Simplifying the Design of Lipschitz Observers by Applying a Novel Batch Pole Assignment Approach

Panos C. Papageorgiou* Antonio T. Alexandridis*

**Department of Electrical and Computer Engineering, University of Patras, Rion-Patras, 26504, Greece. (e-mail: papageorgiou@ece.upatras.gr; a.t.alexandridis@ece.upatras.gr).*

Abstract: A novel, analytic design method for full state observers of nonlinear Lipschitz systems with their nonlinear term bounded, is considered. In standard procedures, the Lipschitz constant of the nonlinear term imposes strict restrictions on the observer gain matrix stable selection and introduces a further uncertainty in the design, caused by the heuristic manner of this selection. As shown in the paper, when the system nonlinear term is additionally bounded, which is a common situation for many real world systems such as manipulators in robotics and generators in power systems, the Lipschitz constant restriction is fully relaxed. Under these circumstances, a direct design approach is proposed that assigns the observer linear part eigenvalues at a common, specific, negative real position on the left of the system poles. The whole procedure is conducted by simply solving a Lyapunov-type equation that simultaneously constructs the suitable corresponding gain matrix of the observer. The validity of the method and the enhanced observer performance are verified by simulation results conducted on a fundamental power system example.

Keywords: Observer design, Lyapunov methods, Observers for linear systems, Lipschitz observers.

1. INTRODUCTION

Over the last decades observer-based designs have been an active topic of research, constituting also an indispensable tool in modern control theory and engineering applications. Either disturbance or system state estimators have been largely applied in many cases of monitoring and feedback control problems, as a mean to attain otherwise inaccessible variables. However, state estimation designs have gradually attracted more interest as they serve directly the purpose of reconstructing the desired system states through measuring only the output of the plant, under certain observability conditions [Marquez, (2003)]. In fact, this kind of observer schemes have been successfully used in many industrial configurations [Chen, et al. (2000)], in mechanical and electronic implementations [Zhou et al. (2018), Li et al. (2015)], in power systems [Jiang et al. (2004)], etc.

Since state estimators are designed based on a mathematical model, the problem of developing a suitable observer greatly depends on the type and structure of the considered system. Certainly, observer designs for linear systems are the most straightforward to implement, as they are based on well-known concepts of linear estimation theory, requiring only the establishment of observability conditions [Dorf and Bishop, (2017)]. Several observers of full- or reduced-order (Luenberger-type) have been extensively used [Sage, (1981)] in linear system models. Furthermore, other state estimators of even more minimal-order have also been designed to address the combined output feedback and observer pole-assignment problem [Alexandridis and Paraskevopoulos (1996), Alexandridis, (1999)]. All the previously considered designs involve pole placement techniques suitably applied to ensure the well-known time separation principle. According to this approach, the system stabilization is accomplished by ensuring fast decays of the state estimation error by assigning the observer poles three to ten times left from the ones of the plant but are not extended to nonlinear cases.

As the large majority of real-world applications and practical configurations are described by nonlinear models, the problem of observer synthesis for this kind of systems is undoubtedly a fundamental one. At the same time, the obvious difficulties encountered in this case, mainly due to different existing nonlinearities in the original model, require careful handling in the development of state estimation schemes [Hassan and Hammuda, (2019)]. It is noted that there are two major approaches established in the literature. The first one is based on finding a suitable transformation for the examined nonlinear model in order to linearize its original plant and then to implement standard linear observer design techniques [Krener and Isidori, (1983)]. In this case, however, the convenient use of well-established linear system theory tools is often negated by the need of adopting strict assumptions regarding the existence of a nonlinear transformation, while finding the suitable transformation itself is typically a non-trivial task [Karagiannis et al. (2008)]. Furthermore, the resulting observer design is heavily dependent from the system operating point and therefore, there are often cases where this method cannot be effectively applied. On the other hand, according to the second basic approach of developing nonlinear estimators, the original dynamic model formulation is retained and the state estimator design is attempted to be directly applied [Afri et al. (2017)].

The method suffers from the fact that there is not exist a universal manner or a general method to obtain the solution.

Nevertheless, there are cases of nonlinear dynamic systems where the linear and the nonlinear parts co-exist and can be clearly separated, a fact which can be exploited in the observer design procedure. A fairly large class of systems falling into the latter category are characterized as Lipschitz ones, since the nonlinear part of them has to satisfy the well-known Lipschitz condition with respect to the state variables. In terms of estimation scheme development, there are two basic concepts of design, namely the indirect and the direct approach. The first one considers the complete dynamics as obtained by an appropriate coordinate transformation and by exploiting the notion of uniform observability, the nonlinear system is rewritten in canonical form, where both linear and nonlinear parts have a specific structure, i.e. triangular one [Gauthier et al. (1992)]. High gain state estimators [Khalil and Praly, (2014)] are the most representative designs implemented in this case, although several other types have also been proposed, including sliding-mode observers [Deng et al (2018)], fuzzy-based estimators [Tanaka et al (1998)] or even geometrical ones [Hammouri et al. (2018)]. However, the indirect design methods present several drawbacks with most important among them the lack of a general desired canonical form and in some cases, the computational difficulties introduced by complex transformations.

In contrast, according to the second approach, the linear and nonlinear parts of the system in state space can be directly split, with the linear one assumed to be observable and the nonlinear one either locally or globally Lipschitz [Thau, (1973)]. This method is in fact the most preferable one and is based on a conventional way of the observer synthesis that initially ignores the characteristics of the nonlinear term in order to apply a Lyapunov-based method on the linear part. Then the derived solutions are heuretically examined in order to verify whether or not they satisfy certain limitations associated with the given Lipschitz constant [Rajamani, (1998)] and the whole design is far from pole-assignment techniques. This standard procedure presents two obvious disadvantages. Firstly, the boundary conditions imposed do not provide any insight for designing an observer with predefined dynamic behavior, i.e. decay characteristics for the linear part. Secondly, the boundary condition related to the Lipschitz constant can sometimes lead to very conservative results regarding the asymptotic stability of the system, since it rigidly restricts the solution of the Lyapunov-type equation via a sequence of norm inequalities [Rajamani, (1998)]. In this frame, the challenging task of designing reliable observers with desirable damping, while ensuring strong stability conditions in a non-conservative and more accurate manner remains unaddressed.

In this paper, the class of nonlinear systems with bounded Lipschitz terms, (sinusoidal or others) are considered. For this kind of bounded systems, a direct observer design is proposed that employs a novel systematic method in order to directly construct the observer gain matrix regardless from the Lipschitz constant value. This is derived through the solution of a Lyapunov matrix equation, which enables to assign the real part of the observer linear-part eigenvalues in a batch, common way at predefined positions anywhere on the left of the original plant poles. The method is accomplished by a rigorous stability analysis based on the results of the contemporary study [Alexandridis, (2020)], which provides a complete theoretical framework to analyze nonlinear systems by combining Lyapunov-based methods with the (ISS) notion [Sontag, (2008)]. This approach can verify global stability and state convergence properties under certain boundedness conditions and is suitably used in the present case to relax and overcome other strict limitations.

The remainder of this paper is organized in the following structure: in Section 2, useful preliminary concepts regarding the Lipschitz-class systems description and the considered observer design are introduced. The main results of the analysis are derived in Section 3, where the pole-assignment method and the stability analysis for the proposed observer design are described in detail. In Section 4, the simulated results of an illustrative real-world example are presented. Finally, in Section 5, some notable remarks extracted from the presented analysis are summarized in the form of conclusions.

2. PRELIMINARY CONCEPTS AND SYSTEM FORMULATION

2.1 Lipschitz Nonlinear System Description

In the context of this paper, a fairly large class of autonomous dynamic systems is considered, in which the existing linear and nonlinear parts can be directly separated, while the output vector is the result of a linear combination of state variables. This type of nonlinear dynamic systems can be described in the following form

$$\dot{x}(t) = Ax(t) + Bu(t) + g(x) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ represent the state, input and measured output vectors of the system, respectively. Also, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ are linear time invariant (LTI) matrices. Additionally, function $g(x)$ is continuous with respect to the state vector and includes all nonlinearities in the formulation.

The system of the form of (1)-(2) is considered to be globally Lipschitz in $\mathbb{R}^n$, with respect to x, if there exists a constant $\gamma > 0$ satisfying:

$$\|g(x_1) - g(x_2)\| \le \gamma \|x_1 - x_2\| \quad \forall x_1, x_2 \in \mathbb{R}^n \tag{3}$$

with $\|\cdot\|$ denoting the Euclidean norm in $\mathbb{R}^n$. The positive scalar γ satisfying (3) is defined as the Lipschitz constant. Note that if condition (3) holds in a subspace M of $\mathbb{R}^n$, i.e. $M \subset \mathbb{R}^n$, then function $g(x)$ is said to be locally Lipschitz in a region M that includes the origin.

It is worth noting that many nonlinear functions satisfy either locally or globally the Lipschitz condition of (3). In this frame, the development of the proposed approach for Lipschitz-type systems serves a rather general purpose, as the method itself can be applied to a large number of real control problems and practical implementations.

2.2 Observer Modeling

Considering the Lipschitz system of (1)-(2), the following observer form is introduced and adopted in the proposed design

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + g(\hat{x}) + LC\left[x(t) - \hat{x}(t)\right] \quad (4)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (5)$$

where $L \in \mathbb{R}^{n\times p}$ is the observer gain matrix, $\hat{x}(t)$ represents the vector containing the estimates of $x(t)$ whereas (C, A) is an observable pair, with C being of full row rank. Defining the state estimation error $\tilde{x} = x - \hat{x}$, the following system dynamics are obtained

$$\dot{\tilde{x}}(t) = [A - LC]\tilde{x}(t) + G(\tilde{x}) \quad (6)$$

where

$$G(\tilde{x}) = g(x) - g(\hat{x}) . \quad (7)$$

3. MAIN RESULTS

The basic objective of the proposed approach is to apply a suitable techniques on the linear part of the observer dynamics of (6), in a way that exactly assigns the observer linear part poles in a batch and easy manner at the left half plane. Simultaneously, in the scope of the present study, is to provide less conservative results compared to the ones obtained by conventional stability analysis methods for Lipschitz observers [Rajamani, (1998)]. To this end, keeping in mind along with condition (3), that the case of bounded Lipschitz systems is considered, we assume

$$\|G(\tilde{x})\| = \|g(x) - g(\hat{x})\| \le d \quad (8)$$

with d being a positive scalar.

In the sequel it is proven that the implied boundedness (8) of the Lipschitz nonlinear terms can decisively relax the strong restrictions imposed by considering only the Lipschitz condition (3). The main results are presented in Theorem 1, where a batch procedure is established for the pole-assignment implementation. It is further proven that the proposed design is adequate to ensure asymptotic stability to the origin for the original estimation-error dynamics as given by (6). This is accomplished by exploiting suitably the ISS property wherein the bounded nonlinear term is considered as an external input.

To proceed with our main results, we firstly recall the following helpful result from [Alexandridis, (1996)].

Lemma 1. If a matrix $S \in \mathbb{R}^{n\times n}$ is decomposed into

$$S = T + U, \text{ where } T = \frac{1}{2}(S + S^H) \text{ and } U = \frac{1}{2}(S - S^H),$$

with S^H being the complex conjugate transpose of matrix S, then the eigenvalues of S are located inside the region

$$\{\lambda(S) \in C : \lambda_{\min}(T) \le \operatorname{Re}\lambda(S) \le \lambda_{\max}(U)\} . \blacksquare \quad (9)$$

Now we are ready to proceed with our main result.

Theorem 1. Consider the Lipschitz system of (1)-(2) satisfying (3) and (8), and the corresponding observer of (4)-(5). Let matrix A have distinct eigenvalues and the pair (C, A) be observable. Then, for any scalar β satisfying $\beta > |\operatorname{Re}\lambda(A)|_{\max}$, there always exists a positive definite solution, $P = P^T > 0$, of the Lyapunov equation

$$[A^T + \beta I]P + P[A^T + \beta I]^T = 2C^T C \quad (10)$$

which i) assigns all the eigenvalues of $(A - LC)$ at the left half-plane with real part $\operatorname{Re}\lambda(A - LC) = -\beta$, ii) determines the observer gain matrix as

$$L = P^{-1}C^T \quad (11)$$

and iii) guarantees that the estimation error $\tilde{x} = x - \hat{x}$ trajectories of (6) converge globally asymptotically to the origin.

Proof: Let matrix $P = P^T > 0$ be the solution of

$$(A - LC)P + P(A - LC)^T = -Q \quad (12)$$

with $Q = Q^T > 0$, for some suitable L. Let the unique square root matrix $P^{1/2}$, which is also positive definite and Hermitian. By defining matrix S as

$$S = P^{1/2}[2(A - LC)]P^{-1/2}$$

and by substituting S in T of Lemma 1, it is easily obtained:

$$T = P^{1/2}(-P^{-1}Q)P^{-1/2} .$$

Note that $P^{-1}Q$ is also a Hermitian matrix and thus, it has only real eigenvalues. Moreover, by inspecting the last two equations, it is easily observed that S is similar to $2(A - LC)$, as matrix T is also similar to $-P^{-1}Q$. Therefore, by employing the results of Lemma 1, the following inequality holds:

$$\lambda_{\min}(-P^{-1}Q) \le \operatorname{Re}\lambda(A - LC) \le \lambda_{\max}(-P^{-1}Q) . \quad (13)$$

Now, selecting

$$Q = 2\beta P \quad (14)$$

all the eigenvalues of $(-P^{-1}Q)$ are on the left half-plane at

$$\lambda(-P^{-1}Q) = -2\beta . \quad (15)$$

Therefore, the minimum eigenvalue identifies with the maximum one, i.e.

$$\lambda_{\min}(-P^{-1}Q) = \lambda_{\max}(-P^{-1}Q) = -2\beta .$$

As a result, inequality (13) suggests that all eigenvalues of the system matrix $(A - LC)$ are assigned to

$$\operatorname{Re}\lambda(A - LC) = -\beta . \quad (16)$$

Obviously, the appropriate selection of β strictly defines where the real part of all eigenvalues are assigned. Hence, taking into account (14), Lyapunov equation (12) gives:

$$(A-LC)^T P + P(A-LC) = -2\beta P \tag{17}$$

Defining L, by (11), equation (17) becomes:

$$[A^T - C^T(CP^{-1})]P + P[A-(CP^{-1})^T C] = -2\beta P \,. \tag{18}$$

After some simple manipulations, (18) is restated to

$$[-(A^T+\beta I)]P + P[-(A^T+\beta I)]^T = -2C^T C$$

which eventually results in the form of (10).

For the Lyapunov equation (18) the assumption that the original system matrix A has distinct eigenvalues, provides the form of A as $A = U\Lambda U^{-1}$, with Λ being the diagonal matrix containing its distinct eigenvalues and U being the modal matrix of A. Therefore, it holds:

$$[-(A+\beta I)] = U[\Lambda - \beta \mathrm{I}]U^{-1}$$

and under the condition

$$\beta > \left|\operatorname{Re}\lambda(\mathrm{A})\right|_{\max} \tag{19}$$

matrix $[-(A+\beta I)]$ is always Hurwitz and under the assumption that the pair (C, A) is observable there always exists a positive definite matrix P solving (18). Equivalently, this means that under condition (19) a positive definite solution of (12) exists.

To complete the proof of points (i) and (ii) of Theorem 1, it is worth noting that (18) is equivalent to (10), and, to continue our proof of point (iii) of Theorem 1, the following positive definite Lyapunov function is first considered for system (6):

$$H(\tilde{x}) = \tilde{x}^T P \,\tilde{x} \,. \tag{20}$$

Obviously, system (6) is Lyapunov asymptotically stable if the time derivative of (20) is negative definite. Since

$$\dot{H}(\tilde{x}) = -\tilde{x}^T Q\,\tilde{x} + 2\tilde{x}^T PG(\tilde{x}) \tag{21}$$

we can see that for the right-hand side of (21) it holds:

$$\begin{aligned} -\tilde{x}^T Q\,\tilde{x} + 2\tilde{x}^T PF &\le -\lambda_{\min}(Q)\|\tilde{x}\|^2 + 2\lambda_{\max}(P)\|\tilde{x}\|\|G\| \\ &\le -\lambda_{\min}(Q)\|\tilde{x}\|\left(\|\tilde{x}\| - \frac{2\lambda_{\max}(P)\|G\|}{\lambda_{\min}(Q)}\right) . \end{aligned} \tag{22}$$

One can easily establish that the negative definiteness of (21) can only be assumed if the last term of (22) is a positive one. This holds true by taking into account (8) and (14):

$$\dot{V}(\tilde{x}) \le -2\beta\lambda_{\min}(P) \,,\, \forall \|\tilde{x}\| > \frac{\lambda_{\max}(P)}{\beta\lambda_{\min}(P)} d \,. \tag{23}$$

Clearly, (23) establishes that (6) is ISS with respect to an external input G. Then, according to the ISS definition [Sontag, (2008)], the estimation-error is bounded for any bound d of the observer-error equation and as has been proven there always exists an ISS Lyapunov function $V(\tilde{x})$ such that

$$\dot{V}(\tilde{x}) \le -\alpha(V(\tilde{x})) + \sigma(d) < -\alpha(V(\tilde{x})) + d \tag{24}$$

where $\alpha \,, \sigma \in K_\infty$.

Now, following a similar procedure as that presented in the theoretical part of [|Alexandridis, (2020)], there exists a lower bounded storage function $W(\tilde{x})$ of the form:

$$W(\tilde{x}) = \begin{cases} \frac{1}{2}\left[a(V(\tilde{x})) + \sigma(d)\right]^2 , \text{for } V(\tilde{x}) > a^{-1}(d) \\ 0, \qquad\qquad\qquad\qquad \text{for } V(\tilde{x}) \le a^{-1}(d) \end{cases} \tag{25}$$

Defining $\Omega = \left\{\tilde{x} \in \mathbb{R}^n : V(\tilde{x}) \le a^{-1}(d)\right\}$ one can see that for the time derivative of the C^1 function W, it is calculated $\dot{W} < 0$ outside of Ω, i.e. when $V(\tilde{x}) > a^{-1}(d)$ and $\dot{W} = 0$ inside Ω.

Furthermore, considering the structure of (6), the solution of $\dot{\tilde{x}} = 0$, determines the origin as the only one equilibrium for the first linear part in the right-hand side of (6). On the contrary, the last term in (6), as given by (7), can certainly provide additional equilibria, since it is a nonlinear Lipschitz function. However, the unique solution satisfying simultaneously both parts, i.e. $f(\tilde{x}, G) = 0$ is only the zero point. Hence, the largest invariant set E of (6) contains only the origin as the unique equilibrium. Then by applying the LaSalle Invariance Principle for system (6) with storage function W given by (25), it is directly proven that every trajectory $\tilde{x}(t)$ converges asymptotically to the largest invariant set E included in Ω, as $t \to \infty$ and consequently, the observer-error state trajectories globally converge to the origin. Therefore, the proof is completed. ■

Clearly, the above analysis results in a method of achieving an easy batch stable eigenvalue assignment of the considered observer design for Lipschitz-type systems. The advantages introduced by this method are duly highlighted in the following Remarks.

Remark 1. It is well-known that following the standard approach, asymptotic stability for the observer design of Lipschitz systems is established if a certain bound condition relating the Lipschitz condition γ with the pair (P, Q) of Lyapunov equation (12) is satisfied [Marquez, (2003)], i.e.

$$\gamma < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \tag{26}$$

It is noted that inequality (24) constitutes a hard relation between all these parameters, including the observer gain matrix L, as well. Restriction (26) cannot be easily combined with pole-assignment techniques and only the trial-and-error technique can be used. In our case, considering the special case where bounded Lipschitz terms exist, condition (19) substitutes (26).

Remark 2. Apparently, the proposed design can be applied to pure LTI systems as well, with even more straightforward manner, since the absence of nonlinearities renders the handling of such systems considerably easier. In this case, global exponential stability along with estimation-error state convergence to the origin equilibrium can be easily realized.

4. ILLUSTRATIVE EXAMPLE

In order to verify the theoretical results and to validate the efficiency of the presented approach, various simulation scenarios were conducted by considering a typical power system application. It is noticed that observer designs have been essentially used in power system applications [Alexandridis and Galanos, (1989)]. In the present case, the proposed observer design was implemented on a single generator infinite bus power system model. For comparison reasons, the dynamic model of the examined system along with the considered parameters are taken as in [Alexandridis, (2020)]. The state-space representation takes the form of (1)-(2), with a state vector $x=[x_1 \quad x_2]^T=[\delta \quad \Delta\omega]^T$ and $u=[0 \quad P_m]^T$, where state δ is the rotor angle and $\Delta\omega$ represents the difference between the rotor speed, ω, and the synchronous speed ω_0, i.e. $\Delta\omega=\omega-\omega_0$. The single external uncontrolled input enforced on the system is the mechanical power applied to the rotor, denoted by P_m. Additionally, the respective system, input, and output matrices are provided as:

$$A=\begin{bmatrix}0 & 1\\ 0 & -0.83\end{bmatrix},\ B=\begin{bmatrix}0 & 0\\ 0 & 52.36\end{bmatrix},\ C=\begin{bmatrix}0 & 1\end{bmatrix}.$$

The Lipschitz function is expressed by

$$g(x)=[0 \quad -274.89\sin(x_1)]^T$$

with a Lipschitz constant, $\gamma=274.89$. Also, for the open-loop system, it is easily calculated $|\mathrm{Re}\,\lambda(\mathrm{A})|_{\max}=0.83$. However, since $\|G(t,x)\|=\|g(t,x_1)-g(t,x_2)\|$, with function $g(x)$ being of sinusoidal type, there is always an upper bound for $\|G\|$, i.e. *d,* as suggested by (8) and therefore, the results of Theorem 1 can be applied in this case.

The examined model and the corresponding observer of (4)-(5) are formulated in Matlab/Simulink environment and the resulting dynamic responses are evaluated with a twofold aim, namely to assess the pole-placement technique capability in assigning the observer poles in a desired position and simultaneously to examine the system stable behavior as established by the ISS property. For the simulation purposes the input of the system was kept constant at $P_m=2.625$ pu while the initial conditions were set to $\delta_0=0$ rad $\Delta\omega_0=12$ rad/s.

In Figures 1 and 2, the actual and estimated states of δ and $\Delta\omega$ are presented whereas the observer-error for both states, are depicted in Fig. 3 and 4. For comparison purposes, the simulation was conducted by considering various values of the scalar β. The eigenvalues of system matrix $(A-LC)$ are presented in Table I for all examined cases.

TABLE I
SYSTEM EIGENVALUES

Examined case	Eig(*A-LC*)
$\beta=5$	$[-5+i4.5662 \quad -5-i4.5662]^T$
$\beta=10$	$[-10+i9.5760 \quad -10-i9.5760]^T$
$\beta=15$	$[-15+i14.5791 \quad -15-i14.5791]^T$

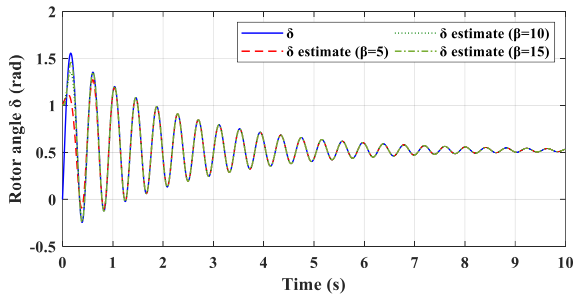

Figure 1. Actual and estimated states of rotor angle for various values of β.

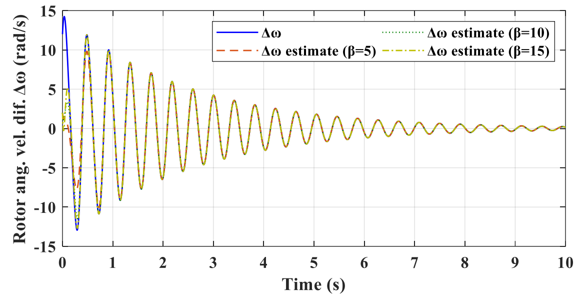

Figure 2. Actual and estimated states of rotor angular velocity difference for various values of β.

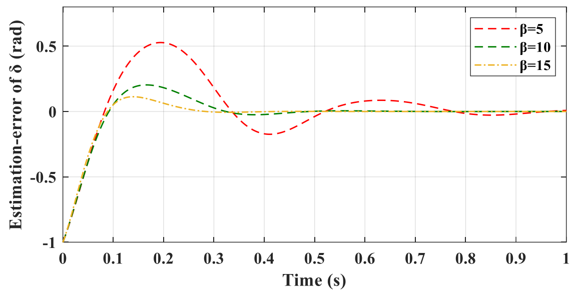

Figure 3. Estimation error of rotor angle for the considered values of β.

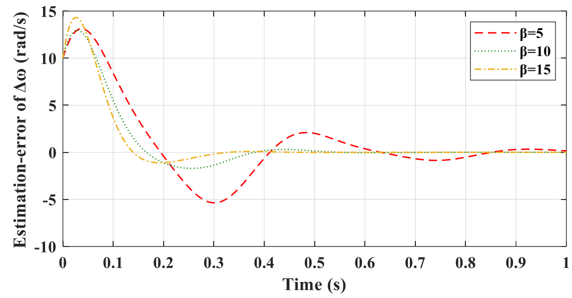

Figure 4. Estimation error of rotor angular velocity difference for the considered values of β.

It is easily established that the implemented observer design achieves accurate and fast estimation of the actual system states without exhibiting significant transients. It is also important to note that with greater values of β the system response displays even better characteristics as a result of the exact pole-placement this approach offers since then larger real parts of system eigenvalues result.

However, it is noted that as the theoretical analysis suggests, a selection of β should be made in accordance to the boundary condition (19). To evaluate this, in Fig. 5 and 6 the actual and estimated states of δ and $\Delta\omega$ are provided with two considered values of β, placed very closely to the boundary determined for this example, i.e. $\beta>0.8$.

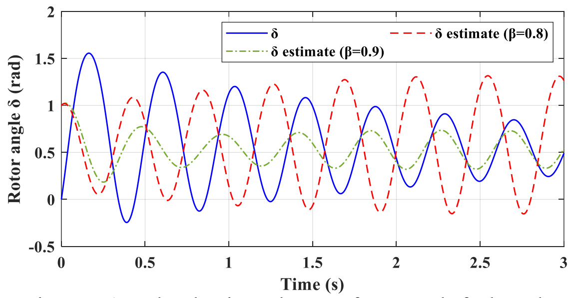

Figure 5. Actual and estimated states of rotor angle for boundary β.

By selecting β=0.9 the dynamic responses of the estimated states converge to the actual ones after a transient period. Conversely, the choice of adopting an even marginally less value than the one permitted, results in significantly more deteriorated performance of the system, rendering it unstable.

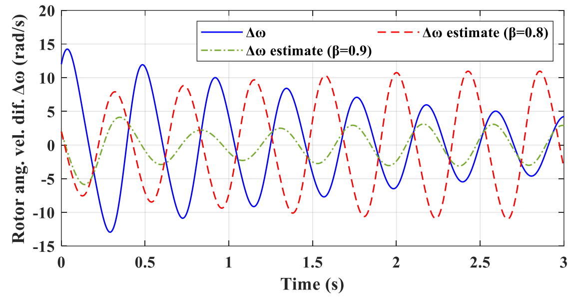

Figure 6. Actual and estimated states of rotor angular velocity difference for boundary β.

6. CONCLUSIONS

In this paper, the challenging task of designing reliable and efficient observers for a class of Lipschitz nonlinear systems with bounded nonlinear terms is considered. In this frame, a novel observer design is proposed that achieves exact damping values of the observer eigenvalues at desired positions on the left halfplane. In this way, it is allowed to predefine how fast the dynamic response of the system will be and thus, it enables observer synthesis according to custom needs and requirements. Furthermore, the implementation of the presented approach guarantees asymptotic stability of the estimation-error to the origin, according to boundary conditions extracted by the pole-assignment technique itself and the stability results as derived by establishing the ISS property applied on bounded systems of this type. The theoretical analysis is fully verified and the enhanced dynamic properties introduced by the deployed method are evaluated by simulating an example based on a real-world power system application. This certainly highlights the wide applicability of the presented approach, although the extension of this method to cover a larger class of systems, such as one-sided Lipschitz ones might be an interesting concept for future consideration.

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