsymbol{x^{\tau}}P_1\boldsymbol{x},$$

where the constant matrix P_1 is

$$P_1 = \frac{1}{2}\begin{bmatrix} \theta B\boldsymbol{k_i} & B\boldsymbol{k_i} & \varepsilon_1 I_{n^*} \\ B\boldsymbol{k_i} & B\boldsymbol{k_p} + \theta B\boldsymbol{k_d} & \theta I_{n^*} \\ \varepsilon_1 I_{n^*} & \theta I_{n^*} & I_{n^*} \end{bmatrix}, \tag{19}$$

θ is a constant defined by

$$\theta = \frac{2(\underline{b}^2\bar{\boldsymbol{k}}_{\boldsymbol{d}}\bar{\boldsymbol{k}}_{\boldsymbol{p}} + \bar{b}\,\|\boldsymbol{k_i}\|) - M_1M_2 + N_1^2 - N_1N_2M_2}{4\underline{\boldsymbol{b}}\bar{\boldsymbol{k}}_{\boldsymbol{p}} + M_2^2},$$

and ε_1 is an arbitrarily small positive constant.

Similar to the proof in Theorem 1, the following four inequities are true:

$$\theta > 0, \tag{20}$$
$$\theta < \underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{d}}, \tag{21}$$

$$\begin{aligned} &4(-\bar{b}\,\|\boldsymbol{k_i}\| + \theta\underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{p}} - \frac{N_1^2}{2})(-\theta + \underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{d}}) \\ >&(\theta M_2 + M_1 + N_1N_2)^2, \end{aligned} \tag{22}$$
$$-\bar{b}\,\|\boldsymbol{k_i}\| + \theta\underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{p}} > 0, \tag{23}$$

and consequently, the matrices P_1 is positive definite.

Denote $\bar{\boldsymbol{k}}_{\boldsymbol{i}} = \min_j k_i^j$.

By simple calculation, the differential operator L acts on the function V along the trajectories of (18) is as follows:

$$
\begin{aligned}
&LV(\boldsymbol{e_i}, \boldsymbol{e}, \boldsymbol{e_d}) \\
=&\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}} F + \frac{1}{2} G^{\boldsymbol{\tau}} \frac{\partial^2 V}{\partial \boldsymbol{x}^2} G \\
=&-\varepsilon_1 [\boldsymbol{e_i^\tau} B \boldsymbol{k_i} \boldsymbol{e_i} + \boldsymbol{e_i^\tau}(-B\boldsymbol{k_p} + \beta_1)\boldsymbol{e} + \boldsymbol{e_i^\tau}(-B\boldsymbol{k_d} + \alpha_1)\boldsymbol{e_d}] \\
&-\boldsymbol{e^\tau}\left[-\boldsymbol{k_i} + \theta(B\boldsymbol{k_p} - \frac{\beta_1 + \beta_1^{\boldsymbol{\tau}}}{2}) - \frac{\beta_2^{\boldsymbol{\tau}}\beta_2}{2}\right]\boldsymbol{e} \\
&+\boldsymbol{e^\tau}(\theta\alpha_1 + \beta_1^{\boldsymbol{\tau}} + \beta_2^{\boldsymbol{\tau}}\alpha_2 + \varepsilon_1 I_{n^*})\boldsymbol{e_d} \\
&-\boldsymbol{e_d}^{\boldsymbol{\tau}}\left(-\theta I_{n^*} + B\boldsymbol{k_d} - \frac{\alpha_1 + \alpha_1^{\boldsymbol{\tau}} + \alpha_2^{\boldsymbol{\tau}}\alpha_2}{2}\right)\boldsymbol{e_d} \\
\leq&-\varepsilon_1 \underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{i}} \|\boldsymbol{e_i}\|^2 + \varepsilon_1(\bar{b}\|\boldsymbol{k_p}\| + M_1)\|\boldsymbol{e_i}\|\|\boldsymbol{e}\| \\
&+\varepsilon_1(\bar{b}\|\boldsymbol{k_d}\| + M_2)\|\boldsymbol{e_i}\|\|\boldsymbol{e_d}\| - [-\bar{b}\|\boldsymbol{k_i}\| + \theta\underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{p}} \\
&-\frac{N_1^2}{2}]\|\boldsymbol{e}\|^2 + (\theta M_2 + M_1 + N_1N_2 + \varepsilon_1)\|\boldsymbol{e}\|\|\boldsymbol{e_d}\| \\
&-(-\theta + \underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{d}} - \frac{N_2^2}{2})\|\boldsymbol{e_d}\|^2 \\
=&-[\|\boldsymbol{e_i}\|, \|\boldsymbol{e}\|, \|\boldsymbol{e_d}\|] Q_1 [\|\boldsymbol{e_i}\|, \|\boldsymbol{e}\|, \|\boldsymbol{e_d}\|]^{\boldsymbol{\tau}},
\end{aligned}
$$

where Q_1 is a symmetric matrix, expressed by

$$
\begin{bmatrix}
\underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{i}}\varepsilon_1 & \frac{-(\bar{b}\|\boldsymbol{k_p}\| + M_1)\varepsilon_1}{2} & \frac{-(\bar{b}\|\boldsymbol{k_d}\| + M_2)\varepsilon_1}{2} \\
* & -\bar{b}\|\boldsymbol{k_i}\| + \theta\underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{p}} - \frac{N_1^2}{2} & -\frac{\theta M_2 + M_1 + N_1N_2 + \varepsilon_1}{2} \\
* & * & -\theta + \underline{b}\bar{\boldsymbol{k}}_{\boldsymbol{d}} - \frac{N_2^2}{2}
\end{bmatrix}
$$

Then, we could deduce that the constant matrix Q_1 is positive definite through the same approach used in the previous proof.

Similar to the proof of Theorem 1, the conclusion of Theorem 2 could be verified.

5. CONCLUSION

In this paper, we have provided a theoretical analysis together with an explicit parameters' design method on PID control for nonlinear stochastic systems with structural uncertainties including input channel uncertainty. It has been shown that based on some prior knowledge about the upper bounds of the derivatives for both the drift and diffusion terms, a three dimensional set can be obtained from which the PID parameters can be chosen arbitrarily to globally stabilize the nonlinear uncertain systems with exponentially vanishing regulation errors in the mean square sense. It has been also shown that for a class of nonlinearly coupled multi-agent stochastic systems, uncoupled PID controllers can be designed to achieve the desired control performance. We remark that the results of this paper include the authors' previously established results as special cases (Zhang and Guo (2019b), Zhang and Guo (2019a)). For further investigation, it would be interesting to consider more complicated situations such as inputs saturation and time-delays, etc.

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