New Tuning Methods of Both PID and ADRC for MIMO Coupled Nonlinear Uncertain Systems ⋆

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Abstract: This paper proposes new tuning methods for both the famous proportional-integral-derivative (PID) control and the active disturbance rejection control (ADRC) of multi-input multi-output (MIMO) coupled nonlinear uncertain systems. Firstly, a quantitative lower bound to the bandwidth of the parallel extended state observers (ESOs) of ADRC is given, which is not necessarily of high gain. Then, inspired by an inherent but less noticed relationship between PID and ADRC, a new and concrete PID tuning rule is introduced, which can achieve both the strong robust decoupling control and good tracking performance of the MIMO closed-loop systems. Finally, the theoretical results, which reveal that why and how both PID control and ADRC can effectively deal with decoupling problem for MIMO coupled nonlinear uncertain systems, are verified by simulations.

Keywords: Multi-input Multi-output (MIMO) Coupled Nonlinear Uncertain Systems, Proportional-Integral-Derivative (PID), Active Disturbance Rejection Control (ADRC), Extended State Observer (ESO), Disturbance Estimation

1. INTRODUCTION

It is well-known that the classical PID (proportional-integral-derivative), which has nearly 100 years of history, is still the most widely and successfully used controller in engineering practice by far even though numerous advanced control techniques have been proposed (see Samad (2017)). Even though the classical PID control achieves such an amazing success in practice, the design of PID parameters is still lack of theoretical support and most methods are based on the experience and experiments (see O'Dwyer (2006)). Recently, some theoretical studies about the global convergence of the PID controller for nonlinear uncertain systems are proposed (see Zhao and Guo (2017), Zhao and Guo (2019), Zhang and Guo (2019)). In Zhao and Guo (2017), for second order nonlinear uncertain dynamical systems, necessary and sufficient conditions on the selection of the PID parameters have been discussed and provided. Moreover, Zhang and Guo (2019) extends the results of Zhao and Guo (2017) to higher dimensional uncertain systems and improves the results significantly. Although these results have provided open unbounded manifolds for the selection of the PID parameters, how to design a concrete tuning rule, which can guarantee both strong robustness as well as good tracking performance, especially for MIMO coupled nonlinear uncertain systems, is still a challenging problem to be resolved.

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The active disturbance rejection control(ADRC) was originally proposed by Han in 1998 (see Han (1998)). Because of its strong ability in dealing with a vast range of uncertainties/disturbances and great transient response, ADRC has become quite attractive to applied researchers (see Huang and Xue (2014), Xue et al. (2015), Sun et al. (2016), Chen et al. (2017), Zheng and Gao (2018)). For MIMO coupled nonlinear uncertain systems, the key of ADRC is to estimate and compensate for the total disturbance, which lumps the coupling effect, the internal uncertainties and the external disturbances, by several parallel designed extended state observers (ESOs) in real time. Although there have been some literatures for the theoretical analysis about ADRC (see Guo and Zhao (2013), Xue and Huang (2015), Xue and Huang (2017), Chen et al. (2020)), research on how to tune the ADRC parameters to achieve satisfactory performance of the closed-loop system under practical restrictions still lacks, especially for MIMO coupled nonlinear uncertain systems.

Recently, a parameter formula connecting PID and ADRC is studied for a kind of single-input single-output (SISO) second-order uncertain systems (see Zhong et al. (2020)). What marvelous is that, for MIMO coupled nonlinear uncertain systems, both PID control and ADRC take simple laws to solve decoupling problem, which do not depend on the coupled nonlinear model. A comprehensive understanding why PID and ADRC are widely effective for real world MIMO coupled nonlinear uncertain systems may help to further push forward the product quality in the widespread practice. This is the motivation of the paper.

In this paper, new and simple tuning methods of both PID controller and ADRC for MIMO coupled nonlinear uncertain systems are given. With the aid of the condition, provided in Zhang and Guo (2019), a quantitative lower bound for the bandwidth of the parallel extended state observers (ESOs), the core in ADRC design, is obtained. This lower bound, which is not necessarily of high gain, can be used as the necessary condition to design ADRC parameter for MIMO coupled nonlinear uncertain systems. Then, inspired by the design of a reduced-order ESO based linear ADRC, a new and concrete tuning rule for PID parameters is provided for MIMO coupled nonlinear uncertain systems. It is proved that the PID controller, tuned by the new tuning rule, also has the capability of estimating and compensating for the coupling, uncertainties and disturbances such that the strong robustness and great transient performance can be achieved. Furthermore, what striking is that the P-term and the D-term also contribute to the estimation and compensation for the unknown nonlinear disturbances, rather than the single I-term of PID controller.

The rest of the paper is organized as follows. Section 2 presents the detailed problem description. The main results are introduced in Section 3. Section 4 gives some simulations to verify the theoretical analysis. Finally, Section 5 concludes the paper.

2. PROBLEM FORMULATION

Consider the following MIMO coupled nonlinear uncertain system:

$$\begin{cases} \dot{X}_1 = X_2, \\ \dot{X}_2 = F(X_1, X_2, t) + U(t), \end{cases} \tag{1}$$

where $X_1 = [x_{11}, x_{12}, ..., x_{1n}]^T \in R^n$ and $X_2 = [x_{21}, x_{22}, ..., x_{2n}]^T \in R^n$ are the system states and can be measured, $U(t) \in R^n$ is the control input, $F(X_1, X_2, t) : R^n \times R^n \times R^+ \to R^n$ is an unknown nonlinear vector function of the state (X_1, X_2) and time t, which consists of the coupling effect of the state, the uncertainty of the system and the external disturbances.

Remark 1. The system (1) can present lots of practical systems, such as moving bodies in R^3 and multi-agent systems (see Yuan et al. (2018)), in which each agent can be described as:

$$\begin{cases} \dot{x}_{1j} = x_{2j}, \\ \dot{x}_{2j} = f_j(X_1, X_2, t) + u_j, j = 1, ..., m, \end{cases} \tag{2}$$

where m is the number of agents.

The control objective is to make the controlled variable X_1 of any initial value track a given bounded reference signal $Y^*(t) \in R^n$, which satisfies

$$\lim_{t\to\infty} Y^*(t) = Y^{**}, \lim_{t\to\infty} \dot{Y}^*(t) = 0, \lim_{t\to\infty} \ddot{Y}^*(t) = 0,$$

where Y^{**} is a constant vector.

To make the closed-loop system have a good tracking performance, the following desired transient process $r(t) = [r_1, ..., r_n]^T \in R^n$ can be designed:

$$\begin{aligned} &\ddot{r}_i = -2c_{r_i}\dot{r}_i - c_{r_i}^2(r_i - y_i^*(t)), \\ &r_i(0) = x_{1i}(0), \dot{r}_i(0) = x_{2i}(0), i \in \underline{n}, \end{aligned} \tag{3}$$

where $c_{r_i} > 0$ is a parameter, $\underline{n}$ denotes the set $\{1, ..., n\}$.

Denote the tracking error as $e_i(t) = r_i(t) - x_{1i}(t), i \in \underline{n}$. Then, the PID law and the ADRC law are described as follows.

First, the classical PID $U_{PID} = [u_{PID_1}, ..., u_{PID_n}]^T$ for the MIMO coupled nonlinear uncertain system (1) takes the form:

$$u_{PID_i} = k_{p_i}e_i + k_{d_i}\dot{e}_i + k_{I_i}\int_0^t e_i(\tau)d\tau + \ddot{r}_i, i \in \underline{n}, \tag{4}$$

where $k_{p_i}, k_{d_i}, k_{I_i}$ are the parameters to be determined in the paper and $\ddot{r}_i$ is a feedforward term. Note that the control signal u_{PID_i} only utilizes the i-th tracking error e_i and $\ddot{r}_i$, which makes the PID a simple decoupling control law.

On the other hand, based on the idea of ADRC, the unknown nonlinear vector function $F(X_1, X_2, t)$, which consists of the coupling effect of the state, the internal uncertainties and the external disturbances, can be regarded as the total disturbance of the system and treated as an extended state vector to be timely estimated and compensated for, by n parallel ESOs.

According to the measurement X_2, the following n parallel reduced-order ESOs can be designed (see Huang and Xue (2014)):

$$\begin{cases} \dot{\xi}_i = -\omega_o\xi_i - \omega_o^2 x_{2i} - \omega_o u_i, \quad \xi_i(0) = -\omega_o x_{2i}(0), \\ \hat{f}_i = \xi_i + \omega_o x_{2i}, \quad i \in \underline{n} \end{cases} \tag{5}$$

where $\hat{f}_i$ is an estimate of $f_i(X_1, X_2, t)$, $\hat{f}_i(0) = 0$ and ω_o is the bandwidth of the parallel ESOs (5) for estimating the total disturbance F, which can be tuned.

Then, to track the transient process $r(t)$, the corresponding ADRC law $U = [u_1, ..., u_n]^T$ takes the following structure:

$$u_i = k_{ap_i}e_i + k_{ad_i}\dot{e}_i - \hat{f}_i + \ddot{r}_i, i \in \underline{n} \tag{6}$$

where $k_{ap_i}, k_{ad_i} > 0$ are parameters to be designed. In (6), $\hat{f}_i$ is used to timely compensate for the total disturbance. Specially, it can be seen from (6) that, similar to the PID (4), the ADRC law u_i, which is independent on the coupled model, also handles decoupling control problem only using the signals x_{1i} and x_{2i}.

Remark 2. The transient process $r(t)$ and the feedforward $\ddot{r}$, which can improve the dynamic accuracy of the closed-loop system, will play a prominent part in the design of both controllers to deduce an inherent relationship between the PID (4) and the ADRC (6).

In the next section, a lower bound to the bandwidth ω_o of the parallel ESOs (5) is quantitatively proposed. Then, inspired by the structure of the ADRC (5) and (6), a new and concrete tuning rule for the classical PID controller (4) with guaranteed performance is given.

3. MAIN RESULTS

Define $F(X_1, X_2, t)$ as follows:

$$
\begin{aligned}
\mathcal{F} =& \Big\{ F \in C^1(R^n \times R^n \times R^+) \Big| F(X_1, X_2, t) = H(X_1, X_2) + W(t), \\
& \left\| \frac{\partial H}{\partial X_1} \right\| \le L_1, \left\| \frac{\partial H}{\partial X_2} \right\| \le L_2, \left\| W(t) \right\| \le L_3, \left\| \dot{W}(t) \right\| \le L_4, \\
& \lim_{t \to \infty} W(t) \ exists, \forall (X_1, X_2) \in R^n \times R^n, \forall t \in R^+ \Big\},
\end{aligned} \tag{7}
$$

where L_1, L_2, L_3 and L_4 are positive constants, $C^1(R^n \times R^n \times R^+)$ is denoted as the space of all functions from $R^n \times R^n \times R^+$ to R^n, which have continuous partial derivatives with respect to (X_1, X_2, t), $W(t)$ is unknown disturbance.

3.1 A quantitative lower bound to the bandwidth of the parallel ESOs (5) with guaranteed performance

In this subsection, a quantitative lower bound to the bandwidth ω_o of the parallel ESOs (5) is given. To that end, denote $\underline{k}_{ap} = \min_{i \in \underline{n}} k_{ap_i}$, $\underline{k}_{ad} = \min_{i \in \underline{n}} k_{ad_i}$, $\bar{k}_{ap} = \max_{i \in \underline{n}} k_{ap_i}$, $\bar{k}_{ad} = \max_{i \in \underline{n}} k_{ad_i}$ and

$$
\Omega = \left\{ \omega \in R \middle| n_0 \omega^4 + n_1 \omega^3 + n_2 \omega^2 + n_3 \omega + n_4 = 0 \right\},
$$

where $n_0 = \underline{k}_{ad}^2$ and

$$
\begin{aligned}
n_1 &= 2\underline{k}_{ad}[\underline{k}_{ad}(\underline{k}_{ad} - L_2) - \bar{k}_{ap} + \underline{k}_{ap} - L_1], \\
n_2 &= \underline{k}_{ad}^2(\underline{k}_{ad} - L_2)^2 + 2\underline{k}_{ad}(\underline{k}_{ad} - L_2)[2(\underline{k}_{ap} - L_1) \\
&\quad - \bar{k}_{ap}] - L_2(L_1 \bar{k}_{ad} + L_2 \bar{k}_{ap}) + \bar{k}_{ap}^2 \\
&\quad + (\underline{k}_{ap} - L_1)(\underline{k}_{ap} - L_1 - 2\bar{k}_{ap}), \\
n_3 &= 2(\underline{k}_{ap} - L_1)(\underline{k}_{ad} - L_2)[\underline{k}_{ad}(\underline{k}_{ad} - L_2) - \bar{k}_{ap} + \underline{k}_{ap} \\
&\quad - L_1] - L_2^2 \bar{k}_{ap}(\bar{k}_{ad} - L_2) - L_1^2 \bar{k}_{ad} \\
&\quad - L_1 L_2[2\bar{k}_{ap} - L_1 + \bar{k}_{ad}(\underline{k}_{ad} - L_2)], \\
n_4 &= (\underline{k}_{ap} - L_1)^2(\underline{k}_{ad} - L_2)^2 - L_1(\bar{k}_{ap} - L_1)[L_1 \\
&\quad + L_2(\underline{k}_{ad} - L_2)].
\end{aligned} \tag{8}
$$

Define

$$
\begin{aligned}
\omega_o^* &= \max \left\{ 0, \frac{L_1 - \underline{k}_{ap}}{\underline{k}_{ad}}, L_2 - \underline{k}_{ad}, \bar{\omega}_o \right\}, \\
\bar{\omega}_o &= \begin{cases} 0, & \Omega = \varnothing \quad or \quad \max\{\Omega\} \le 0, \\ \max\{\Omega\}, & \max\{\Omega\} > 0, \end{cases}
\end{aligned} \tag{9}
$$

where $\varnothing$ represents the empty set.

The following theorem will show that ω_o^* is a suitable lower bound to the parallel ESOs parameter in order to guarantee the global boundness and global attractivity of the steady state.

Theorem 1. ***(Stability)***. Consider the ADRC controlled MIMO coupled nonlinear uncertain system (1),(5) and (6), where $F \in \mathcal{F}$. Then, for any given L_1, L_2, k_{ap_i} and $k_{ad_i}, i \in \underline{n}$, the closed-loop system will be bounded and satisfy

$$
\lim_{t \to \infty} X_1(t) = Y^{**}, \lim_{t \to \infty} X_2(t) = 0,
$$

for any initial value $(X_1(0), X_2(0)) \in R^n \times R^n$ and any Y^{**}, as long as the bandwidth of the parallel ESOs satisfies $\omega_o > \omega_o^*$.

Remark 3. From (9), it shows that the lower bound ω_o^* is only decided by the constants $L_1, L_2, \underline{k}_{ap}, \underline{k}_{ad}, \bar{k}_{ap}, \bar{k}_{ad}$ and independent of the unknown disturbance $W(t)$, initial values of the state and the reference signal $Y^*(t)$.

Proof.

Denote $e = [e_1, ..., e_n]^T$. Substituting equation (6) into (5), it can be deducted that

$$
\hat{F}_i(s) = -\omega_o k_{ad_i} E_i(s) - \omega_o s E_i(s) - \frac{\omega_o k_{ap_i} E_i(s)}{s}, \tag{10}
$$

where $E_i(s)$ and $\hat{F}_i(s)$ are the Laplace transform of $e_i(t)$ and $f_i(t)$, respectively. Take the inverse Laplace transform for (10), there is

$$
\hat{f}_i = -\omega_o k_{ad_i} e_i - \omega_o \dot{e}_i - \omega_o k_{ap_i} \int_0^t e_i(\tau) d\tau. \tag{11}
$$

Thus, the ADRC law (6) can be rewritten as

$$
U = k_p e + k_d \dot{e} + k_I \int_0^t e(\tau) d\tau + \ddot{r}, \tag{12}
$$

where $k_p = k_{ap} + \omega_o k_{ad}, k_d = k_{ad} + \omega_o I_n, k_I = \omega_o k_{ap}$, I_n represents n-dimensional identity matrix,

$$
k_{ap} = \begin{bmatrix} k_{ap_1} & 0 & \cdots & 0 \\ 0 & k_{ap_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & k_{ap_n} \end{bmatrix}, k_{ad} = \begin{bmatrix} k_{ad_1} & 0 & \cdots & 0 \\ 0 & k_{ad_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & k_{ad_n} \end{bmatrix}.
$$

Since $\lim_{t \to \infty} W(t)$ exists, there exists a constant vector C, such that $\lim_{t \to \infty} W(t) = C$. Denote

$$
\begin{aligned}
e_I(t) &= \int_0^t e(\tau) d\tau + \frac{H(Y^{**}, 0) + C}{k_i}, \quad e_D(t) = \dot{e}(t), \\
G(e, e_D) &= -H(Y^{**} - e, -e_D) + H(Y^{**}, 0).
\end{aligned}
$$

From (7), there is $G \in \mathcal{F}$ and $G(0,0) = 0$. Thus, the closed-loop system (1) and (12) can be rewritten as

$$
\begin{cases} \dot{e}_I = e, \\ \dot{e} = e_D, \\ \dot{e}_D = -k_I e_I - k_p e - k_d e_D + G(e, e_D) + \Delta(\cdot), \end{cases} \tag{13}
$$

where

$$
\Delta(\cdot) = G(e + Y^{**} - r, e_D - \dot{r}) - G(e, e_D) + C - W(t)
$$

and

$$
\|G(e + Y^{**} - r, e_D - \dot{r}) - G(e, e_D)\| \le L_1 \|Y^{**} - r\| + L_2 \|\dot{r}\|.
$$

Thus,

$$
\|\Delta\| \le L_1 \|Y^{**} - r\| + L_2 \|\dot{r}\| + \|C\| + L_3.
$$

Furthermore, $(0,0,0)$ is an equilibrium of (13), when t approaches infinity.

Based on the analysis in Zhang and Guo (2019), $G(e, e_D)$ can be expressed as

$$
\begin{aligned}
& G(e, e_D) \\
=& \Big\{ \int_0^1 \frac{\partial G(\bar{e}, 0)}{\partial \bar{e}} d\lambda \Big\} e + \Big\{ \int_0^1 \frac{\partial G(e, \bar{e}_D)}{\partial \bar{e}_D} d\lambda \Big\} e_D \\
\triangleq& b(e) e + a(e, e_D) e_D,
\end{aligned} \tag{14}
$$

where $\bar{e} = \lambda e, \bar{e}_D = \lambda e_D, \lambda \in [0,1], \|b(e)\| \le L_1, \|a(e, e_D)\| \le L_2$.

Hence, the closed-loop system (13) turns into

$$
\begin{cases} \dot{e}_I = e, \\ \dot{e} = e_D, \\ \dot{e}_D = -k_I e_I - \phi(e) e - \psi(e, e_D) e_D + \Delta(\cdot), \end{cases} \tag{15}
$$

where

$$
\phi(e) = k_p - b(e), \psi(e, e_D) = k_d - a(e, e_D).
$$

Denote $\underline{k}_p = \lambda_{\min}(k_p), \underline{k}_d = \lambda_{\min}(k_d), \underline{k}_I = \lambda_{\min}(k_I), \bar{k}_p = \lambda_{\max}(k_p), \bar{k}_d = \lambda_{\max}(k_d), \bar{k}_I = \lambda_{\max}(k_I)$, where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of the corresponding matrix, respectively.

Similar to Zhang and Guo (2019), the following positive define matrix P is considered:

$$P = \frac{1}{2}\begin{bmatrix} \mu k_I & k_I & \delta I_n \\ k_I & k_p + \mu k_d & \mu I_n \\ \delta I_n & \mu I_n & I_n \end{bmatrix}, \tag{16}$$

where μ is a constant defined by

$$\mu = \frac{2((\underline{k}_p - L_1)(\underline{k}_d - L_2) + \bar{k}_I) - L_1 L_2}{4(\underline{k}_p - L_1) + L_2^2},$$

and δ is a positive constant, which is small enough.

Then, design the following Lyapunov function:

$$V(e_I^T, e^T, e_D^T) = [e_I^T, e^T, e_D^T]P[e_I^T, e^T, e_D^T]^T. \tag{17}$$

It can be obtained that

$$\lambda_{\min}(P)\left\|[e_I^T, e^T, e_D^T]\right\|^2 \leq V \leq \lambda_{\max}(P)\left\|[e_I^T, e^T, e_D^T]\right\|^2.$$

The time derivative of $V(e_I^T, e^T, e_D^T)$ along the trajectories of (15) is

$$\begin{aligned} \dot{V} =& -[e_I^T, e^T, e_D^T]B(\cdot)[e_I^T, e^T, e_D^T]^T + \\ & [e_I^T, e^T, e_D^T][\delta\Delta^T, \mu\Delta^T, \Delta^T]^T, \\ \leq& -[\|e_I\|, \|e\|, \|e_D\|]Q[\|e_I\|, \|e\|, \|e_D\|]^T + \\ & [\|e_I\|, \|e\|, \|e_D\|][\delta, \mu, 1]^T\|\Delta\|, \end{aligned} \tag{18}$$

where

$$B(\cdot) = \begin{bmatrix} \delta \underline{k}_I & \frac{\delta\phi}{2} & \frac{\delta\psi}{2} \\ \frac{\delta\phi}{2} & w_{22} & -\frac{\mu a + b^T + \delta I_n}{2} \\ \frac{\delta\psi}{2} & -\frac{\mu a + b^T + \delta I_n}{2} & w_{33} \end{bmatrix},$$

$$w_{22} = -k_I + \mu(k_p - \frac{b + b^T}{2}),$$

$$w_{33} = k_d - \mu I_n - \frac{a + a^T}{2},$$

and Q is a positive define matrix, expressed by

$$Q = \begin{bmatrix} \delta k_I & \frac{-\delta(\bar{k}_p + L_1)}{2} & \frac{-\delta(\bar{k}_d + L_2)}{2} \\ \frac{-\delta(\bar{k}_p + L_1)}{2} & -\bar{k}_I + \mu(\underline{k}_p - L_1) & -\frac{\mu L_2 + L_1 + \delta}{2} \\ \frac{-\delta(\bar{k}_d + L_2)}{2} & -\frac{\mu L_2 + L_1 + \delta}{2} & -\mu + \underline{k}_d - L_2 \end{bmatrix}.$$

From (18), there is

$$\begin{aligned} \dot{V} &\leq -\lambda_{\min}(Q)\|[e_I^T, e^T, e_D^T]\|^2 + c_0\|[e_I^T, e^T, e_D^T]\|\|\Delta\| \\ &\leq -c_1 V + c_2\sqrt{V}\|\Delta\|, \end{aligned}$$

where $c_0 = \max\{\delta, \mu, 1\}, c_1 = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, c_2 = \frac{c_0}{\sqrt{\lambda_{\min}(P)}}$.

Since $\|\Delta\|$ is bounded, there exists a constant M_0, such that $\|\Delta\| \leq M_0$. Then, with the help of Gronwall Lemma Corduneanu (2008), it can be obtained that

$$\sqrt{V} \leq e^{\frac{-c_1 t}{2}}\sqrt{V(e_I^T(0), e^T(0), e_D^T(0))} + \frac{c_2 M_0}{c_1}(1 - e^{\frac{-c_1 t}{2}}).$$

Therefore, $V(e_I^T, e^T, e_D^T)$ is bounded and e_I^T, e^T and e_D^T are bounded. Since $\lim_{t\to\infty} r(t) = Y^{**}$, $\lim_{t\to\infty} \dot{r}(t) = 0$ and $\lim_{t\to\infty} W(t) = C$, it can be obtained that $\lim_{t\to\infty} \Delta(\cdot) = 0$.

Therefore, there exists $T > 0$, such that for any $t > T$, $\varepsilon > 0$, there is

$$\dot{V} \leq -c_1 V + \varepsilon.$$

In sum, $\lim_{t\to\infty} X_1(t) = Y^{**}$, $\lim_{t\to\infty} X_2(t) = 0$. ∎

Denote the estimation error of the parallel ESOs (5) as $e_{f_i} = \hat{f}_i - f_i(X_1, X_2, t)$. Since $\ddot{e}_i(t) + k_{ad_i}\dot{e}_i(t) + k_{ap_i}e_i(t) = 0$ is the ideal tracking performance for a double integrator system, determined by the given pole-placement via feedback parameters k_{ap_i} and k_{ad_i}, Theorem 2 will show that by increasing the parallel ESOs parameter $\omega_o > \omega_o^*$, the real closed-loop tracking performance can be closer to the ideal one in the whole time-domain.

Theorem 2. ***(Tracking Performance).*** Consider the ADRC controlled MIMO coupled nonlinear uncertain system (1),(5) and (6), where $F \in \mathcal{F}$. Then, there exists a positive constant η, which is independent of ω_o, such that for all $\omega_o > \omega_o^*$, the tracking error has the following property:

$$\begin{aligned} &|\ddot{e}_i(t) + k_{ad_i}\dot{e}_i(t) + k_{ap_i}e_i(t)| = |e_{f_i}(t)| \\ &\leq |e_{f_i}(0)|e^{-\omega_o t} + \frac{\eta}{\omega_o}, \quad t \geq 0. \end{aligned} \tag{19}$$

The proof of Theorem 2 is omitted here due to space limitation.

3.2 *A new and concrete tuning rule for PID*

In this subsection, via an inherent but rarely noticed relationship between PID and ADRC, a novel and concrete tuning rule for the parameters of the PID controller (4) will be proposed.

The formula (11) in the proof of Theorem 1 shows that the output of the ESO (5) can be rewritten as a linear combination of the three terms in PID. Hence, setting

$$k_{p_i} = k_{ap_i} + \omega_o k_{ad_i}, k_{d_i} = k_{ad_i} + \omega_o, k_{I_i} = \omega_o k_{ap_i},$$

then, the PID (4) has the same function of the reduced-order ESO based linear ADRC law (6). Therefore, a new tuning rule for the PID law (4), inspired by ADRC design, can be proposed as follows:

$$\begin{aligned} &k_{p_i} = k_{ap_i} + \omega_o k_{ad_i}, \quad k_{d_i} = k_{ad_i} + \omega_o, \quad k_{I_i} = \omega_o k_{ap_i}, \\ &k_{ap_i} > 0, \quad k_{ad_i} > 0, \quad \omega_o > \omega_o^*. \end{aligned} \tag{20}$$

From this meaningful relationship between ADRC (6) and PID (4), similar to Theorem 1 and Theorem 2, the following corollary gives the properties of the MIMO closed-loop system based on the PID (4) and the new tuning rule (20).

Corollary 1. Consider the PID controlled MIMO coupled nonlinear uncertain system (1),(4) and (20), where the unknown nonlinear function $F \in \mathcal{F}$. Then, there exists a positive constant η, which is the same as in Theorem 2, such that for any given $L_1, L_2, k_{ap_i}, k_{ad_i} > 0$, any initial value $(X_1(0), X_2(0)) \in R^n \times R^n$ and any Y^{**},the closed-loop system has the following properties:

(1) $\lim_{t\to\infty} X_1(t) = Y^{**}$, $\lim_{t\to\infty} X_2(t) = 0$.

(2) $|\ddot{e}_i(t) + k_{ad_i}\dot{e}_i(t) + k_{ap_i}e_i(t)| \leq |f_i(0)|e^{-\omega_o t} + \frac{\eta}{\omega_o}, t \geq 0$,

whenever $\omega_o > \omega_o^*$.

To further compare the estimation capacity of the ESOs (5) and the integral term of the PID controller (4), Theorem 3 is given as follows:

Theorem 3. Consider the ADRC controlled MIMO closed-loop system (1),(5) and (6) and the PID controlled MIMO closed-loop system (1),(4) and (20). For any unknown nonlinear $F \in \mathcal{F}$, the setting time of the ESO (5), which depends on the parameter ω_o, is shorter than that of the integral term of the PID controller (4), when $\omega_o > \omega_o^*$.

The proof of Theorem 3 is omitted here due to space limitation.

4. SIMULATIONS

In this section, some simulations on a moving body in R^3 are given.

Assume that the unknown coupled nonlinear dynamics is represented by vector $F = [f_1, f_2, f_3]^T$,

$$\begin{aligned} f_1(t) =& x_{11} + 3sin(x_{12})cos(x_{13}) + 0.5x_{21} + sin(x_{23}) \\ &+ cos(x_{22}) - 1, \\ f_2(t) =& 2x_{12} + sin(x_{11}) + x_{13} + x_{22} \\ &+ sin(x_{23})cos(x_{21}) + 1, \\ f_3(t) =& x_{13} + sin(x_{12}) + x_{11} + 0.5x_{23} \\ &+ sin(x_{22})cos(x_{21}) + w_1(t), \end{aligned} \tag{21}$$

where

$$w_1(t) = \begin{cases} cos(t), & \text{if } t < 4, \\ cos(4), & \text{else.} \end{cases}$$

$X_1(0) = 0, X_2(0) = 0$ and the discontinuous reference signal is

$$Y^*(t) = \begin{cases} [2, 0, -3]^T, & \text{if } t < 4, \\ [2, 1, -3]^T, & \text{else.} \end{cases}$$

The ideal transient process $r(t)$ is:

$$\begin{aligned} &\ddot{r}_i = -2c_{r_i}\dot{r}_i - c_{r_i}^2(r_i - y_i^*(t)), \\ &c_{r_i} = 5, r_i(0) = 0, \dot{r}_i(0) = 0, i = 1, 2, 3. \end{aligned} \tag{22}$$

According to Theorem 1, it can be calculated that the lower bound of the bandwidth of ESOs is $\omega_o^* = 4.5$. Then, the parameters of the ADRC (6) and PID (4) are set as

$$\begin{aligned} &k_{ap_i} = 1, k_{ad_i} = 2, \omega_o = 15, \\ &k_{p_i} = 31, k_{d_i} = 17, k_{I_i} = 15, i = 1, 2, 3. \end{aligned}$$

Figures 1 $\sim$ 3 are the response curves of the state (X_1, X_2) based on the ADRC (5) and (6), and the PID controller (4) and (20). From Fig. 1 $\sim$ 3, it can be seen that under the new tuning rule (20), the dynamic response of the PID controlled system is similar to that of the ADRC (5),(6) controlled system. Moreover, both of them can make the tracking error converge to zero and have nice tracking performance.

Figures 4$\sim$6 compare the estimations of the unknown nonlinear coupled function vector F, given by the parallel ESOs (5), the single I-term of the PID controller (4) tuned by the new rule (20), and the linear combination of the three terms of P-I-D (11) of PID controller. In Fig. 4$\sim$6, the real value of F of the ADRC based closed-loop system (1),(5) and (6) is expressed by the blue line, and F of the PID based closed-loop system (1), (4) and (20) is expressed by the red dash line.

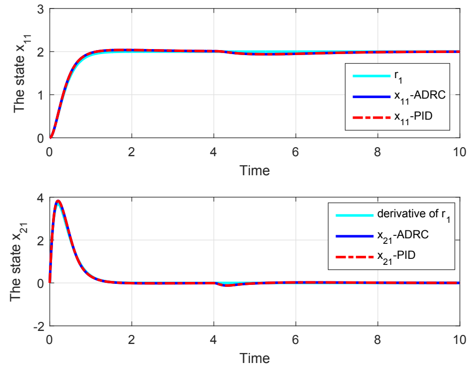

Fig. 1. The response curves of the states x_{11} and x_{21}

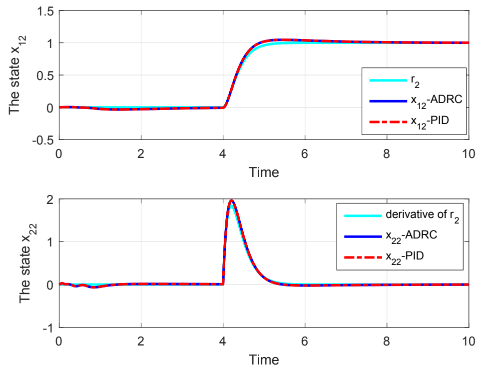

Fig. 2. The response curves of the states x_{12} and x_{22}

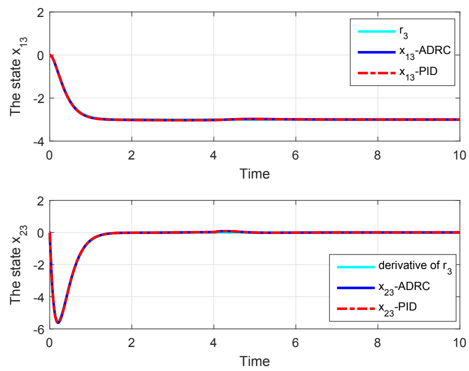

Fig. 3. The response curves of the states x_{13} and x_{23}

Figures 4$\sim$6 further verify that compared to the single I-term of the PID, the parallel ESOs (5) can track various unknown nonlinear disturbances more quickly. Moreover, under the new tuning rule (20), inspired by the ADRC (5) and (6), the combination of the three terms of PID (11) has the strong ability to estimate the unknown coupled nonlinear function F.

5. CONCLUSION

In this paper, new design methods for both PID and ADRC of MIMO coupled nonlinear uncertain systems are presented. On the one hand, a lower bound ω_o^* to the bandwidth ω_o, which can be quantitatively calculated, is introduced for ensuring the global boundness and global

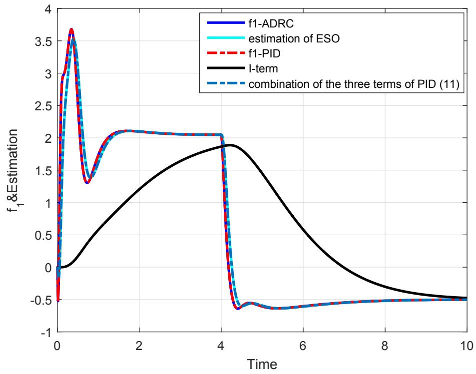

Fig. 4. The estimations of the unknown dynamics f_1

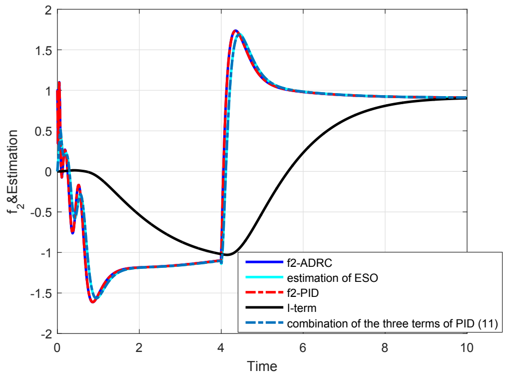

Fig. 5. The estimations of the unknown dynamics f_2

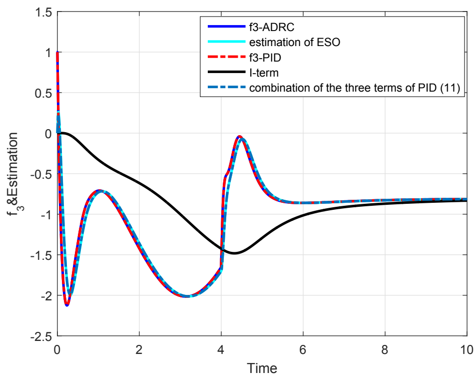

Fig. 6. The estimations of the unknown dynamics f_3

attractivity of the ADRC, indicating that the parallel ESOs (5) are not necessary of high gain. It is further illustrated that the tracking performance in the whole time-domain can be improved by increasing $\omega_o > \omega_o^*$. On the other hand, a novel and concrete PID controller tuning rule (20), stimulated by the design of the linear ADRC based on a reduced-order ESO, is given, such that both the strong robustness and nice tracking performance can be achieved. Moreover, it is proved that the parallel ESOs (5) can estimate the coupling uncertain disturbances more quickly than the single I-term.

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