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Decentralized Control Design for Adaptive Structures with Tension-only Elements *

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Abstract: Adaptivity in civil engineering structures is realized by the integration of sensors, actuators and a control scheme. The large dimensions of such structures cause high installation effort for cabling and challenges in control through transmission delays. Furthermore, modern lightweight structures typically include elements that bear tension forces only, leading to a nonlinear model for control design. In this contribution, we propose a decentralized control scheme for civil engineering structures which can handle nonlinearities through tension-only elements in control design. A large structure is subdivided into local substructures, incorporating nonlinear elements each. The Craig-Bampton model order reduction is applied to the substructures, which can only be conducted by intelligent separation of degrees of freedom for inner and boundary nodes, where degrees of freedom influenced by tension-only elements are set as boundary nodes. Linearizing input transformations are designed by means of local substructures, forcing the nonlinear model to the dynamics of a local linear target system. Linear feedback controllers can be designed based on the linear target system. These decentralized linearizing input transformations and feedback controllers are applied to the combined nonlinear structure. This approach is illustrated numerically on an adaptive high-rise structure.

Keywords: adaptive structures, tension-only-elements, decentralized control

1. INTRODUCTION

Adaptive structures featuring sensors, actuators and control algorithms, are an opportunity to reduce the immense resource and energy consumption as well as waste and emission production of the construction sector (UN Environmental Program (2011)). Assuming a proceeding urbanization and a substantial growth of world's population, our planet cannot meet the needs of the predicted construction boom. Thus, efficient and save application of civil engineering adaptive structures are indispensable (Sobek and Teuffel (2001)). Such structures have been researched since years (Yao (1972)) and overviews are given, among others, in Korkmaz (2011) or Housner et al. (1997).

Just as traditional structures, adaptive buildings can include elements, which only bear tension forces, e. g. realized by flat steel or cables to stiffen the main load carrying elements. Applying compression forces to such elements is impossible due to slack, under which these elements do not contribute to a structure's stiffness any more, described in Alart et al. (2007). In this contribution, link elements, which bear normal forces and no bending forces, are regarded as tension-only elements. Tension-only elements can be controlled by a linearizing input transformation, forcing a structure onto desired (linear) target dynamics. Subsequently, linear control techniques can be used for vibration damping, as introduced in Wagner et al. (2019). A further aspect of adaptive structures is the perpetual operation. At any time, they need to withstand loads to guarantee an undamaged structure and safety for the inhabitants. Decentralized control units contribute to this goal by distributing the control hardware across the large structure.

Fig. 1. Rendering of an adaptive high-rise structure (right) and an access and supply tower (ILEK).

In control of nonlinear systems, input transformations are a powerful method, especially linearizing a system's dynamics onto an integrator chain, describe in the theory of flat systems (Fliess et al. (1995)). Another method regarding input output linearization for multi-input/multi-output systems is given in Kravaris and Soroush (1990),

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where static state feedback is calculated. Comparison to other input/output transformations is given and the theory is applied to a numerical example of a semi-batch reactor. An approximate input/output linearization is designed in Hauser et al. (1992) for systems that do not have a well defined relative degree. Coming to decentralized control, such approaches are used since quite some time, e. g. Babuska and Craig Jr (1993) makes use of a decentralized control approach, based on local reduced order models, to save the ancient limited computation power to design a global controller. A more recent article researching on decentralized control in civil engineering structures is presented in Bakule et al. (2016). A model order reduction (MOR) after Guyan is applied to submodels to design a switching control for earthquake excited structures. The authors Monajemi-Nezhad and Rofooei (2007) present a sliding mode control for a multistory building and proof global system's stability, where decentralized controllers are designed for substructures of the building. The application of substructuring and MOR for nonlinear system to achieve decentralized models is also found in literature. Bathe and Gracewski (1981) shows substructuring via static condensation by separating the linear and nonlinear part of the nonlinear system equations. The authors Kuether et al. (2017) perform substructuring of mechanical models with geometric nonlinearities as a non-intrusive method on top of commercial finite element (FE) software. They separate the nonlinear part and derive a transformation to achieve a reduced order model accounting for geometric nonlinearities in an example of coupling elastic plates. Wenneker and Tiso (2014) use two substructuring methods, within the Craig-Bampton method to calculate reduced order models, which accounts also for geometric nonlinearities. An insight in the literature on decentralized control of nonlinear systems, especially for civil engineering systems is given in the following. A decentralized model predictive control approach for a discrete time system is proposed by Magni and Scattolini (2006), where local controllers help to reduce the larger optimization problem. Udwadia et al. (2014) introduces a decentralized control approach for nonlinear systems via sliding mode control. In a first step, local asymptotically stable controllers are designed for a subsystem using only local information and in a second step interconnections are considered. In Lynch et al. (2002), different decentralized controllers are applied to a civil engineering structural system under earthquake excitations, which show acceptable control performance.

Main Contribution and Overview This article contributes to model order reduction (MOR) of substructured systems with tension-only elements by extending the Craig-Bampton method to partly nonlinear systems. Furthermore, a decentralized nonlinear control approach is introduced based on linearizing input transformations, which can globally stabilize the structure.
In section 2, the nonlinear equations of motion are derived, which contain the effects of tension-only elements. The system is substructured and a MOR is performed by means of the Craig-Bampton method, preserving the nonlinear properties of the tension-only elements. Section 3 gives the equations for the linearizing input function for nonlinear systems and in section 4 performance of the decentralized nonlinear control approach is illustrated using an exemplary civil engineering structure.

2. SYSTEM MODELING

In this section, a truss structure with tension-only elements is modeled and via nonlinear MOR a simulation model is derived. For decentralized nonlinear control design, local models are deduced by substructuring.

2.1 Nonlinear Equations of Motion

Civil engineering structures are commonly modeled by means of the FE method. A second order mechanical model with possibly large, but limited number of degrees of freedom (DOF) $\boldsymbol{q}(t) \in \mathbb{R}^n$ is achieved

$$\begin{aligned} \boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{D}\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) &= \boldsymbol{f}(t), \quad t > 0, \\ \boldsymbol{q}(0) = \boldsymbol{q}_0, \qquad \dot{\boldsymbol{q}}(0) &= \boldsymbol{q}_1, \end{aligned} \tag{1}$$

where the positive definite mass and stiffness matrices $\boldsymbol{M}$, $\boldsymbol{K} \in \mathbb{R}^{n\times n}$ are determined by FE analysis. External forces and the system's input are summarized in $\boldsymbol{f}(t)$. The damping matrix is approximated by the Rayleigh approach (Rayleigh (1877),O'Kelly and Caughey (1965)) for classical and proportional damping

$$\boldsymbol{D} = \alpha_1 \boldsymbol{M} + \alpha_2 \boldsymbol{K}, \tag{2}$$

with the coefficients α_1 and α_2, which need to be determined experimentally. The external forces on the structure are specified by

$$\boldsymbol{f}(t) = \boldsymbol{F}\boldsymbol{u}(t) + \boldsymbol{z}(t), \tag{3}$$

with the input matrix $\boldsymbol{F} \in \mathbb{R}^{n\times m}$ and the input forces $\boldsymbol{u}(t) \in \mathbb{R}^m$, which are realized by structure-integrated actuators. Actuator dynamics do not significantly influence the closed loop system behavior. External disturbances are captured in $\boldsymbol{z}(t)$ and neglected in presentation of the feedback control for decentralized control because disturbance dynamics are not known.

In case of nonlinear system behavior, the linear equations of motion are not sufficient. Within an FE software, structural analysis model equations are solved iteratively and therefore not exportable for control design. However, for the purpose of control design for nonlinear systems, the reduced order model equations are required. Starting from the linear equations (1) without tension-only elements, the nonlinear behavior of tension-only elements is included. Such elements only transmit normal forces, meaning tension and compression forces. If such an element is compressed, its contribution to the stiffness and damping matrix vanishes. Therefore, the stiffness k_i of a tension-only element i is equal to the stiffness of the linear model in case of tension and zero in case of compression:

$$k_i(\boldsymbol{q}(t)) = \begin{cases} k_i, & \Delta l_i(\boldsymbol{q}(t)) \geq 0 \\ 0, & \Delta l_i(\boldsymbol{q}(t)) < 0 \end{cases} \qquad i = 1, ..., n_\mathrm{t}. \tag{4}$$

The condition to switch between tension and compression is stated by means of the length difference of an element,

$$\Delta l_i(\boldsymbol{q}(t)) = \sqrt{\Delta\tilde{\boldsymbol{q}}_i^\intercal(t)\ \Delta\tilde{\boldsymbol{q}}_i(t)} - \sqrt{\Delta\boldsymbol{q}_{i,0}^\intercal\ \Delta\boldsymbol{q}_{i,0}}. \tag{5}$$

The second term is the initial element length with $\Delta\boldsymbol{q}_{i,0} = \boldsymbol{q}_{i,1,0} - \boldsymbol{q}_{i,2,0}$ where $\boldsymbol{q}_{i,1,0}$ and $\boldsymbol{q}_{i,2,0}$ are the initial positions of the nodes element i is attached to. The first term covers the displacements according to movements of the structure

$$\Delta\tilde{\boldsymbol{q}}_i(t) = \tilde{\boldsymbol{q}}_{i,1}(t) - \tilde{\boldsymbol{q}}_{i,2}(t), \text{ with } \tilde{\boldsymbol{q}}_{i,1}(t) = \boldsymbol{q}_{i,1,0} + \boldsymbol{q}_{i,1}(t), \tag{6}$$

$\tilde{\boldsymbol{q}}_{i,2}(t)$ respectively. The relative displacements of the attached nodes $\boldsymbol{q}_{i,1}(t) \subset \boldsymbol{q}(t)$ and $\boldsymbol{q}_{i,2}(t) \subset \boldsymbol{q}(t)$ are a subset of the DOF vector. Based on the changes in the elements' stiffness, the stiffness matrix is assembled, depending on the actual $\boldsymbol{q}(t)$. Damping of the structure is approximated as in (2), where the damping matrix is dependent on $\boldsymbol{q}(t)$, as inherited from $\boldsymbol{K}(\boldsymbol{q}(t))$. The inertia $\boldsymbol{M}$ is assumed to remain unchanged because an element is physically still available and changes in mass distribution are supposed to be small. For example, if a tension-only element is under tension, the inertia matrix coming from the FE tool fits perfectly. If such an element is slack, it only hangs on the upper node and the inertia matrix is slightly different. However, this is neglectable because the main effect which appears in the inertia matrix is dominated by the location of an element within the structure. Therefore, the nonlinear equations of motion yield:

$$\begin{aligned} &\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{D}(\boldsymbol{q}(t))\dot{\boldsymbol{q}}(t) + \boldsymbol{K}(\boldsymbol{q}(t))\boldsymbol{q}(t) = \boldsymbol{F}\boldsymbol{u}(t), t > 0, \\ &\quad \boldsymbol{q}(0) = \boldsymbol{q}_0, \qquad \dot{\boldsymbol{q}}(0) = \boldsymbol{q}_1. \end{aligned} \tag{7}$$

2.2 Simulation Model

For simulation purposes, a reduced order model is used to achieve acceptable simulation time. To preserve the nonlinear model behavior, MOR is conducted by means of a transformation to a lower dimensional space. In a proper orthogonal decomposition (POD), a simulation data matrix $\boldsymbol{y}_{\mathrm{data}}$ is factorized by a singular value decomposition

$$\boldsymbol{y}_{\mathrm{data}} = \boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{W}^*, \tag{8}$$

where $\boldsymbol{V} \in \mathbb{R}^{n_1 \times n_1}$ is a unitary matrix, which comprises the left eigenvectors of $\boldsymbol{y}_{\mathrm{data}}$. $\boldsymbol{\Sigma} \in \mathbb{R}^{n_1 \times n_2}$ covers the singular values and $\boldsymbol{W}^* \in \mathbb{R}^{n_2 \times n_2}$ the conjugate transpose of a unitary matrix $\boldsymbol{W}$, where the columns are the right eigenvectors of $\boldsymbol{y}_{\mathrm{data}}$. MOR is performed by transforming $\boldsymbol{q}(t) = \boldsymbol{V}_{\mathrm{r}}\boldsymbol{\zeta}(t)$ with $\boldsymbol{\zeta}(t) \in \mathbb{R}^{n_{\mathrm{r}}}$, a column matrix $\boldsymbol{V}_{\mathrm{r}} \subset \boldsymbol{V}$ and left-multiplying by $\boldsymbol{V}_{\mathrm{r}}^{\mathsf{T}}$

$$\begin{aligned} &\boldsymbol{M}_{\mathrm{r}}\ddot{\boldsymbol{\zeta}}(t) + \boldsymbol{V}_{\mathrm{r}}^{\mathsf{T}}\boldsymbol{D}(\boldsymbol{V}_{\mathrm{r}}\boldsymbol{\zeta}(t))\boldsymbol{V}_{\mathrm{r}}\dot{\boldsymbol{\zeta}}(t) + \boldsymbol{V}_{\mathrm{r}}^{\mathsf{T}}\boldsymbol{K}(\boldsymbol{V}_{\mathrm{r}}\boldsymbol{\zeta}(t))\boldsymbol{V}_{\mathrm{r}}\boldsymbol{\zeta}(t) \\ &= \boldsymbol{F}_{\mathrm{r}}\boldsymbol{u}(t), \quad t > 0, \quad \boldsymbol{\zeta}(0) = \boldsymbol{V}_{\mathrm{r}}^{-1}\boldsymbol{q}_0, \quad \dot{\boldsymbol{\zeta}}(0) = \boldsymbol{V}_{\mathrm{r}}^{-1}\boldsymbol{q}_1. \end{aligned} \tag{9}$$

2.3 Substructuring

For decentralized control, the system needs to be substructured in local models. The i^{th} substructure for $i = 1...n_{\mathrm{S}}$ is derived by substructuring $\boldsymbol{q}(t) = \boldsymbol{C}_{\mathrm{q}}^{i\,\mathsf{T}}\boldsymbol{q}^i(t)$ in (7) with $\boldsymbol{q}^i(t) \in \mathbb{R}^{n^i}$. The binary selection matrix $\boldsymbol{C}_{\mathrm{q}}^i$ chooses the states comprising this substructure. The decentralized nonlinear model equations for the i^{th} substructure yield

$$\begin{aligned} &\boldsymbol{M}^i\ddot{\boldsymbol{q}}^i(t) + \boldsymbol{K}^i(\boldsymbol{q}^i(t))\boldsymbol{q}^i(t) = \boldsymbol{F}^i\boldsymbol{u}^i(t), \quad t > 0, \\ &\quad \boldsymbol{q}^i(0) = \boldsymbol{C}_{\mathrm{q}}^i\boldsymbol{q}_0, \quad \dot{\boldsymbol{q}}^i(0) = \boldsymbol{C}_{\mathrm{q}}^i\boldsymbol{q}_1, \end{aligned} \tag{10}$$

with $\boldsymbol{M}^i = \boldsymbol{C}_{\mathrm{q}}^i\boldsymbol{M}\boldsymbol{C}_{\mathrm{q}}^{i\,\mathsf{T}}$, $\boldsymbol{K}^i(\boldsymbol{q}^i(t)) = \boldsymbol{C}_{\mathrm{q}}^i\boldsymbol{K}(\boldsymbol{q}^i(t))\boldsymbol{C}_{\mathrm{q}}^{i\,\mathsf{T}}$. The damping for the control design model is introduced in section 2.4. The local input forces $\boldsymbol{u}^i(t)$ capture the m^i actuators of a substructure, achieved by the transformation $\boldsymbol{u}(t) = \boldsymbol{C}_{\mathrm{u}}^i\boldsymbol{u}^i(t)$. The binary selection matrix for the input is $\boldsymbol{C}_{\mathrm{u}}^i$ and thus $\boldsymbol{F}^i = \boldsymbol{C}_{\mathrm{q}}^i\boldsymbol{F}\boldsymbol{C}_{\mathrm{u}}^{i\,\mathsf{T}}$ holds.

2.4 MOR for Substructures

For control design, a low order model is desirable, which captures the main characteristics of a substructure. For linear MOR for mechanical systems, the Craig-Bampton method is well established, where the DOF vector is separated into boundary and inner nodes (see Fig 2). It can be

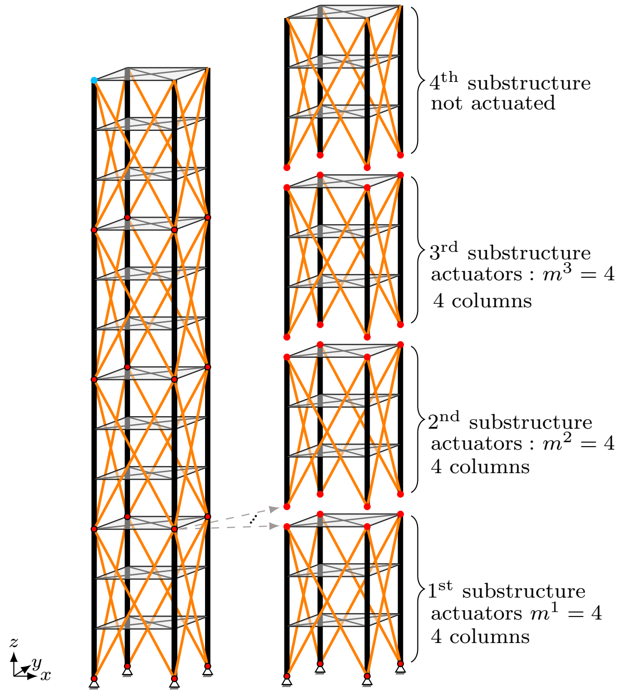

Fig. 2. Sketch of a full/substructured high-rise building including tension-only elements (orange) with boundary nodes (red) and the DOFs used in the results in section 4 (light blue).

extended to the case presented here, where some elements exhibit nonlinear dynamics. Thereby, analogously to the linear Craig-Bampton approach, boundary nodes $\boldsymbol{q}_{\mathrm{b}}^i(t) \in \mathbb{R}^{n_{\mathrm{b}}^i}$ are selected, comprising not only physical boundaries, but also nodes where actuators are attached to and nodes, where nonlinear elements are attached to. This is important because after the transformation, all boundary nodes of $\boldsymbol{q}_{\mathrm{b}}(t)$ are still available and can be accessed for nonlinear evaluation during control. The dynamics of the remaining inner nodes $\boldsymbol{q}_{\mathrm{i}}^i(t) \in \mathbb{R}^{n_{\mathrm{i}}^i}$, where only linear elements are attached to, are approximated via modal analysis. Therefore, the nonlinear substructures can be reduced, while the nonlinearities and physical coupling to other structures are preserved and captured completely in $\boldsymbol{K}_{\mathrm{bb}}^i(\boldsymbol{q}^i(t))$, which is dependent on the boundary nodes $\boldsymbol{q}_{\mathrm{b}}^i(t)$. Without loss of generality, $\boldsymbol{q}^i(t)$ is assumed to be in the right order through proper choice of $\boldsymbol{C}_{\mathrm{q}}^i$. The rearranged equations of motion from (7) are stated as:

$$\begin{bmatrix} \boldsymbol{M}_{\mathrm{bb}}^i & \boldsymbol{M}_{\mathrm{bi}}^i \\ \boldsymbol{M}_{\mathrm{ib}}^i & \boldsymbol{M}_{\mathrm{ii}}^i \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_{\mathrm{b}}^i(t) \\ \ddot{\boldsymbol{q}}_{\mathrm{i}}^i(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{\mathrm{bb}}^i(\boldsymbol{q}^i(t)) & \boldsymbol{K}_{\mathrm{bi}}^i \\ \boldsymbol{K}_{\mathrm{ib}}^i & \boldsymbol{K}_{\mathrm{ii}}^i \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_{\mathrm{b}}^i(t) \\ \boldsymbol{q}_{\mathrm{i}}^i(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \tag{11}$$

The quasi-stationary solution of the second line of (11) leads to the rigid-body modes of the substructure

$$\boldsymbol{q}_{\mathrm{i}}^i(t) = \boldsymbol{\Phi}_{\mathrm{c}}^i\boldsymbol{q}_{\mathrm{b}}^i(t) \quad \text{with} \quad \boldsymbol{\Phi}_{\mathrm{c}}^i = -\boldsymbol{K}_{\mathrm{ii}}^{i\;-1}\boldsymbol{K}_{\mathrm{ib}}^i. \tag{12}$$

The inversion is guaranteed due to the positive definiteness and linearity of $\boldsymbol{K}_{\mathrm{ii}}^i$. The inner nodes with linear connections are approximated through a reduced number of modal coordinates $\boldsymbol{\eta}^i(t) \in \mathbb{R}^{n_{\mathrm{i,r}}^i}$, which are calculated by solving the general eigenvalue problem of the second line of (11) for $\boldsymbol{q}_{\mathrm{b}}^i(t) = \boldsymbol{0}$. A low dimensional column space of the eigenvector matrix of the inner DOF $\boldsymbol{\Phi}^i = [\boldsymbol{\varphi}_1, ..., \boldsymbol{\varphi}_{n_{\mathrm{i,r}}^i}]$ is chosen for transformation. The Craig-

Bampton transform applied to nonlinear systems yields:

$$\begin{bmatrix}\boldsymbol{q}_\mathrm{b}^i(t)\\ \boldsymbol{q}_\mathrm{i}^i(t)\end{bmatrix} \approx \boldsymbol{T}^i\boldsymbol{q}_\mathrm{c}^i(t),\ \ \boldsymbol{T}^i = \begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{\Phi}_\mathrm{c}^i & \boldsymbol{\Phi}^i\end{bmatrix},\ \ \boldsymbol{q}_\mathrm{c}^i(t) = \begin{bmatrix}\boldsymbol{q}_\mathrm{b}^i(t)\\ \boldsymbol{\eta}^i(t)\end{bmatrix}, \tag{13}$$

with transformed coordinates $\boldsymbol{q}_\mathrm{c}^i(t) \in \mathbb{R}^{n_\mathrm{c}^i}$. The order n_c^i of the reduced model is determined by the number of boundary nodes, defined by the system's structure including the nonlinear elements, and by the number of modal coordinates used for reducing the dynamics of the inner nodes. Applying this transformation to (11) and left multiplying with $\boldsymbol{T}^i$ leads to

$$\begin{aligned}\boldsymbol{M}_\mathrm{c}^i\ddot{\boldsymbol{q}}_\mathrm{c}^i(t) + \boldsymbol{K}_\mathrm{c}^i(\boldsymbol{q}_\mathrm{c}^i(t))\boldsymbol{q}_\mathrm{c}^i(t) &= \boldsymbol{F}_\mathrm{c}^i\boldsymbol{u}^i(t), \qquad t>0,\\ \boldsymbol{q}_\mathrm{c}^i(0) = \boldsymbol{q}_{\mathrm{c},0}^i\ , \qquad \dot{\boldsymbol{q}}_\mathrm{c}^i(0) &= \boldsymbol{q}_{\mathrm{c},1}^i\ ,\end{aligned} \tag{14}$$

with $\boldsymbol{M}_\mathrm{c}^i = \boldsymbol{T}^{i\mathsf{T}}\boldsymbol{M}^i\boldsymbol{T}^i$, the blockdiagonal matrix $\boldsymbol{K}_\mathrm{c}^i(\boldsymbol{q}_\mathrm{c}^i(t)) = \boldsymbol{T}^{i\mathsf{T}}\boldsymbol{K}^i(\boldsymbol{T}^i\boldsymbol{q}_\mathrm{c}^i(t))\boldsymbol{T}^i$ and $\boldsymbol{F}_\mathrm{c}^i = \boldsymbol{T}^{i\mathsf{T}}\boldsymbol{F}^i$. In contrast to the Craig-Bampton reduction for linear systems, the extension to structures with nonlinear elements requires a modified selection of boundary nodes. Additionally – at each time step – the nonlinear dynamics have to be evaluated in the original state space and subsequently transformed into the reduced space. In (14), this holds for $\boldsymbol{K}_\mathrm{c}^i(\boldsymbol{q}_\mathrm{c}^i(t))$. This kind of nonlinear MOR by means of the Craig-Bampton method values the physical coupling and the preservation of nonlinearities, especially beneficial for large structures including a limited number of, in this case, tension-only elements. The damping matrix is defined as in (2)

$$\boldsymbol{D}_\mathrm{c}^i(\boldsymbol{q}_\mathrm{c}^i(t)) = \alpha_1\boldsymbol{M}_\mathrm{c}^i + \alpha_2\boldsymbol{K}_\mathrm{c}^i(\boldsymbol{q}_\mathrm{c}^i(t)). \tag{15}$$

with α_1 and α_2 according to (2), which is equivalent to an earlier definition.

2.5 Elimination of Rigid-Body Modes

In general, substructuring may yield modules that are not fixed by supports any more, leading to rigid-body modes. In the previous section, the method of Craig-Bampton regards these rigid-body modes in $\boldsymbol{\Phi}_\mathrm{c}^i$. However, in Craig and Bampton (1968) the boundary nodes are not distinguished between constraint coordinates and rigid-body coordinates, contrary to Hurty (1965). In a civil engineering sense, the subsystem is statically overdetermined and the number of statical overdeterminacy is equal to the number of rigid-body coordinates $n_{\mathrm{c,rb}}^i$, which is the same as the number of uncontrollable zero eigenvalues of the substructured model. The concept of statical indeterminacy is not defined for nonlinear dynamic structures, such as (14). Since the nonlinear system is feedback controlled, the assumption of reaching $\boldsymbol{q}_\mathrm{b}^i(t) = \boldsymbol{0}$ is justified. Therefore, (14) is evaluated for $\boldsymbol{K}_\mathrm{c}^i(\boldsymbol{0})$ and $\boldsymbol{D}_\mathrm{c}^i(\boldsymbol{0})$ to determine the rigid-body and constraint coordinates of this linearized system, which leads to a transformation matrix $\boldsymbol{\Phi}_\mathrm{l}^i$. Using the transformation $\boldsymbol{q}_\mathrm{c}^i(t) = \boldsymbol{\Phi}_\mathrm{l}^i\boldsymbol{q}_\mathrm{l}^i(t)$ with the new DOF vector $\boldsymbol{q}_\mathrm{l}(t) \in \mathbb{R}^{n_\mathrm{l}}$ and $n_\mathrm{l}^i = n_\mathrm{c}^i - n_{\mathrm{c,rb}}^i$, the model for local control design is:

$$\begin{aligned}&\boldsymbol{M}_\mathrm{l}^i\ddot{\boldsymbol{q}}_\mathrm{l}^i(t) + \boldsymbol{D}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t))\dot{\boldsymbol{q}}_\mathrm{l}^i(t) + \boldsymbol{K}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t))\boldsymbol{q}_\mathrm{l}^i(t)\\ &= \boldsymbol{F}_\mathrm{l}^i\boldsymbol{u}^i(t)\ , \quad t>0\ , \quad \boldsymbol{q}_\mathrm{l}^i(0) = \boldsymbol{q}_{\mathrm{l},0}^i\ , \quad \dot{\boldsymbol{q}}_\mathrm{l}^i(0) = \boldsymbol{q}_{\mathrm{l},1}^i,\end{aligned} \tag{16}$$

with the matrices without static modes for mass $\boldsymbol{M}_\mathrm{l}^i = \boldsymbol{\Phi}_\mathrm{l}^{i\mathsf{T}}\boldsymbol{M}_\mathrm{c}^i\boldsymbol{\Phi}_\mathrm{l}^i$, stiffness $\boldsymbol{K}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t)) = \boldsymbol{\Phi}_\mathrm{l}^{i\mathsf{T}}\boldsymbol{K}_\mathrm{c}^i(\boldsymbol{\Phi}_\mathrm{l}^i\boldsymbol{q}_\mathrm{l}^i(t))\boldsymbol{\Phi}_\mathrm{l}^i$ and damping $\boldsymbol{D}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t)) = \boldsymbol{\Phi}_\mathrm{l}^{i\mathsf{T}}\boldsymbol{D}_\mathrm{c}^i(\boldsymbol{\Phi}_\mathrm{l}^i\boldsymbol{q}_\mathrm{l}^i(t))\boldsymbol{\Phi}_\mathrm{l}^i$, and input matrix $\boldsymbol{F}_\mathrm{l}^i(t) = \boldsymbol{\Phi}_\mathrm{l}^{i\mathsf{T}}\boldsymbol{F}_\mathrm{c}^i(t)$.

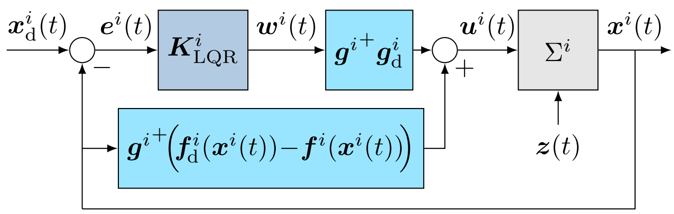

Fig. 3. Structure of the control loop for subsystem Σ^i.

System (16) is transferred to general state space representation using state $\boldsymbol{x}^i(t) = [\boldsymbol{q}_\mathrm{l}^i(t), \dot{\boldsymbol{q}}_\mathrm{l}^i(t)]^\mathsf{T} \in \mathbb{R}^{2n_\mathrm{l}^i}$:

$$\dot{\boldsymbol{x}}^i(t) = \boldsymbol{f}^i(\boldsymbol{x}^i(t)) + \boldsymbol{g}^i(\boldsymbol{x}^i(t))\boldsymbol{u}^i(t),\ \ t>0,\ \ \boldsymbol{x}^i(0) = \boldsymbol{x}_0^i,$$

$$\boldsymbol{f}^i(\boldsymbol{x}^i(t)) = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ -\boldsymbol{K}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t)) & -\boldsymbol{D}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t))\end{bmatrix}\boldsymbol{x}^i(t),\ \ \boldsymbol{g}^i = \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{F}_\mathrm{l}^i\end{bmatrix}, \tag{17}$$

where $\boldsymbol{g}^i$ is not state-dependent, in this case. This system is assigned as control design model in the following section.

3. CONTROL DESIGN

The term decentralized control refers to a separate control design for each substructure, derived in 2.3-2.5. At first, a linear control scheme is given for comparison, designed for the linear system using $\boldsymbol{K}(\boldsymbol{0})$ and $\boldsymbol{D}(\boldsymbol{0})$. Second, a nonlinear control scheme is presented, where the nonlinear substructure is forced onto the dynamics of a linear substructure by use of a linearizing input transformation. Afterward, linear control schemes can be applied to the linearized system.

3.1 Linear Decentralized Control

To show the performance of the decentralized nonlinear control scheme introduced in 3.2, a linear decentralized approach is given. The tension-only elements are treated as linear link elements, thus (17) is evaluated for $\boldsymbol{q}_\mathrm{l}^i(t) = \boldsymbol{0}$ to achieve linear substructured models. We use here a linear quadratic regulator (LQR), with the control law

$$\boldsymbol{u}^i(t) = \boldsymbol{K}_{\mathrm{lqr,l}}^i(\boldsymbol{x}_{\mathrm{l}0}^i - \boldsymbol{x}_\mathrm{l}^i(t)), \tag{18}$$

where $\boldsymbol{x}_{\mathrm{l}0}^i$ is the desired local state. The feedback matrix $\boldsymbol{K}_{\mathrm{lqr,l}}^i$ is determined such that the cost function

$$J = \int_0^\infty \boldsymbol{x}^{i\mathsf{T}}(t)\,\boldsymbol{Q}\,\boldsymbol{x}^i(t) + \boldsymbol{u}^{i\mathsf{T}}(t)\,\boldsymbol{R}\,\boldsymbol{u}^i(t)\mathrm{d}t \tag{19}$$

is minimal under given weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$.

3.2 Decentralized Nonlinear Control

To damp oscillations in the system given by (17), a linearizing input transformation is designed similar to the procedure for global controller design in Wagner et al. (2019). The behavior of an exemplary system with tension-only elements is similar to that of a linear structure. In general, for nonlinear system's feedback, linearization is a proper choice to transform a system to an integrator chain (Fliess et al. (1995)), where control is fairly simple. However, an integrator chain is a theoretic system behavior and for systems as introduced here, it might be better to choose a different target system. Therefore, the linearizing input transformation is used, which forces the dynamics of

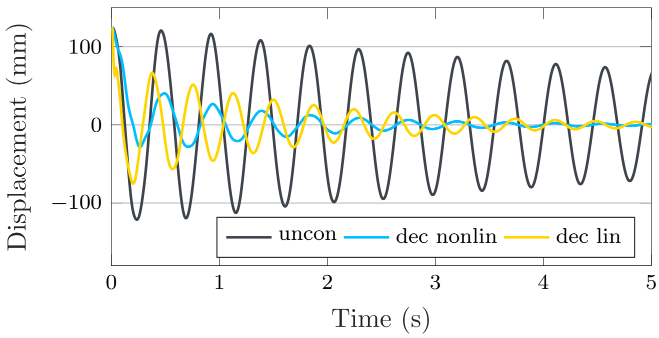

Fig. 4. Decentralized nonlinear control (light blue) applied to the full system, compared to the uncontrolled system (gray) and decentralized linear control (yellow).

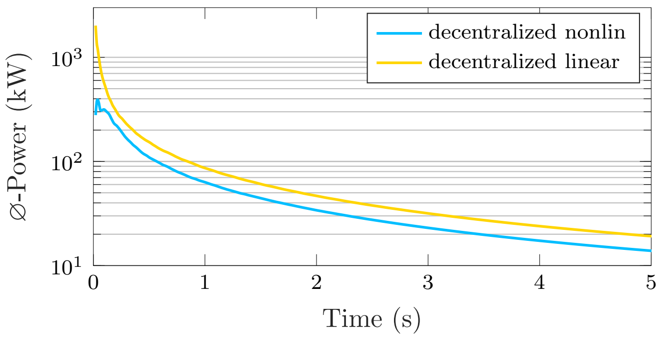

Fig. 5. Average power of the actuators on a logarithmic scale for decentralized nonlinear control (light blue) and of the decentralized linear control (yellow).

the nonlinear substructure onto desired (linear) target dynamics. The decentralized target dynamics can in general be formulated as

$$\dot{\boldsymbol{x}}^i(t) = \boldsymbol{f}_\mathrm{d}^i(\boldsymbol{x}^i(t)) + \boldsymbol{g}_\mathrm{d}^i(\boldsymbol{x}^i(t))\boldsymbol{w}^i(t),\ t>0,\ \boldsymbol{x}^i(0) = \boldsymbol{x}_0^i \quad (20)$$

and more detailed given in (23). The virtual input $\boldsymbol{w}^i(t) \in \mathbb{R}^{m^i}$ is the control input of the target system. The system is desired to be stable also for $\boldsymbol{w}^i(t) = \boldsymbol{0}$.

Linearizing Input Transformation The input signals of a substructure are calculated by setting the nonlinear system dynamics (17) equal to the target dynamics (20). Solving for input $\boldsymbol{u}^i(t)$ yields

$$\begin{aligned}\boldsymbol{u}^i(t) = {} & \boldsymbol{g}^{i+}(\boldsymbol{x}^i(t))\Big(\boldsymbol{f}_\mathrm{d}^i(\boldsymbol{x}^i(t)) - \boldsymbol{f}^i(\boldsymbol{x}^i(t))\Big) \\ & + \boldsymbol{g}^{i+}(\boldsymbol{x}^i(t))\,\boldsymbol{g}_\mathrm{d}^i(\boldsymbol{x}^i(t))\boldsymbol{w}^i(t),\end{aligned} \quad (21)$$

where $(\cdot)^+$ denotes the Moore-Penrose pseudo-inverse (acc. to Penrose (1955)). If the input function $\boldsymbol{g}^i$ is quadratic with full rank, in the case of an equal number of inputs and states, $\boldsymbol{g}^{i+}$ is the inverse. The (pseudo)-inverse exists due to all input directions being linearly independent for a reasonable choice of actuated elements. Plugging (21) into the original dynamics (17), the nonlinear structure is forced to follow the target dynamics. The block diagram in Fig. 3 visualizes the control scheme with the linearizing part from (21) in light blue and the feedback control, given in the next paragraph, in dark blue. The desired state is $\boldsymbol{x}_\mathrm{d}^i(t)$, which should not be confused with the desired dynamics. A feedback control can be designed for the target system to compensate modeling errors and inaccuracies using $\boldsymbol{w}^i(t)$. Linear control strategies can be applied. We design an LQR such that:

$$\boldsymbol{w}^i(t) = \boldsymbol{K}_{\mathrm{lqr,d}}^i(\boldsymbol{x}_{\mathrm{d}0}^i - \boldsymbol{x}_\mathrm{d}^i(t)), \quad (22)$$

with the desired state $\boldsymbol{x}_{\mathrm{d}0}^i$. The feedback matrix is determined according to (19). The weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are chosen as diagonal matrices of the form $k_{\mathrm{Q/R}}^i\boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix. For $\boldsymbol{R}$, this is reasonable, since each actuator input should be weighted the same. For $\boldsymbol{Q}$, each state is weighted the same, as damping is desired to be high, which is related to the velocities $\dot{\boldsymbol{q}}_1^i(t)$, while displacements $\boldsymbol{q}_1^i(t)$ are required to be small such that functionality and user comfort are not affected. Because the final LQR design is invariant to a scaling of $\boldsymbol{Q}$ and $\boldsymbol{R}$, there remains only one parameter to choose for each module – the quotient $k_{\mathrm{lqr}}^i = k_\mathrm{Q}^i/k_\mathrm{R}^i$. It has to be chosen according to the desired compromise between aggressiveness and robustness of the controller and can be different for each module.

Linear Target System For proof of concept as aimed in this contribution, we choose a target system by defining the functions in (20):

$$\boldsymbol{f}_\mathrm{d}^i(\boldsymbol{x}^i(t)) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{K}_\mathrm{d}^i & -\boldsymbol{D}_\mathrm{d}^i \end{bmatrix}\boldsymbol{x}(t), \quad \boldsymbol{g}_\mathrm{d}^i = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{F}_\mathrm{d}^i \end{bmatrix}. \quad (23)$$

This presents a linear mechanical system with desired linear stiffness $\boldsymbol{K}_\mathrm{d}^i$, damping $\boldsymbol{D}_\mathrm{d}^i$ and input matrix $\boldsymbol{F}_\mathrm{d}^i$. The choice of the target system is a design parameter in the control design, as are the parameters in the additional linear feedback control (22). By choosing the target system, the oscillation frequency and damping of the controlled system are implicitly influenced.

Application to Global System To apply the decentrally designed control inputs to the global system, the decentralized states $\boldsymbol{q}_\mathrm{l}^i(t)$ are related to the global state $\boldsymbol{q}(t)$ by means of the transformations matrices introduced before

$$\boldsymbol{q}_\mathrm{l}^i(t) = \boldsymbol{\Phi}_\mathrm{l}^{i+}\boldsymbol{T}^{i+}\boldsymbol{C}_\mathrm{q}^i\boldsymbol{q}(t). \quad (24)$$

The velocities are calculated the same and further combined to the state $\boldsymbol{x}^i(t)$. The decentralized inputs $\boldsymbol{u}^i(t)$ from n_S subsystems are summarized to the global input

$$\boldsymbol{u}(t) = \sum_{i=1}^{n_\mathrm{S}} \boldsymbol{C}_\mathrm{u}^i\boldsymbol{u}^i(t). \quad (25)$$

The need to evaluate the nonlinear functions $\boldsymbol{K}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t))$ and $\boldsymbol{D}_\mathrm{l}^i(\boldsymbol{q}_\mathrm{l}^i(t))$ during each time step of the control cycle leads to increasing resource demand for input calculation.

4. NUMERICAL RESULTS

4.1 System Description

The effectiveness of the decentralized nonlinear input transformation is shown by means of an exemplary adaptive high-rise structure, as depicted in Fig. 1. This adaptive building will be constructed on the site of the University of Stuttgart. The actuation is realized using hydraulic cylinders. In Fig. 2, a sketch of this structure and its division into four substructures is illustrated. Each substructure comprises three stories and one control unit. All diagonal bracings are tension-only elements and are depicted in orange. Vertical elements are modeled as beam elements and all remaining elements as common link elements. The necessary material and geometry parameters are given in

Tab. 1. The connection of the structure to the ground are hinged supports. The boundary nodes, depicted in red in Fig. 2 and defined in 2.4, cover all nodes, where the subsystems are connected to each other and elements with integrated actuators and tension-only elements are attached to. Actuators, realized as hydraulic cylinders, are placed in the columns of the first three substructures, for which controllers are designed. More details about the intergated actuator are given in Weider et al. (2019). States are assumed to be known, however, a decentralized observer could be used to estimate the states Warsewa et al. (2020). To eliminate the static modes from the control design model, the systems are linearized with $\boldsymbol{q}_{\mathrm{b}}^i(t) = \mathbf{0}$. The first system does not incorporate static modes because supports are available. Substructure two and three are statical indeterminate ($n_{\mathrm{c,rb}}^2 = n_{\mathrm{c,rb}}^3 = 10$). The desired target system is chosen as a linear mechanical system with $\boldsymbol{K}_{\mathrm{d}}^i = \boldsymbol{K}_{\mathrm{l}}^i(\mathbf{0})/100$ and the desired damping $\boldsymbol{D}_{\mathrm{d}}^i$ is selected as in (2) with constants $\alpha_{\mathrm{d},1} = 0.05$ and $\alpha_{\mathrm{d},2} = 0.01$. The mass matrices and the input matrices remain unchanged w.r.t (17). Thus, (16) for $i = \{1, 2, 3\}$ is evaluated for $\boldsymbol{q}_{\mathrm{l}}^i(t) = \mathbf{0}$. An LQR is designed for each substructure with actuators and applied to the simulation model. Good parameters can be identified in a closed loop simulation, we chose $k_{\mathrm{lqr,d}}^1 = 10^{13}, k_{\mathrm{lqr,d}}^2 = 10^{16}, k_{\mathrm{lqr,d}}^3 = 10^2$. Other choices might yield a similar performance. The feedback matrices for the linear decentralized feedback controller in (18) are determined via individual weighting in LQR design, leading to $k_{\mathrm{lqr,l}}^1 = 10^9, k_{\mathrm{lqr,l}}^2 = 10^{15} k_{\mathrm{lqr,l}}^3 = 10^9$. Results are depicted for the node at the top marked in blue in Fig. 2. For presentation of the decentralized method and to better compare it to a linear approach, disturbances are neglected and simulations are conducted with an initial condition by bending the structure to one side. For simulation, the reduced order model of the overall structures with tension-only elements from (9) is used. The simulation model's order is $n_{\mathrm{r}} = 18$, which describes the original model sufficiently (Wagner et al. (2019)).

4.2 Control Results

The uncontrolled nonlinear system almost exhibits a linear oscillation behavior, which was discussed in Wagner et al. (2019). The results of the decentralized nonlinear control applied to the nonlinear simulation system are depicted in Fig. 4 for the DOF in x-direction for the node at the top in comparison to the uncontrolled system. Oscillations are strongly damped within the controlled substructures as well as for the locally uncontrolled forth substructure. Through the physical coupling of the substructures, motions of uncontrolled parts directly affect other parts such that oscillations are mitigated within the whole structure. Due to the decentralized character of the control approach, the oscillation frequency of the undamped structure is not preserved in the decentralized nonlinear approach. This is similar to Wagner et al. (2019). The cumulated average actuator power totals 14.1 kW as can be seen in Fig. 5. A decentralized linear approach is applied and results are depicted in Fig. 4, which is outperformed by the decentralized nonlinear control. Cumulated average control power rises up to 19.1 kW despite less damping, which proofs the nonlinear approach more efficient. On top, Fig. 5 highlights for the decentralized nonlinear approach, that the peak of the average power needed at the beginning of control, is decreased significantly. Therefore, the size of installed hydraulic accumulators can be reduced, saving in installation cost and complexity. Moreover, higher weighting of the states in LQR design in the linear decentralized control, which usually increases damping, leads to instability of the linearly controlled system with tension-only elements.

Table 1. Geometry and Material Parameters.

Description	**Formula sign**	**Value**	**Unit**
Density	ρ	7850	kg/m^3
Young's modulus	E	210×10^9	N/m^2
Damping Coefficient	α_1	0.05	
Damping Coefficient	α_2	0.001	
Vertical columns, quadratic hollow profile			
Length	L_{v}	3	m
Width	w_{v}	0.3	m
Wall thickness	t_{v}	0.01	m
Poisson's ratio	ν	0.3	
Horizontal link elements, 2 rectangular hollow profiles			
Length	L_{h}	4.75	m
Width	w_{h}	0.2	m
Height	h_{h}	0.12	m
Wall thickness	t_{h}	0.008	m
Horizontal diagonal link elements			
Length	L_{hd}	6.72	m
Width	h_{hd}	0.01	m
Height	h_{hd}	0.06	m
Diagonal link elements, tension-only			
Length	L_{vd}	10.18	m
Width	w_{vd}	0.15	m
Height	h_{vd}	0.012	m

5. CONCLUSION

In this contribution, a method for decentralized control of civil engineering structures with tension-only elements was presented. A system was substructured while preserving nonlinear properties of tension-only element. By means of proper choice of the boundary nodes, a Craig-Bampton MOR can be performed preserving nonlinear system characteristics. For this, the concept of boundary nodes is extended to include not only the coupling nodes of substructures, but also nodes of actuated and of nonlinear, e. g. tension-only elements. Through this method, controller design models can be synthesized for decentralized control of large scale structures. A linearizing input transformation can be applied to each substructure, forcing its dynamics onto a desired target dynamics. Afterward, linear control theory can be used to control the linear target system, e. g. an LQR. This concept is exemplary applied to the linearized substructure with tension-only elements. It outperforms a decentralized approach using a linear control design model. The global damping is increased while energy consumption decreased. This highlights the importance of modeling nonlinear effects properly, e. g. tension-only elements.

In further research, a decentralized nonlinear observer needs to be included and results should be validated on the experimental setup. A more elaborate choice of the target system could further increase the control performance. Stability issues need to be investigated for the decentralized control scheme.

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