

Control System Analysis and Design of Quadcopter in the Presence of Unmodelled Dynamics and Disturbances

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Abstract: UAVs especially quadcopters have recently caught the attention of researchers and manufacturers due to their various commercial and military applications like surveillance, photography and many others. They have small sizes since have low cost, easy manufacturing, extreme maneuverability and VTOL capabilities. This paper addresses the problem of unmodelled dynamics and disturbances while designing an appropriate control law for the quadcopter UAV having very coupled nonlinear dynamics. Most of the controllers available in the literature ignore Coriolis terms in the model and small signal approximations are made to linearize or simplify the model about certain operating conditions. But such control system has a very limited performance and fails to deliver the desired results even for small disturbances and parametric variations since the assumptions no longer remain valid. We have derived an extensive nonlinear model of quadcopter with least approximations in terms of linear velocities in body frame, position in the inertial frame, the Euler angles and their rates. We have designed a feedback linearization based nonlinear controller using a novel approach. This has further been cascaded with sliding mode control and backstepping based control to handle uncertainties. The simulation results of this controller have also been included for a known quadcopter model.

Keywords: Nonlinear Control; Feedback Linearization; Quadcopter; Unmodelled Dynamics and Disturbances;

1. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) are flying vehicles without having a human pilot onboard (Valavanis and Vachtsevanos (2015)). Quadcopters are a subclass of rotorcraft UAVs with four propellers mounted on a + or × shaped rigid frame for thrust and orientation control. Their sizes are much smaller than the conventional aircraft hence the advantages of low power consumption, manufacturing cost indoor usage. They have geometrical symmetry which makes the modelling and controller design easy as compared to other rotorcraft. Their extreme maneuverability and Vertical Take-off and Landing (VTOL) features make it even superior to other UAVs. These are the reasons that we have considered control system design problem of quadcopter.

In the literature, quadcopters have a wide range of control system applications like cooperative and formation control Ali and Montenegro (2016), Yamamoto et al. (2017) and altitude control. Furthermore, many controller design schemes have been proposed like fuzzy control Sun and Liu (2017), linear parameter varying (LPV) control Cisneros et al. (2016), adaptive control Lin et al. (2013), Ding et al. (2017), Song and Wang (2018), predictive control Tanveer et al. (2014), Alexis et al. (2012), neural network control-

Hwang (2012), nonlinear control methods Kumar et al. (2017)-Mellinger and Kumar (2011) and sliding mode control Falcón et al. (2018). While these are all excellent contributions we saw a need of improvising the model and derive the model with lesser approximations and in a form that is easily controllable. Also the linear system approximation has a very limited performance region so we developed a nonlinear control law. Our contributions in this paper are that we have derived the quadcopter nonlinear model with least small signal approximations and assumptions in section (2). Feedback linearization based control of the derived model has been designed in section (3). Finally the simulation results of the proposed control scheme and future work has been discussed in section (??) and (7) respectively.

2. QUADCOPTER MODELLING

For quadcopter modelling we have considered three coordinate frames that are inertial, vehicle and body frame, which have been explained in the Fig. 1 and Fig. 2

The state vector is defined as

$$\mathcal{X} \triangleq \begin{bmatrix} x & y & z & \phi & \theta & \psi & \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}$$

Where x, y and z are the components of position vector of O_b in inertial frame along \hat{i}^i, \hat{j}^i and \hat{k}^i respectively as

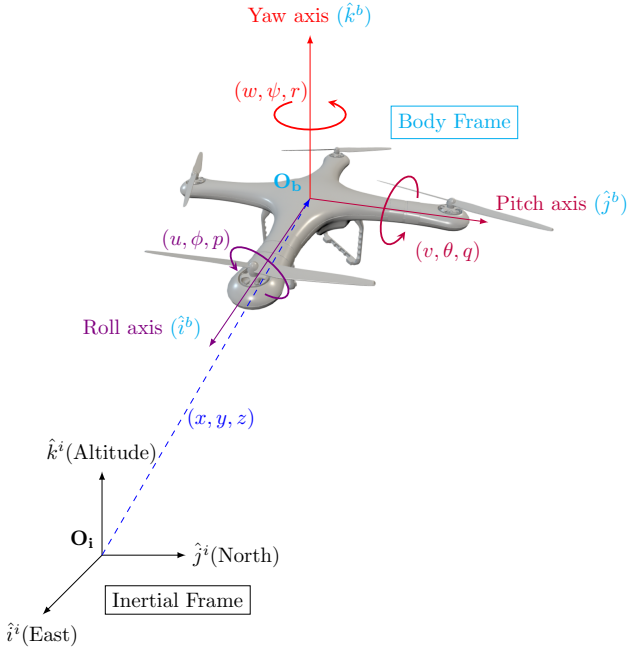


Fig. 1. The **Body Frame** and Inertial Frame

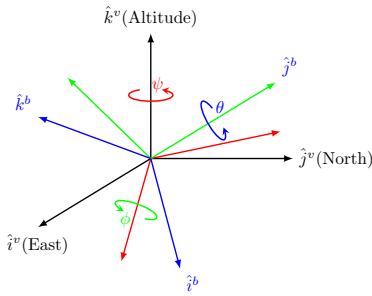


Fig. 2. Transformation to **Body Frame** from Vehicle Frame

shown in the Fig. 1. And ϕ, θ and ψ are the Euler angles that define the transformation from vehicle frame to body frame as expressed in Fig. 2. O_i is the origin of inertial frame located at the ground control station where O_b is the origin of vehicle frame and body frame located at the center of mass of the quadcopter. It should be noted that inertial frame and vehicle frame have parallel coordinate axis only the origin is located elsewhere.

We have derived the expression for the transformation from vehicle to body frame. Take $c_x \triangleq \cos x$ and $s_x \triangleq \sin x$ then

$$R = \begin{bmatrix} c_\phi c_\theta - s_\phi s_\psi s_\theta & -c_\phi s_\psi & c_\phi s_\theta + c_\theta s_\phi s_\psi \\ c_\theta s_\psi + c_\psi s_\phi s_\theta & c_\phi c_\psi & s_\psi s_\theta - c_\psi c_\theta s_\phi \\ -c_\phi s_\theta & s_\phi & c_\phi c_\theta \end{bmatrix} \quad (1)$$

This is a very peculiar transformation belonging to *special orthogonal group* ($SO(3)$) hence if $R = R_{v \rightarrow b}$ then $R_{b \rightarrow v} = R^{-1} = R^T$ since for $R \in SO(3)$

$$R = \{R \in \mathbb{R}^3 \mid R^T R = R R^T = I, \det(R) = 1\}$$

Now the acceleration according to Newton's second law in inertial frame can be written as

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R^T \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix} \quad (2)$$

Where T is the total thrust force produced by all the rotors. And m is the total mass of quadcopter with g being the gravitational constant. If we express the roll, pitch and yaw rates i.e the angular velocities in body frame as p, q and r respectively then these quantities in terms of the Euler angle rates can be given as

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \cos \phi \\ 0 & 1 & \sin \phi \\ \sin \theta & 0 & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (3)$$

Say the angular velocities vector $\omega = [p \ q \ r]^T$ and from (3)

$$R_\omega = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \cos \phi \\ 0 & 1 & \sin \phi \\ \sin \theta & 0 & \cos \phi \cos \theta \end{bmatrix}$$

In the linearized models R_ω is assumed to be identity using small signal approximation, but as mentioned earlier we shall not be making any assumptions. Differentiating (3) both sides we get

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = R_\omega \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} \sin \phi \sin \theta \dot{\phi} \dot{\psi} - \cos \theta \cos \phi \dot{\theta} \dot{\psi} - \sin \theta \dot{\theta} \dot{\phi} \\ \cos \phi \dot{\phi} \dot{\psi} \\ \cos \theta \dot{\theta} \dot{\phi} - \sin \phi \cos \theta \dot{\phi} \dot{\psi} - \sin \theta \cos \phi \dot{\theta} \dot{\psi} \end{bmatrix}$$

Say

$$A = \begin{bmatrix} \sin \phi \sin \theta \dot{\phi} \dot{\psi} - \cos \theta \cos \phi \dot{\theta} \dot{\psi} - \sin \theta \dot{\theta} \dot{\phi} \\ \cos \phi \dot{\phi} \dot{\psi} \\ -\sin \phi \cos \theta \dot{\phi} \dot{\psi} - \sin \theta \cos \phi \dot{\theta} \dot{\psi} + \cos \theta \dot{\theta} \dot{\phi} \end{bmatrix} \quad (4)$$

Then

$$\dot{\omega} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = R_\omega \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + A \quad (5)$$

Remark 1. The term A in (5) is often ignored in literature to simplify Euler angle dynamics, this might make sense for linearized model controller designs, but in some cases it is ignored even when nonlinear controllers are being designed. See Joukhadar et al. (2019) and Navabi and Mirzaei (2016). This is because term R_ω is taken as an identity matrix using small signal approximations for the Euler angles. This is a very impractical supposition since ϕ and θ are never small enough to be taken as close to zero while performing practical maneuvers. This is the reason controller designed for a linearized model has a very limited performance region while controllers designed using nonlinear model without unrealistic suppositions show excellent performance for a wide region. But we have not taken R_ω as identity to develop a model with least possible assumptions.

The angular acceleration $\dot{\omega} = [\dot{p} \ \dot{q} \ \dot{r}]^T$ can be given by Newton's second law and adding the Coriolis terms we get

$$J \dot{\omega} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} - \omega \times J \omega \quad (6)$$

Here ' \times ' represents the cross product. τ_ϕ, τ_θ and τ_ψ are the moments along \hat{b}_1, \hat{b}_2 and \hat{b}_3 respectively as shown in Fig. (??). J is the matrix of inertia and can be expressed as as a diagonal matrix due to the symmetry of quadcopter as follows

$$J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}$$

J_{xx}, J_{yy} and J_{zz} are the moment of inertia of quadcopter about \hat{b}_1, \hat{b}_2 and \hat{b}_3 respectively. Using (2), (5) and (6) we have the system dynamics as follows

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + R^T \begin{bmatrix} 0 \\ 0 \\ \frac{T}{m} \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = R_{\omega}^{-1} J^{-1} \left(\begin{bmatrix} \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix} - \omega \times J\omega \right) - R_{\omega}^{-1} A$$

The (7) describe the nonlinear model of quadcopter in terms of position in inertial frame linear velocities in body frame, Euler angles and their rates.

3. FEEDBACK LINEARIZATION BASED CONTROL

Quadcopter is an under-actuated system there are only four inputs and twelve state variables. We can effectively control four states at a time. We aim to control position and yaw or heading angle that is $[x^{des}, y^{des}, z^{des}]^T$ and ψ^{des} as shown in Fig.(3). In (7) we can see that there is

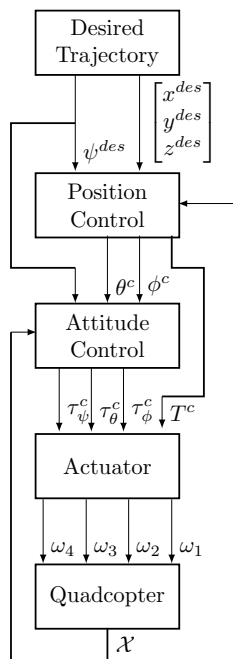


Fig. 3. Control Scheme for Quadcopter

only one input variable T and three output variables $[x \ y \ z]$ in the first equation, so for feedback linearization we desire to transform this system in the form

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} u_{\phi} \\ u_{\theta} \\ u_{\psi} \end{bmatrix} \quad (9)$$

Where $u_x, u_y, u_z, u_{\phi}, u_{\theta}$ and u_{ψ} are the inputs we can control as we desire. From (7) and (8) we can write

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + R^T \begin{bmatrix} 0 \\ 0 \\ \frac{T}{m} \end{bmatrix}$$

$$R \begin{bmatrix} u_x \\ u_y \\ u_z + g \end{bmatrix} = RR^T \begin{bmatrix} 0 \\ 0 \\ \frac{T}{m} \end{bmatrix}$$

$$\begin{bmatrix} c_{\phi}c_{\theta} - s_{\phi}s_{\psi}s_{\theta} & -c_{\phi}s_{\psi} & c_{\phi}s_{\theta} + c_{\theta}s_{\phi}s_{\psi} \\ c_{\theta}s_{\psi} + c_{\psi}s_{\phi}s_{\theta} & c_{\phi}c_{\psi} & s_{\psi}s_{\theta} - c_{\psi}c_{\theta}s_{\phi} \\ -c_{\phi}s_{\theta} & s_{\phi} & c_{\phi}c_{\theta} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z + g \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{T}{m} \end{bmatrix}$$

The above system can be solved for the commanded values of ϕ, θ and T and they come out to be as follows

$$\begin{aligned} \phi^c &= \tan^{-1} \frac{u_x s_{\psi^c} - u_y c_{\psi^c}}{u_z + g} \\ \theta^c &= \tan^{-1} \frac{u_x c_{\psi^c} + u_y s_{\psi^c}}{(u_z + g)c_{\phi^c} + (s_{\psi^c}u_x - c_{\psi^c}u_y)s_{\phi^c}} \quad (10) \\ T^c &= \begin{bmatrix} c_{\psi^c}s_{\theta^c} + c_{\theta^c}s_{\phi^c}c_{\psi^c} \\ s_{\psi^c}s_{\theta^c} - c_{\theta^c}s_{\phi^c}s_{\psi^c} \\ c_{\phi^c}c_{\theta^c} \end{bmatrix}^T \begin{bmatrix} u_x \\ u_y \\ u_z + g \end{bmatrix} \end{aligned}$$

Remark 2. The typical feedback linearization techniques use complex Lie algebra and there are often some approximations or limitations involved taking into account the fact that this is an underactuated system with transcendental and bilinear coupled state dynamics. But novelty and contribution of our controller lies in the fact that we have used the very basic definition of feedback linearization process and derived the control inputs utilizing a special property ($R^T = R^{-1}$) of the $SO(3)$ group of matrices. This way the expressions for control inputs are way simpler and easy to derive as compared to typical methods. See Chang and Eun (2014).

In (10) $\psi^c = \psi^{des}$. Now from (7) and (9) we can write

$$\begin{bmatrix} u_{\phi} \\ u_{\theta} \\ u_{\psi} \end{bmatrix} = R_{\omega}^{-1} J^{-1} \left(\begin{bmatrix} \tau_{\phi} \\ \tau_{\theta} \\ \tau_{\psi} \end{bmatrix} - \omega \times J\omega \right) - R_{\omega}^{-1} A$$

$$\Rightarrow \begin{bmatrix} \tau_{\phi}^c \\ \tau_{\theta}^c \\ \tau_{\psi}^c \end{bmatrix} = \omega \times J\omega + J \left(A + R_{\omega} \begin{bmatrix} u_{\phi} \\ u_{\theta} \\ u_{\psi} \end{bmatrix} \right) \quad (11)$$

The commanded values in (10) and (11) ensure that the system follows (8) and (9). Now the states of the system follow simple double integrator system dynamics so no integral action is needed for zero steady state error and desired tracking performance can be achieved by applying proportional derivative control strategies to $u_x, u_y, u_z, u_{\phi}, u_{\theta}$ and u_{ψ} as described in (??). For proportional derivative (PD) control we simply use the strategy in (Khalil (2002)) that is for any system of the form

$$\ddot{s} = u_s \quad (12)$$

We define an error

$$e \triangleq s^{des} - s$$

For tracking this error must exponentially go to zero that means the error must satisfy

$$\ddot{e} + k_{ps}e + k_{vs}\dot{e} = 0 \quad (k_{ps}, k_{vs} > 0)$$

$$\Rightarrow (\ddot{s}^{des} - u_s) + k_{ps}(s^{des} - s) + k_{vs}(\dot{s}^{des} - \dot{s}) = 0$$

Then our control input must be

$$u_s = \ddot{s}^{des} + k_{ps}(s^{des} - s) + k_{vs}(\dot{s}^{des} - \dot{s}) \quad (13)$$

From the linearized model in (8) and (9) we can see that all the states in \mathcal{Q} follow the dynamics similar to (12), hence the control in (13) can be extended to find the linearized control inputs as follows,

The Position Control Inputs:

$$\begin{aligned} u_x &= \ddot{x}^{des} + k_{px}(x^{des} - x) + k_{vx}(\dot{x}^{des} - \dot{x}) \\ u_y &= \ddot{y}^{des} + k_{py}(y^{des} - y) + k_{vy}(\dot{y}^{des} - \dot{y}) \\ u_z &= \ddot{z}^{des} + k_{pz}(z^{des} - z) + k_{vz}(\dot{z}^{des} - \dot{z}) \end{aligned} \quad (14)$$

The Attitude Control Inputs:

$$\begin{aligned} u_\phi &= \ddot{\phi}^{des} + k_{p\phi}(\phi^{des} - \phi) + k_{v\phi}(\dot{\phi}^{des} - \dot{\phi}) \\ u_\theta &= \ddot{\theta}^{des} + k_{p\theta}(\theta^{des} - \theta) + k_{v\theta}(\dot{\theta}^{des} - \dot{\theta}) \\ u_\psi &= \ddot{\psi}^{des} + k_{p\psi}(\psi^{des} - \psi) + k_{v\psi}(\dot{\psi}^{des} - \dot{\psi}) \end{aligned} \quad (15)$$

3.1 Simulation Results

Tracking for Step Input The step tracking of quadcopter using feedback linearization and the conventional PID control designed for a linearized model are being compared here. It is very interesting to note that the Feedback Linearization based control shows slightly better performance for a smaller step signal as shown in Fig. (4) but it outclasses the tracking performance when there is a larger step as shown in Fig. (5). The reason is when linearizing the model small signal approximations etc are made which hold true for small slowly varying reference signals but no longer remain valid for fast changing reference signals.

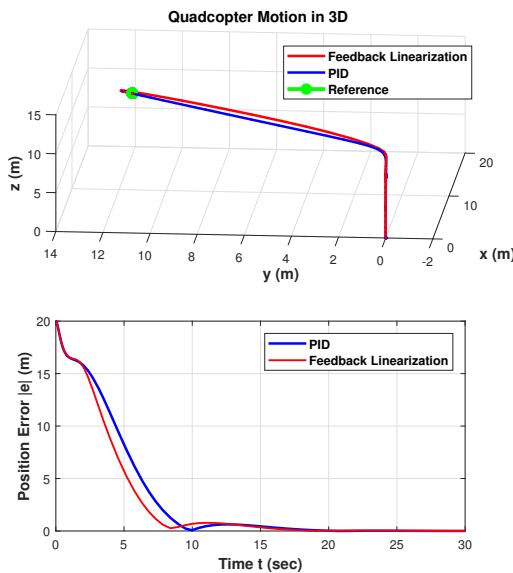


Fig. 4. Position Tracking for a Smaller Step Input

Increasing the step size further leads the PID controller towards instability while the feedback linearization based control shows excellent performance. This can be easily verified in the simulations. This behavior is also intuitively predictable seeing that as the step size increases the more conventional PID strays from the small signal

approximations and hence away from its limited region of convergence.

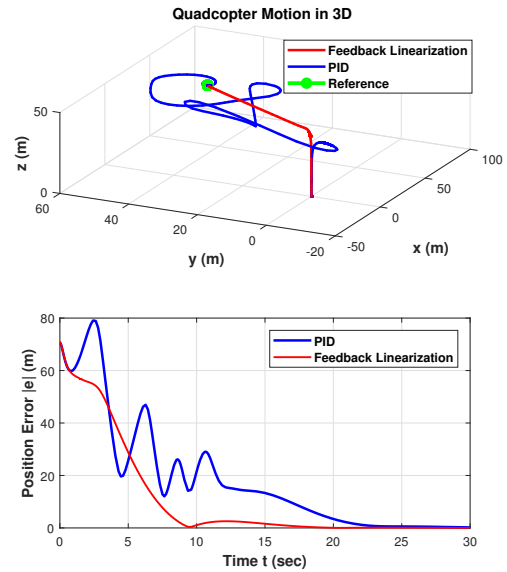


Fig. 5. Position Tracking for a Larger Step Input

Tracking for Sinusoidal Input A similar performance can be seen for a sinusoidal input or a circular trajectory in Fig. (6). That is for sinusoids of smaller amplitude or frequency the Feedback linearization shows similar or slightly better performance than conventional PID but again way better performance for sinusoids of larger amplitude or frequency.

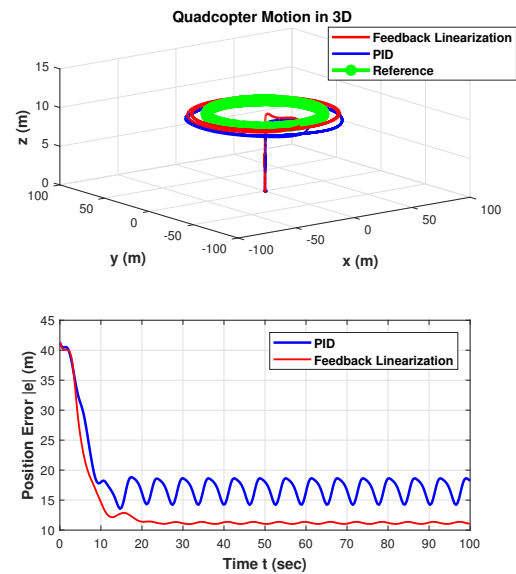


Fig. 6. Position Tracking for Sinusoidal Input

4. CONTROL IN THE PRESENCE OF UNMODELLED DYNAMICS AND DISTURBANCES

The unmodelled dynamics and disturbances play a very important role in the stability and tracking performance as

discussed in (Rohrs et al. (1985)). No matter how accurate our model is, there are always uncertainties involved when dealing with the practical systems. Therefore our control system must robust enough to handle a fair amount of them. There are always external disturbances like wind conditions and friction and unmodelled dynamics like rotor drag involved in a quadcopter flight which can be modelled as bounded uncertainties in the model.

The feedback linearization based control designed in the previous section has a great advantage that the unmodelled dynamics and disturbances can be included in the model and handled very easily with even a simple Lyapunov-based control scheme for the linearized inputs. If we include external disturbances and other bounded uncertainties in the system dynamics we can describe the model using (8) and (9) as

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} u_x + \delta_x(\mathcal{X}, t) \\ u_y + \delta_y(\mathcal{X}, t) \\ u_z + \delta_z(\mathcal{X}, t) \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} u_\phi + \delta_\phi(\mathcal{X}, t) \\ u_\theta + \delta_\theta(\mathcal{X}, t) \\ u_\psi + \delta_\psi(\mathcal{X}, t) \end{bmatrix} \quad (17)$$

For each state ξ in \mathcal{Q} , each $\delta_\xi(\mathcal{X}, t)$ in (17) and (16) represents a bounded uncertainty discussed above given as $\Delta_\xi \geq |\delta_\xi(\mathcal{X}, t)|$. Where Δ_ξ is the known uncertainty upper bound. Now for the controller design we again consider a system of the form

$$\ddot{s} = u_s + \delta_s(\mathcal{X}, t) \quad (18)$$

We define an error

$$e \triangleq s - s^{des} \implies \ddot{e} = u_s + \delta_s(\mathcal{X}, t) - \ddot{s}^{des}$$

For tracking of (18), this error must go to zero exponentially, so we can define $e_1 = e$ and $e_2 = \dot{e}$ and represent the above error dynamics equation in state space form as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= u_s + \delta_s - \ddot{s}^{des} \end{aligned} \quad (19)$$

Now we shall use two advanced nonlinear control strategies for uncertainty handling that are backstepping and sliding mode control.

5. BACKSTEPPING AND LYAPUNOV REDESIGN CONTROL

The control discussed here combines backstepping and Lyapunov redesign from (Khalil (2002)). First we define a Lyapunov function for e_1 and treat e_2 as input to stabilize it then we use backstepping to change the variable and find a control that stabilizes the composite Lyapunov function of e_1 and new variable.

Lets take the Lyapunov function for e_1 as,

$$V_1(e_1) = \frac{1}{2}e_1^2$$

Then $\dot{V}_1(e_1) = e_1\dot{e}_1 = e_1e_2$. For exponential stability of e_1 , we take $e_2 = \rho(e_1) = -e_1$. Therefore,

$$\dot{V}_1(e_1) = -e_1^2 < 0$$

Now we define the new variable $\zeta = e_2 - \rho(e_1) = e_2 + e_1$. Then,

$$\dot{\zeta} = u_s + \delta_s - \ddot{s}^{des} + \zeta - e_1$$

The composite Lyapunov function of e_1 and ζ can be written as,

$$V_c(e_1, \zeta) = \frac{1}{2}e_1^2 + \frac{1}{2}\zeta^2$$

For the system to be globally asymptotically stable the derivative this composite Lyapunov function must be negative definite that is,

$$\dot{V}_c(e_1, \zeta) = -e_1^2 + \zeta(u_s + \delta_s - \ddot{s}^{des} + \zeta)$$

If we select

$$u_s = \ddot{s}^{des} - (1 + \kappa_s)\zeta + v_s \quad (\kappa_s > 0)$$

Then,

$$\begin{aligned} \dot{V}_c(e_1, \zeta) &= -e_1^2 - \kappa_s\zeta^2 + \zeta(v_s + \delta_s) \\ &\leq -e_1^2 - \kappa_s\zeta^2 + \zeta v_s + \Delta_s |\zeta| \end{aligned} \quad (20)$$

Take,

$$v_s = -\eta(e) \text{sat} \left(\eta(e) \frac{\zeta}{\epsilon} \right) \quad (21)$$

Where $\eta(e) \geq |\delta_s| + \eta_{os}$, hence we can take $\eta(e) = \Delta_s + \eta_{os}$ the η_{os} and ϵ are positive constants. Here,

$$\text{sat}(\xi) \triangleq \begin{cases} \text{sgn}(\xi) & \text{if } \xi \geq 1 \\ \xi & \text{if } \xi < 1 \end{cases}, \quad (22)$$

$$\text{sgn}(\zeta) \triangleq \begin{cases} -1 & \text{if } \zeta < 0 \\ 0 & \text{if } \zeta = 0 \\ 1 & \text{if } \zeta > 0 \end{cases} \quad (23)$$

This piece-wise definition of v_s has been chosen to stabilize and remove chattering from error dynamics which is always present when the control uses signum nonlinearity. That is,

For this choice of v_s in (21) the derivative of composite Lyapunov function from (20) can be expressed as,

When $\eta(s)|\zeta| \geq \epsilon$,

$$\dot{V}_c(e_1, \zeta) \leq -e_1^2 - \kappa_s\zeta^2 - \eta_{os} |\zeta| < 0$$

When $\eta(s)|\zeta| < \epsilon$,

$$\dot{V}_c(e_1, \zeta) \leq -e_1^2 - \kappa_s\zeta^2 + \frac{\epsilon}{4}$$

This ensures ultimate boundedness of the solution, but ϵ small enough we can make the system very close to globally asymptotically stable. Extending this strategy to our system in (17) and (16) the linearized control inputs for quadcopter for (10) and (11) can be written as,

The Position Control Inputs:

$$\begin{aligned} u_x &= \ddot{x}^{des} + (1 + \kappa_x)\zeta_\phi + v_x \\ u_y &= \ddot{y}^{des} + (1 + \kappa_y)\zeta_\phi + v_y \\ u_z &= \ddot{z}^{des} + (1 + \kappa_z)\zeta_\phi + v_z \end{aligned} \quad (24)$$

The Attitude Control Inputs:

$$\begin{aligned} u_\phi &= \ddot{\phi}^{des} + (1 + \kappa_\phi)\zeta_\phi + v_\phi \\ u_\theta &= \ddot{\theta}^{des} + (1 + \kappa_\theta)\zeta_\theta + v_\theta \\ u_\psi &= \ddot{\psi}^{des} + (1 + \kappa_\psi)\zeta_\psi + v_\psi \end{aligned} \quad (25)$$

Where for each state ξ in \mathcal{Q} , κ_ξ is a positive constant that can be tuned and,

$$\begin{aligned} \zeta_\xi &\triangleq (\xi^{des} + \dot{\xi}^{des} - \xi - \dot{\xi}) \\ v_\xi &\triangleq -\eta_\xi \text{sat} \left(\frac{\eta_\xi \zeta_\xi}{\epsilon_\xi} \right) \\ \eta_\xi &\triangleq \Delta_\xi + \eta_{o\xi} \end{aligned} \quad (26)$$

Here $\eta_{o\xi}$ is a positive constant. The above mentioned (24), (25) and (26) describe the backstepping control design for the feedback linearized inputs of a quadcopter.

5.1 Simulation Results

We have compared the results of tracking performance of backstepping control designed above in Fig. (7). We can see that the error is converging for our controller while diverging for conventional PID.

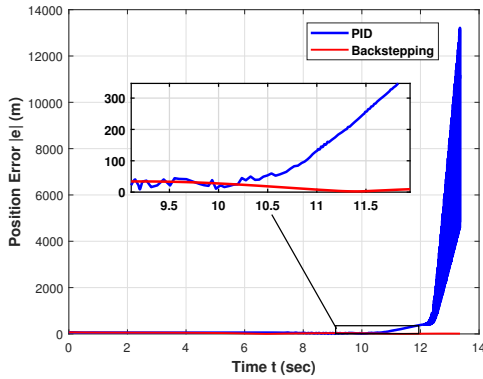


Fig. 7. Position Tracking for Sinusoidal Disturbance [$z = z + \sin(x)$]

6. SLIDING MODE CONTROL

In sliding mode control we design a sliding surface and ensure the system stability on that surface. We design a control law that ensures the system's stability by pushing it on the sliding surface. As suggested in (Khalil (2002)), we consider a sliding surface for the system in (19) as

$$\mathcal{S} = \alpha_s e_1 + e_2$$

The Lyapunov function for the sliding surface can be taken as

$$V(\mathcal{S}) = \frac{1}{2} \mathcal{S}^2$$

Then

$$\begin{aligned} \dot{V}(\mathcal{S}) &= \mathcal{S} (\alpha_s e_2 + \delta_s - \ddot{s}^{des} + u_s) \\ &\leq \mathcal{S} (\alpha_s e_2 + \Delta_s - \ddot{s}^{des} + u_s) \end{aligned}$$

By taking

$$u_s = -\beta(e) \text{sat} \left(\frac{\mathcal{S}}{\epsilon} \right) \quad (27)$$

Where $\beta(e) \geq |\alpha_s e_2 + \delta_s - \ddot{s}^{des}| + \beta_o$, hence can be taken as $\beta(e) = |\alpha(\dot{s} - \dot{s}^{des}) - \ddot{s}^{des}| + \Delta_s + \beta_o$, here β_o is a small positive constant.

Then if $|\mathcal{S}| \geq \epsilon$,

$$\dot{V}(\mathcal{S}) < -\beta_o |\mathcal{S}| \leq 0$$

and if $|\mathcal{S}| < \epsilon$

$$\dot{V}(e_1) < -\alpha e_1^2 + |e_1| \epsilon$$

Again we do not actually stabilize the system but achieve ultimate boundedness, whose bound can be decreased by decreasing ϵ . Now again extending this strategy to our perturbed system in (17) and (16) the linearized control inputs for quadcopter for (10) and (11) can be written as, *The Position Control Inputs:*

$$\begin{aligned} u_x &= -\beta_x \text{sat} \left(\frac{\mathcal{S}_x}{\epsilon_x} \right) \\ u_y &= -\beta_y \text{sat} \left(\frac{\mathcal{S}_y}{\epsilon_y} \right) \\ u_z &= -\beta_z \text{sat} \left(\frac{\mathcal{S}_z}{\epsilon_z} \right) \end{aligned} \quad (28)$$

The Attitude Control Inputs:

$$\begin{aligned} u_\phi &= -\beta_\phi \text{sat} \left(\frac{\mathcal{S}_\phi}{\epsilon_\phi} \right) \\ u_\theta &= -\beta_\theta \text{sat} \left(\frac{\mathcal{S}_\theta}{\epsilon_\theta} \right) \\ u_\psi &= -\beta_\psi \text{sat} \left(\frac{\mathcal{S}_\psi}{\epsilon_\psi} \right) \end{aligned} \quad (29)$$

Where for each state ξ in \mathcal{Q} , ϵ_ξ is a small positive that can be adjusted to reduce chattering and,

$$\begin{aligned} \beta_\xi &\triangleq |\alpha_\xi (\dot{\xi} - \dot{\xi}^{des}) - \ddot{\xi}^{des}| + \Delta_\xi + \beta_{o\xi} \\ \mathcal{S}_\xi &\triangleq (\dot{\xi} - \dot{\xi}^{des}) + (\xi - \xi^{des}) \end{aligned} \quad (30)$$

6.1 Simulation Results

We have compared the results of tracking performance of sliding mode control designed above in Fig. (7) with backstepping control designed in the earlier section. While both have converging errors unlike PID transient performance of backstepping is much better but steady state performance of sliding mode control is better.

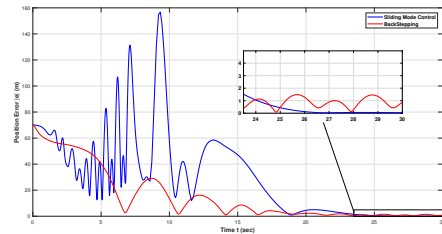


Fig. 8. Position Tracking for Sinusoidal Disturbance [$z = z + \sin(x)$]

7. CONCLUSION & FUTURE WORK

The PID control has its own benefits like easy implementation and tuning, but it fails to operate outside a limited performance region for most of the complex nonlinear systems. It also can not perform well in the presence of uncertainties which are always there in practical systems. Our proposed control schemes, while having much wider performance regions and capability of handling uncertainties, also contains some of the benefits of PID control since we have used a novel feedback linearization scheme. We have stressed upon the use of more accurate models for better control by deriving a model closer to the reality than the ones usually used in the literature. In the future we can design a robust adaptive controller for handling even the unmatched uncertainties. We can design nonlinear control for sampled data systems. We can explore geometric nonlinear control or passivity based control.

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