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Multi-state two-machine permutation flow shop scheduling optimisation with time-dependent energy costs

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Abstract: This paper studies a multi-state two-machine flowshop scheduling problem with variable electricity tariffs. The objective is to process a set of jobs in the available time slots with different energy costs to minimise the total energy consumption costs. For this purpose, two different 0-1 linear programs are proposed for the problem.
Computational experiments are presented and numerical results are discussed and analyzed in order to evaluate their efficiencies.

Keywords: Two-machine flow shop system; state dependent energy consumption; Time-Of-Use energy cost

1. INTRODUCTION

Nowadays, in such a competitive world, the need to manage a production system with less cost and pollution has attracted the attention of many researchers. Also, rising electricity prices in most industrial countries, has encouraged researchers to study the energy consumption minimization of manufacturing systems to decrease production costs and environmental pollution simultaneously. Based on the literature, this objective can be achieved at different levels: machine; product and system. Among these levels, system-level which is addressed in this study is the fastest and cheapest one. At system-level, manufacturers may reduce the energy consumption of their system by using existing decision models and optimization techniques for production planning and scheduling.
In a production system, the total energy consumption can be divided into two main categories based on the system's states. It consists of the amount of energy which consumed during the non-processing states, and the amount of energy consumed during the processing state. Noting that the non-processing states include set-up states, idle state, transition between different states, and off state. The other factors which may modify the energy consumption of the system are consist of the kind of the machines or the jobs, and the processing speed of the machine. Moreover, in the real world, to improve the reliability and efficiency of electrical power grids, electricity suppliers offer variable pricing to balance electricity supply and demand. These variations can also have an impact on the total energy consumption costs of any production system. Based on the literature review, the most common electrical pricings are Time-Of-Use (TOU), Critical Peak Pricing (CPP), and Real-Time Pricing (RTP).
According to the statements above, energy efficient scheduling problem for a production system can be introduced with different objective(s) such as minimizing energy consumption, or (and) minimizing the total energy cost. One of the most popular and efficient ways to minimize the total energy consumption costs of any system consists of managing the energy consumption of the non-processing states and using a scheduling method to modify the order of the processing jobs order and the state of the machine during a given period by passing from peak periods to off-peak ones.
In the following, the summary of some previous studies that have studied the problem of energy efficient scheduling in a production system are represented.

In 2016, Gahm et al. (2016) investigated scheduling approaches which aim to improve energy efficiency. They classified the literature based on three aspects and developed a framework for energy efficient scheduling. Fang et al. (2014) considered the scheduling problem of processing jobs with arbitrary power demands that must be processed at a single uniform speed or speed-scalable machine to minimize total electricity cost under a time of use electricity tariffs. They analyzed the complexity of these two problems in preemptive and non-preemptive cases.
Shrouf et al. (2014) proposed a mathematical model to minimize the energy consumption costs of processing the jobs, considering variable energy prices during one day and different possible states for the machine with different energy consumption. Aghelinejad et al. (2017) studied the same problem as Shrouf et al. (2014). They proposed two mathematical models. The first one considers a predetermined order for the processing jobs, and the second one finds the optimal schedule for the machine state and job's sequence simultaneously. Then, a new heuristic algorithm and a genetic algorithm are proposed to solve the general problem. The complexity of several other energy-oriented

single-machine scheduling problems with the objective of the total energy consumption costs minimization are addressed in Aghelinejad et al. (2019b). A new linear programming model is proposed by Aghelinejad et al. (2019a) for a multi-state two-machine flow-shop scheduling problem under Time-Of-Use electricity tariffs. Gong et al. (2016) proposed a novel production scheduling method for a problem with finite state machine, multiple processes idle modes and time varied electricity price to minimize the energy cost of the considered system. Chen and Zhang (2019) addressed a class of single machine scheduling problems under variable time-of-use tariffs to minimize the total cost of processing all the jobs subject to a minimum level of performance in one of the regular scheduling criteria.

Masmoudi et al. (2017a) and Masmoudi et al. (2017b) addressed a flow-shop system in a lot-sizing problem by considering energy constraints and different energy costs during the planning horizon. Pilerood et al. (2018) proposed a new continuous-time MILP model, as well as a two-stage greedy heuristic for a two-machine flow-shop scheduling problem under time-dependent electricity tariffs. In their cas, the energy consumption depends on the jobs and the machines. Fazli Khalaf and Wang (2018) addressed a two stages stochastic flow shop scheduling problem to minimize the total electricity purchase cost, such that, the energy demand is met by on-site renewable, energy storage, and the power grid. The volatile price, such as day ahead and real time pricing, is applied to the portion supplied by the power grid.
Liu et al. (2018) investigated a permutation flow shop scheduling problem to minimize the total idle energy consumption of the machines. For the cases with two-machines, they proved that the optimal schedule can be found by employing a relaxed Johnson's algorithm. For the cases with multiple machines (more than 2), they proposes a novel NEH heuristic algorithm to obtain an approximate energy-saving schedule. Zhang et al. (2019) established an energy consumption model of machine tools which involves the processing energy, standby energy and set-up energy for a flexible flow shop scheduling problem in the environment of TOU energy prices. They also applied an Improved Strength Pareto Evolutionary Algorithm to obtain the Pareto Front of the makespan and electricity cost. Wang et al. (2018) considered a two-machine permutation flow shop scheduling problem to minimise the total electricity cost of processing jobs under time-of-use electricity tariffs. They formulated the problem as a mixed integer linear programming, and proposed two heuristic algorithms based on Johnson's rule and dynamic programming method.

This study deals with the total energy consumption costs minimization of a two machine flow shop system by using the scheduling method. To the best of our knowledge, they are only few publications in the literature that address the energy efficiency of a multi-state two machine flow shop system with the time-dependent electricity cost. This paper aims to fill this gap within the literature.

The rest of this paper is organized as follows. In section 2, the problem definition and different assumptions are presented. In section 3, two new mathematical models are proposed. The efficiency of the proposed models are evaluated based on several randomly generated instances, and the numerical results are presented in section 4. Finally, section 5 draws the conclusions in addition to the future directions of this study.

2. PROBLEM DEFINITION

This research deals with a two-machine flowshop problem and Time-Of-Use (TOU) energy cost. A given set $J = (1, 2, ..., n)$ of jobs, that are available at time zero, must be proceed during a horizon of time which is devided to T time-slots or periods. These two considered machines have 5 possible states (OFF, ON, Idle, Ton and Toff). It is noteworthy that, the possible transitions between OFF and ON states for turning on and off the machines are named as Ton and Toff states, respectively. The energy consumption of machine i in state $s \in \{OFF,\ ON,\ Idle,\ Ton,\ Toff\}$ is denoted by $E_{i,s}$. The required number of periods to be in state s depends on the machine and it's state which is denoted by $d_{i,s}$. A permutation flow-shop system is considered which means that each job must be processed on machine 1 and then on machine 2, and the jobs must be processed in the same order on every machine. At most one job per period can be processed by each machine; and each job can be processed on one machine at the same time. Once the processing of a job has started, it must be finished without preemption. Let $p_{j,1}$ and $p_{j,2}$ be the processing times of job j on machines 1 and 2, respectively.
As can be seen in Figure 1, the initial and final state of each machine, during the initial and final periods, must be consider as OFF state. The energy consumption of machine i in OFF state is negligible and can be consider equal to 0 ($E_{i,1} = 0$). The transition for turning on the machine i, takes $d_{i,4}$ periods and consumes $E_{i,4}$ units of energy per period. After that, the machine is in ON state and can process a job $j = 1, ..., n$. This process takes $p_{j,i}$ periods and consumes $E_{i,2}$ units of energy per period. Once the selected job is processed completely, during the following periods there are three possibilities for the machine's state: the machine may stay in ON state and process the next job; it can go into the Idle state for one or more periods, the machine can also go into the OFF state. The selection of any of these possibilities depends on the total energy consumption and the total energy cost during the selected periods. It is noteworthy that, in this study, the transition time and energy consumption between Idle and ON states are neglected, and the transition between Idle and OFF states is not allowed. Consequently, when the machine is in the Idle state, for the following period, it can remain in the Idle state or change to ON state.
The goal of this work is to find the most economical production schedule for the given time horizon in terms of energy consumption costs.

3. MATHEMATICAL MODELS

This section starts with the definition of the common parameters and variables of the two presented models.

T : Total number of periods
C_t: Cost of energy in period t

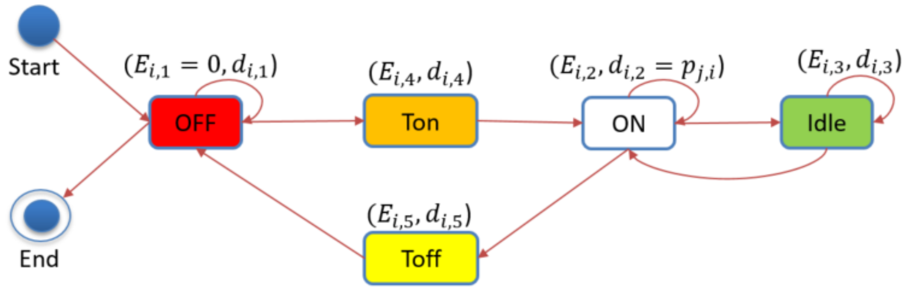

Fig. 1. The possible states and transitions for machine $i \in \{1, 2\}$ (Aghelinejad et al. (2019a)).

n : Number of jobs
$p_{j,i}$: Processing time of job j on machine i (in number of periods)
$E_{i,s}$: Amount of energy that machine i consumes during state s

Decision variables:

$$\alpha_{i,s,t} = \begin{cases} 1 \text{ if machine } i \text{ is in state } s \text{ during period } t \\ 0 \text{ otherwise} \end{cases}$$

Model 1:
The related parameters and variables of the first model are:

s : States of the machine
$d_{i,s}$: Number of periods that must elapse when machine i is in state s

In order to simplify the model, the five possible states of the machine are considered as integer numbers (s=1 for OFF, s=2 for ON, s=3 for Idle, s=4 for Ton and s=5 for Toff).

Decision variable:

$$y_{j,i,t} = \begin{cases} 1 \text{ if job } j \text{ processes on machine } i \text{ during period } t \\ 0 \text{ otherwise} \end{cases}$$

$$Min \sum_{t=0}^{T} C_t \left(\sum_{i=1}^{2} \sum_{s=1}^{5} E_{i,s} \cdot \alpha_{i,s,t} \right) \tag{1}$$

$$\alpha_{i,1,t} = 1 \qquad ; \forall i = 1, 2; \quad t \in \{0, T\} \tag{2}$$

$$\sum_{s=1}^{5} \alpha_{i,s,t} = 1 \qquad ; \forall i = 1, 2; \quad \forall t \in [0, T] \tag{3}$$

$$\alpha_{i,1,t} \leq \alpha_{i,1,t+1} + \alpha_{i,4,t+1} \qquad ; \forall i = 1, 2; \quad \forall t \in [0, T-1] \tag{4}$$

$$\alpha_{i,2,t} \leq \alpha_{i,2,t+1} + \alpha_{i,3,t+1} + \alpha_{i,5,t+1} \quad ; \forall i = 1, 2; \quad \forall t \in [0, T-1] \tag{5}$$

$$\alpha_{i,3,t} \leq \alpha_{i,2,t+1} + \alpha_{i,3,t+1} \qquad ; \forall i = 1, 2; \quad \forall t \in [0, T-1] \tag{6}$$

$$\alpha_{i,4,t} \leq \alpha_{i,4,t+1} + \alpha_{i,2,t+1} \qquad ; \forall i = 1, 2; \quad \forall t \in [0, T-1] \tag{7}$$

$$\alpha_{i,5,t} \leq \alpha_{i,5,t+1} + \alpha_{i,1,t+1} \qquad ; \forall i = 1, 2; \quad \forall t \in [0, T-1] \tag{8}$$

$$\sum_{t'=t+1}^{t+d_{i,4}} \alpha_{i,4,t'} \geq (\alpha_{i,4,t+1} + \alpha_{i,1,t} - 1) \cdot d_{i,4} \quad \forall i = 1, 2; \quad \forall t \in [0, T - d_{i,4}] \tag{9}$$

$$\sum_{t'=t+1}^{t+d_{i,5}} \alpha_{i,5,t'} \geq (\alpha_{i,5,t+1} + \alpha_{i,2,t} - 1) \cdot d_{i,5} \quad \forall i = 1, 2; \quad \forall t \in [0, T - d_{i,5}] \tag{10}$$

$$\alpha_{i,s,t} + \alpha_{i,s,t+d_{i,s}} \leq 1 \quad ; \forall i = 1, 2; \quad \forall t \in [0, T - d_{i,s}], \quad \forall s \in [4, 5]; \tag{11}$$

$$\sum_{j=1}^{n} y_{j,i,t} = \alpha_{i,2,t} \qquad ; \forall i = 1, 2; \quad \forall t \in [0, T] \tag{12}$$

$$\sum_{j=1}^{n} y_{j,i,t} \leq 1 \qquad ; \forall i = 1, 2; \quad \forall t \in [0, T] \tag{13}$$

$$p_{j,1} \cdot y_{j',2,t} \leq \sum_{t'=0}^{t-1} y_{j,1,t'} \qquad ; \forall t \in [0, T], \quad \forall j \in [1, n] \tag{14}$$

$$\sum_{t'=0}^{t-p_{j,i}} y_{j,i,t'} + \sum_{t'=t+p_{j,i}}^{T} y_{j,i,t'} \leq p_{j,i} \cdot (1 - y_{j,i,t}) \quad ; \forall i = 1, 2; \quad \forall t \in [p_j, T - p_j - 1], \quad \forall j \in [1, n] \tag{15}$$

$$\sum_{t=1}^{T} y_{j,i,t} \geq p_{j,i} \qquad ; \forall i = 1, 2; \quad \forall j \in [1, n] \tag{16}$$

$$\alpha_{i,s,t}, \; y_{j,i,t} \in \{0, 1\} \tag{17}$$

In this model, equation (1) represents the objective function that minimize the total energy consumption cost of the system. As it is mentioned before, the objective depends on the unit of electricity price in each period, as well as, the energy consumption of each machine (the machine's state) in each period. Equation (2) ensures that the machines are in OFF state during the initial and final periods. Equation (3) restricts that the machines must be in one of the possible states (ON, OFF, Idle, Ton, and Toff) in each period. Equations (4) to (8) limit the state of the machines in each period regarding to their states in the previous period. Equations (9), (10) and (11) identify lower and upper number of required periods for Ton and Toff states. Equation (12) and (13) state that each machine may process at most one job at a time and only, when the machine is in ON state ($s = 1$). Equation (14) expresses that the jobs must be processed, with the same sequence, at first on machine 1 and then on machine 2. Equation (15) represents that the jobs must be processed non-preemptively. Equation (16) defines the processing time for each job on each machine.

Model 2:
The related parameters and variables for the second model are:

$E_{i,s,s'}$: Amount of energy that machine i consumes in transiting from state s to s'.
$d_{s,s'}$: required period for the transition from state s to s' ($s \neq s'$).

The states of the machine are considered as integer numbers (s=1 for ON, s=2 for OFF, s=3 for Idle).

Decision variables:

$$\beta_{i,s,s',t} = \begin{cases} 1 \text{ if machine } i \text{ in transition from } s \text{ to } s' \text{ at } t \\ 0 \text{ otherwise} \end{cases}$$

$$X_{i,j,t} = \begin{cases} 1 \text{ if machine } i \text{ ends the job } j \text{ at period } t \\ 0 \text{ otherwise} \end{cases}$$

$$Z_{i,j,t} = \begin{cases} 1 \text{ if the job } j \text{ is executed on machine } i \text{ at period } t \\ 0 \text{ otherwise} \end{cases}$$

$$\sigma_{j,k} = \begin{cases} 1 \text{ if the job } j \text{ preceed the job } k \\ 0 \text{ otherwise} \end{cases}$$

$$Min \sum_{i=1}^{2} \sum_{t=0}^{T} C_t \cdot \left(\sum_{s=1}^{3} E_{i,s} \cdot \alpha_{i,s,t} + \sum_{s'=1}^{3} E_{i,s,s'} \cdot \beta_{i,s,s',t} \right) \quad (18)$$

$$\alpha_{i,1,t} = 1 \quad \forall i = 1,2; t \in \{0, T\} \quad (19)$$

$$\sum_{s=1}^{3} \alpha_{i,s,t} + \sum_{s=1}^{3} \sum_{s'=1}^{3} \beta_{i,s,s',t} = 1 \quad \forall i = 1,2; t \in [0, T] \quad (20)$$

$$\alpha_{i,s,t} \leq \sum_{s'=1:d_{ss'}=0}^{3} \alpha_{i,s',t+1} + \sum_{s''=1:d_{ss''}>0}^{3} \beta_{i,s,s'',t+1} \quad (21)$$
$$\forall i = 1,2; t \in [0, T-1]$$

$$\beta_{i,s,s',t} \leq \beta_{i,s,s',t+1} + \alpha_{i,s',t+1} \quad (22)$$
$$\forall i = 1,2; t \in [0, T-1]; s, s' \in [1,3] : d_{ss'} \geq 1$$

$$\beta_{i,s,s',t} + \beta_{i,s,s',t+d_{ss'}} \leq 1 \quad (23)$$
$$\forall i = 1,2; t \in [0, T-1]; s, s' \in [1,3] : d_{ss'} \geq 1$$

$$\sum_{t'=t+1}^{t+d_{ss'}} \beta_{i,s,s',t'} \geq (\alpha_{i,s,t} + \beta_{i,s,s',t+1} - 1) \cdot d_{ss'} \quad (24)$$
$$\forall i = 1,2; t \in [0, T - d_{ss'}]; s, s' \in [1,3] : d_{ss'} \geq 1$$

$$\sum_{j=1}^{n} Z_{i,j,t} = \alpha_{i,2,t} \quad \forall i = 1,2; t \in [0, T] \quad (25)$$

$$\sum_{j=0}^{n} Z_{i,j,t} \leq 1 \quad \forall i = 1,2; t \in [0, T] \quad (26)$$

$$\sum_{t=0}^{T} t \cdot X_{1,j,t} \leq \sum_{t=0}^{T} ((t - p_{j,2}) \cdot X_{2,j,t}) \qquad \forall j \in [1, n] \quad (27)$$

$$\sum_{t=0}^{T} t \cdot X_{i,j,t} \leq \sum_{t=0}^{T} ((t - p_{k,i}) \cdot X_{i,k,t}) + T \cdot (1 - \sigma_{j,k}) \quad (28)$$
$$\forall i = 1,2; j, k \in [1, n] : k \neq j$$

$$\sum_{t=0}^{T} t \cdot X_{i,k,t} \leq \sum_{t=0}^{T} ((t - p_{j,i}) \cdot X_{i,j,t}) + T \cdot \sigma_{j,k} \quad (29)$$
$$\forall i = 1,2; j, k \in [1, n] : k \neq j$$

$$\sigma_{j,k} + \sigma_{k,j} = 1 \quad \forall j, k \in [1, n] : k \neq j \quad (30)$$

$$\sigma_{j,j} = 0 \quad \forall j \in [1, n] \quad (31)$$

$$\sum_{t'=0:t \leq t' < t+p_{j,i}}^{T} X_{i,j,t'} \leq Z_{i,j,t} \quad \forall i = 1,2; t \in [0, T], j \in [1, n] \quad (32)$$

$$\sum_{t=0}^{T} t \cdot X_{i,j,t} \leq T \quad \forall i = 1,2; \quad j \in [1, n] \quad (33)$$

$$\sum_{t=0:t \geq p_{j,i}}^{T} X_{i,j,t} = 1 \quad \forall i = 1,2; j \in [1, n] \quad (34)$$

$$\sum_{t=0:t<p_{j,i}}^{T} X_{i,j,t} = 0 \quad \forall i = 1,2; j \in [1, n] \quad (35)$$

$$\alpha_{i,s,t}, \beta_{i,s,s',t}, Z_{i,j,t}, X_{i,j,t}, \sigma_{j,k} \in \{0, 1\} \quad (36)$$

The same objective function is presented through the equation (18). It must be noted that, in this model the trasition states are expressed by another variables, while in the first model, these states are considered by the same variables. The constraint (19) is equivalent to the equation (2) of the first model and the (20) is equivalent to the (3) one. The constraints (21) and (22) traduce the same aspect of those from (4) to (8). The constraints (23) and (24) are equivalent to those (9), (10), (11), the (25) and (26) are equivalent to (12) and (13), the equations (27) to (31) are equivalent to (14), the equation (32) is equivalent to (15). The remaining constraints define the different variables.

4. NUMERICAL EXPERIMENTS

To compare the performances of the two models established in this study, they are solved using the CPLEX Studio 12.9.0 solver and with a time limitation of $3600s$. The performance evaluation of the proposed models is made on the basis of more than 120 randomly generated instances, and several criteria like objective value, the number of variables and constraints as well as computing time.

For each instance size (n, T), which corresponds to the number of processing jobs (n) and the total number of existing periods (T), ten different instances are randomly generated by changing the processing time of the jobs and the price of energy in the operating horizon. For this purpose, the processing time of the jobs are generated as an integer random number between 1 and 5 periods, and the unit of energy cost per period is generated randomly between 1 and 10. Moreover, to assess how the two parameters n and T separately affect computation time, the instances with size $(n \pm 1, T)$ and $(n, T \pm 5)$ are tested to evaluate the affect of the parameter n and T on the computation time, respectively.

As it is mentioned previously, the number of variables and constraints are considered as the main criteria in this study to compare the performance of the proposed models. The number of variables are computed through the equation (37) for the first model and (40) for the second one.

$$NbVar = N1 + N2 \quad (37)$$

where :

$$N1 = 2 \cdot S \cdot (T + 1) \quad (38)$$
$$N2 = 2 \cdot n \cdot (T + 1) \quad (39)$$

$$NbVar = N3 + N4 + N5 + N6 + N7 \quad (40)$$

where :

$$N3 = 2 \cdot S \cdot (T + 1) \quad (41)$$
$$N4 = S \cdot S \cdot 2 \cdot (T + 1) \quad (42)$$
$$N5 = n \cdot 2 \cdot (T + 1) \quad (43)$$
$$N6 = n \cdot 2 \cdot (T + 1) \quad (44)$$

$$N7 = n \cdot n \tag{45}$$

Knowing that S is equal to 5 for the first model and 3 for the second one. A fictive period is added ($t = 0$) in order to respect the assumption which translate that the initial state of each machine is oFF, so, the total number of periods is equal to $T + 1$. $N1$ represents the variable related to α, $N2$ represents the variable related to y, $N3$ represents the variable related to α, $N4$ related to β, $N5$ related to X, $N6$ related to Z and $N7$ related to σ.

In this paper, the numerical experiment results on the instances with a size from $(4, 30)$ to $(20, 90)$ are presented. It is noteworthy that, the obtained results demonstrated that, CPLEX solver is not able to find the optimal solution for the instances with a size greater than 20 jobs and 90 periods during one hour time limitation.

As a first comparison, based on the obtained results we can conclude that, the first model has less number of variables and constraints than the second model. In average, the first model has 2385 variables and 4005 constraints for the tested instances against 5189 and 4180 for the second model. So, the number of variables for the second model is equal to almost double of the first model.

In this paper, the results are presented in two tables. Table 2 shows the instances where the optimal solutions were found by the both proposed models, and the second table (Table 3) presents the results when at least one model does not found the optimal solution. Therefore, Table 2 compares the models based on their computation time, while Table 3 compares them based on the percentage of the deviation between the obtained solution and the lower bound. The deviation are determined through the equation 46:

$$Dev(\%) = 100 \cdot \frac{(Obtained\ solution - Lower\ bound)}{Obtained\ solution} \tag{46}$$

Based on the presented results, as a conclusion, the first model has less number of variables and constraints than the second model, meanwhile the second one outperforms the first one in terms of computational times ($501.69s$ for Model 1 VS $159.28s$ for Model 2) and the percentage of the deviation (2.32 %VS 1.43%).

5. CONCLUSIONS AND FUTURE DIRECTIONS

In this study, a two machine flow-shop scheduling problem is considered to minimize the total energy consumption costs of the system, when the machines have multi-states and the energy costs are varying within the horizon time. The contribution of this paper consists of proposing two mathematical models for the problem. The performance of these models are evaluated based on several numerical instances. The obtained results show that, for the problems as large as 20 jobs and 90 periods, CPLEX is not able to find the optimal solution during the 3600 seconds time limitation.

Since the considered problem is known to be NP-hard, for future works, it could be interesting to propose some heuristic and meta-heuristic algorithms which are able to solve the medium and large size instances of this problem in a reasonable time. Moreover, considering more assumptions for this problem like set-up time before each job, and the energy constraint at each period can be proposed as future research directions of this study.

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Table 1. Comparison of the two models for the instances with optimal solutions (for the both models)

(n, T)		CPU_1 (s)	CPU_2 (s)	(n, T)		CPU_1 (s)	CPU_2 (s)
	Min	0.36	0.8		Min	88.53	11.9
$(4, 30)$	Average	1.00	2.34	$(10, 55)$	Average	253.85	20.03
	Max	2.16	3.97		Max	593.3	33.33
	Min	0.08	0.15		Min	88.19	16.51
$(5, 25)$	Average	0.39	0.49	$(11, 50)$	Average	218.19	37.86
	Max	0.63	0.92		Max	538.02	72.42
	Min	0.55	0.52		Min	1630.21	133.95
$(5, 30)$	Average	2.37	1.72	$(14, 70)$	Average	2329.10	202.19
	Max	8.02	3.63		Max	3534.09	313.09
	Min	0.69	0.58		Min	765.53	232.75
$(5, 35)$	Average	2.12	2.87	$(15, 65)$	Average	1444.51	903.17
	Max	3.63	9.03		Max	1937.78	2203.69
	Min	0.42	0.37		Min	718.8	36.82
$(6, 30)$	Average	6.11	3.01	$(15, 70)$	Average	2063.20	850.66
	Max	14.55	6.35		Max	3393.14	3161
	Min	27.36	10.23		Min	759.97	22.13
$(9, 50)$	Average	90.11	20.35	$(15, 75)$	Average	1264.47	564.53
	Max	183.01	32.39		Max	1610.26	1393
	Min	7.89	1.17		Min	1284.91	89.28
$(10, 45)$	Average	77.27	9.48	$(16, 70)$	Average	1356.50	239.32
	Max	258	13.35		Max	1428.08	389.36
	Min	16.27	9.74				
$(10, 50)$	Average	155.21	14.86	$(20, 90)_1$		1334.34	397
	Max	565.31	21.34				
				Average		**501.69**	**159.28**

Table 2. Comparison of the two models were feasible solutions are obtained

(n, T)	$Obj\ value_1$	$Obj\ value_2$	Dev_1 (%)	Dev_2 (%)	CPU_1 (s)	CPU_2 (s)
$(15, 70)_1$	2495	**2485**	1.02	0.54	3600	3600
$(15, 70)_6$	2556	**2522**	1.94	0.00	3600	865.34
$(15, 70)_7$	2275	**2235**	1.60	0.00	3600	496.52
$(15, 70)_{10}$	2101	**2057**	2.28	0.00	3600	426.95
$(20, 90)_2$	**2913**	3010	0.62	4.09	3600	3600
$(20, 90)_3$	3213	**3135**	2.77	0.00	3600	2697
$(20, 90)_4$	2811	**2623**	8.44	0.96	3600	3600
$(20, 90)_5$	**2456**	2574	0.76	5.46	3600	3600
$(20, 90)_6$	3091	**3060**	2.35	1.34	3600	3600
$(20, 90)_7$	2645	**2557**	4.05	0.00	3600	3187.42
$(20, 90)_8$	**3273**	3279	1.83	2.01	3600	3600
$(20, 90)_9$	2537	**2519**	4.97	3.60	3600	3600
$(20, 90)_{10}$	**2561**	2722	3.87	8.92	3600	3600
Average			**2.32**	**1.43**	**3600**	**2684.65**

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