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A fully distributed control scheme for power balancing in distribution networks

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Abstract: The progressive diffusion of generation units based on intermittent renewable energy sources, as well as the increasing volatile power demand, calls for a new framework to compensate the power variability in a local fashion. In this context, the European Union instituted the figure of the Balance Responsible Party, i.e. an entity entitled of internally compensating the power fluctuations, exploiting a portfolio of local dispatchable units. Considering a distribution network carrying balance responsibility, this work devises a scalable, fully distributed, multi-layer control strategy for internal power balancing. The proposed scheme features multiple local MPC regulators, performing an autonomous power balancing; a supervisory layer based on *Distributed Consensus ADMM* is introduced to coordinate local regulators when some of them exhausts its local resources. Numerical results eventually show the effectiveness of the approach.

Keywords: Electric power systems, Supervisory control, Predictive control, Networked systems, Smart power applications

1. INTRODUCTION

The wide diffusion of Renewable Energy Sources (RESs) is currently regarded as one of the key solutions to reduce carbon dioxide emissions. In particular, a capillary integration of Energy Resources (ERs) based on RESs in the distribution stage of the power system is advisable to foster local energy consumption, and to mitigate the intermittency of renewable sources. Nonetheless, the spread of solar and wind energy sources, as well as the continuous increase of non-deterministic power demand, e.g. due to electric vehicles' charging stations, have the effect of increasing variability of the power system, undermining the security of supply and the stability of the network itself. To cope with these challenges, a more flexible management of the power system is required, in which generation and demand patterns can be actively and continuously adapted. This need of controllability can be nowadays achieved thanks to Dispatchable distributed Energy Resourcess (DERs), like Battery Energy Storage Systems (BESSs) and micro-Generators (mGENs), able to provide part of their power capabilities to compensate the variability of external loads and RESs, offering the so-called ancillary services to the system operator. In this context, the European Commission identified the necessity of a new market player in the power system operation, named *Balance Responsible Party* (BRP), see Van der Veen and Hakvoort (2016). This actor works as an intermediary between the system operator and a portfolio of grid units, carrying financial responsibility over the imbalances caused by its assets. Each BRP submits to the system operator a power program based on the outcomes of the day-ahead and intra-day energy markets, relying on forecasted power profiles for loads and RESs. As these forecasts can be significantly wrong from reality, causing imbalances in the power system, Hirth and Ziegenhagen (2015), the BRP is entitled to internally address these power deviations in order to avoid penalties. Therefore, this work focuses on the design of a coordination control strategy for multiple DERs connected to a distribution network carrying balance responsibility, to compensate imbalances caused by the non-dispatchable elements therein. The control strategy is required to act promptly, to avoid the propagation of power variability to the main utility, and to be scalable, so that its performances do not depend on the scale of the considered distribution network. The design of a low-level controller using DERs to compensate the fluctuations of an external load is presented in Hong and de León (2017). Centralized Model Predictive Control (MPC) methods have been adopted for the compensation of power imbalances in Hug-Glanzmann (2010); Cominesi et al. (2017), but, in light of the low scalability of centralized approaches, these solutions are applicable only to small scale systems. A well-known method to overcome scalability issues in large-scale networks consists in decomposing the grid in multiple areas and in properly regulating their operations and power exchanges, as discussed in La Bella et al. (2019). Distributed optimization frameworks for the coordination of different grid areas have been proposed

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in the literature as in Hug-Glanzmann and Andersson (2009), for the power flow optimization, and in Baker et al. (2016), using a distributed MPC scheme for the efficient management of BESSs and mGENs. Nonetheless, distributed approaches are characterized by iterative and communicationally-intensive procedures, especially when many variables must be optimized, and therefore they are not considered as the best solution to quickly compensate power imbalances. Because of this, an alternative and novel control architecture is proposed in this paper for the coordination of networks partitioned in areas. To this regard, many methods are discussed in the literature for network partitioning, based on topological properties, as described in Hug-Glanzmann and Andersson (2007). Considering the goal of coordinating different DERs to balance unexpected power variations of non-dispatchable elements, a novel supervised MPC architecture is here proposed. To ensure a prompt control action, not depending on the size of the considered network and on the number of controlled DERs, a two-layer control scheme is designed for a distribution network properly decomposed in several non-overlapping areas. At the lower level, each area is controlled by a decentralized and autonomous MPC regulator, designed to locally compensate the power variability exploiting the DERs therein included. It is assumed that each dispatchable DER unit has already scheduled its power profile based on the day-ahead or intra-day markets, and that part of their power reserves can be offered to compensate power imbalances. The local MPC regulator must act such that, from an external perspective, the active power exchanged by each area with the rest of the network exactly matches the power program. Moreover, local MPC regulators are designed such that they do not need to measure the instantaneous output power of each non-dispatchable element, but just the exchanged power among areas. A supervisory layer is then introduced to ensure that each area is always able to operate autonomously, having enough power reserves to balance the local power variability. This supervisory layer is activated just in case an area needs an external power support from the others. Indeed, in case an area does not have enough resources to address the power variability, the corresponding MPC regulator is designed to send an emergency signal to the supervisory layer, which in turn dispatches the optimal power exchanges between the areas so that the overall power imbalances are compensated. To enhance the scalability of the control architecture, the supervisory layer is implemented as a fully distributed optimization problem, based on the distributed Dual Consensus ADMM (DC-ADMM) algorithm, presented in Chang et al. (2014). The implemented method relies on a properly defined communication network graph, where each agent directly interacts with its neighbors, without needing a total communication among agents or a coordination entity. Each agent of the supervisory layer is implemented on top of each local MPC regulator. A schematic of the proposed control architecture is depicted in Figure 1. The proposed event-triggered hierarchical control architecture has several advantages with respect to traditional approaches, allowing to achieve both a prompt control action and scalability properties. The supervisory layer is formulated to solve a simple optimization problem with reduced area models, which allows, when activated, to find the optimal power exchanges among the areas with only a few iterations. On the other hand, local MPC regulators are autonomously executed at each sampling time, so as to quickly counteract the power variability in each grid area.

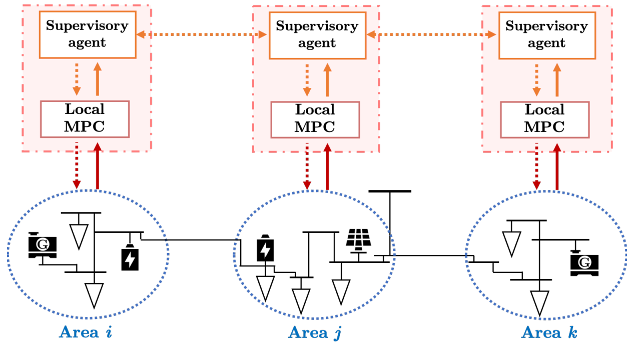


Fig. 1. Schematic of the proposed control architecture.

Symbol	Description
p^g, p^b	mGEN and BESS output power [kW]
e^b	BESS stored energy [kWh]
$r_p^{g\uparrow}$, $r_p^{g\downarrow}$	mGEN upward/downward power reserves [kW]
$r_p^{b\uparrow}$, $r_p^{b\downarrow}$	BESS upward/downward power reserves [kW]
$r_e^{b\uparrow}$, $r_e^{b\downarrow}$	BESS upward/downward energy reserves [kWh]
p^l	Non-dispatchable element output power [kW]
p^a	Area net output power [kW]
d^a	Aggregated disturbance in area [kW]
$r_p^{a\uparrow}$, $r_p^{a\downarrow}$	Total area upward/downward power reserves [kW]
p^p	Power exchanges between areas [kW]
Δp^{req}	Requested power variation by MPC regulator [kW]
Δp^{sup}	Committed power variation by supervisory layer [kW]

Table 1. Main variables and parameters

2. MODEL DESCRIPTION

The main variables and parameters used in the following are reported in Table 1. Fixed quantities, such as pre-scheduled or forecasted power profiles, are marked with an upper hat, e.g. $\hat{p}_i^g$. The deviations of variables with respect to the programmed values are denoted by Δ, e.g. $\Delta p_i^g = p_i^g - \hat{p}_i^g$. Moreover, all the power values are positive if delivered and negative if absorbed, while maximum and minimum limits of each variable are denoted by a bar over or below the variable, respectively. The optimal value of each variable is denoted with the superscript $*$, e.g. Δp_i^{g*}.

The designed control scheme is supposed to be applied to a distribution network, with nodes being collected in $\mathcal{N}$. The network is equipped with several mGENs, BESSs and non-dispatchable elements, whose nodes are collected in the sets $\mathcal{N}_G$, $\mathcal{N}_B$, $\mathcal{N}_L \subseteq \mathcal{N}$, respectively.

The distribution grid is supposed to be pre-partitioned into M distinct and connected areas according to some criteria, meaning that $\mathcal{N} = \mathcal{N}_{a1} \cup ... \cup \mathcal{N}_{aM}$. To identify the nodes of the generic area k where local units are connected, the subsets $\mathcal{N}_{Gk} = \mathcal{N}_G \cap \mathcal{N}_{ak}$, $\mathcal{N}_{Bk} = \mathcal{N}_B \cap \mathcal{N}_{ak}$, and $\mathcal{N}_{Lk} = \mathcal{N}_L \cap \mathcal{N}_{ak}$ are introduced. As a partition of the overall distribution network, each area k is in general connected to the rest of the grid through multiple interconnection points, which are collected in the set $\mathcal{I}_k^p = \{1, ..., n_k^p\}$.

The proposed control structure is supposed to run with sampling time $\tau_s = 90\,$s, both to ensure a prompt balance action and to have sufficient computational time to solve the optimization problems. Denoting by t the generic time index and assuming the proposed algorithm to operate for one day, one has $t = \{1, ..., T\}$, where $T = 24\,\mathrm{h}/\tau_s = 960$. As mentioned, DERs are supposed to have already scheduled their nominal power profiles through the energy market procedures, considering the production costs and the energy prices, e.g. as described in La Bella et al. (2019). However, the remaining upward and downward active power reserves are exploited by the local MPC regulators for balancing the power variability in the correponding area. These are defined as the remaining power margins with respect to the capability limits, as follows

$$\hat{r}^{g\uparrow}_{p,i}(t) = \bar{p}^{g}_{i} - \hat{p}^{g}_{i}(t)\,, \qquad \hat{r}^{g\downarrow}_{p,i}(t) = \hat{p}^{g}_{i}(t) - \underline{p}^{g}_{i}\,, \tag{1}$$

$$\hat{r}^{b\uparrow}_{p,j}(t) = \bar{p}^{b}_{j} - \hat{p}^{b}_{j}(t)\,, \qquad \hat{r}^{b\downarrow}_{p,j}(t) = \hat{p}^{b}_{j}(t) - \underline{p}^{b}_{j}\,, \tag{2}$$

defined $\forall i \in \mathcal{N}_G$ and $\forall j \in \mathcal{N}_B$. The power variations that can be requested by the local MPC regulators to mGENs and BESS, denoted as $\Delta p^g_i(t)$ and $\Delta p^b_i(t)$, must respect the following constraints

$$-\,\hat{r}^{g\downarrow}_{p,i}(t) \;\le\; \Delta p^{g}_{i}(t) \;\le\; \hat{r}^{g\uparrow}_{p,i}(t)\,, \tag{3}$$

$$-\,\hat{r}^{b\downarrow}_{p,i}(t) \;\le\; \Delta p^{b}_{i}(t) \;\le\; \hat{r}^{b\uparrow}_{p,i}(t)\,. \tag{4}$$

Concerning BESSs, it should be considered that the power variation $\Delta p^b_i(t)$ has an impact on the stored energy, and the corresponding constraints must be respected. The dynamics of BESS energy deviation with respect to the pre-scheduled profile is modeled as a pure integrator:

$$\Delta e^{b}_{j}(t+1) \;=\; \Delta e^{b}_{j}(t) \;-\; \tau_s\, \Delta p^{b}_{i}(t)\,. \tag{5}$$

Therefore, defining the available energy reserves available at each time instant as

$$\hat{r}^{b\uparrow}_{e,j}(t) = \bar{e}^{\,b}_{j} - \hat{e}^{\,b}_{j}(t)\,, \qquad \hat{r}^{b\downarrow}_{e,j}(t) = \hat{e}^{\,b}_{j}(t) - \underline{e}^{\,b}_{j}\,, \tag{6}$$

it must be imposed that $\Delta e^b_j(t)$ always respects the following bounds

$$-\,\hat{r}^{b\downarrow}_{e,j}(t) \;\le\; \Delta e^{b}_{j}(t) \;\le\; \hat{r}^{b\uparrow}_{e,j}(t)\,. \tag{7}$$

Based on the power deviations committed by the local MPC regulators, the effective reserves of DERs are varied with respect to the pre-scheduled values as follows

$$\begin{aligned} r^{g\uparrow}_{p,i}(t) &= \hat{r}^{g\uparrow}_{p,i}(t) - \Delta p^{g}_{i}(t)\,, & r^{g\downarrow}_{p,i}(t) &= \hat{r}^{g\downarrow}_{p,i}(t) + \Delta p^{g}_{i}(t)\,,\\ r^{b\uparrow}_{p,j}(t) &= \hat{r}^{b\uparrow}_{p,j}(t) - \Delta p^{b}_{j}(t)\,, & r^{b\downarrow}_{p,j}(t) &= \hat{r}^{b\downarrow}_{p,j}(t) + \Delta p^{b}_{j}(t)\,,\\ r^{b\uparrow}_{e,j}(t) &= \hat{r}^{b\uparrow}_{e,j}(t) - \Delta e^{b}_{j}(t)\,, & r^{b\downarrow}_{e,j}(t) &= \hat{r}^{b\downarrow}_{e,j}(t) + \Delta e^{b}_{j}(t)\,. \end{aligned} \tag{8}$$

The output power of the non-dispatchable elements, i.e. RESs and loads, is modelled as the sum of the forecasted profile and the unexpected power variation, as

$$p^{l}_{m}(t) = \hat{p}^{l}_{m}(t) + \Delta p^{l}_{m}(t)\,, \tag{9}$$

defined $\forall m \in \mathcal{N}_L$.

Since each grid area is equipped with a sub-set of DERs and non-dispatchable units, the expected net power profile of the generic grid area k can be defined as

$$\hat{p}^{a}_{k}(t) = \sum_{i\in\mathcal{N}_{Gk}} \hat{p}^{\,g}_{i}(t) \;+\; \sum_{j\in\mathcal{N}_{Bk}} \hat{p}^{\,b}_{j}(t) \;+\; \sum_{m\in\mathcal{N}_{Lk}} \hat{p}^{\,l}_{m}(t)\,, \tag{10}$$

while the unknown power variability in area k is aggregated in the following variable

$$d^{a}_{k}(t) \;=\; \sum_{m\in\mathcal{N}_{Lk}} \Delta p^{l}_{m}(t)\,. \tag{11}$$

Therefore, the actual power profile of each area is

$$p^{a}_{k}(t) = \hat{p}^{a}_{k}(t) + \sum_{i\in\mathcal{N}_{Gk}} \Delta p^{\,g}_{i}(t) + \sum_{j\in\mathcal{N}_{Bk}} \Delta p^{\,b}_{j}(t) + d^{a}_{k}(t)\,. \tag{12}$$

As mentioned, it is assumed that the continuous measurements of the power exchanged by the non-dispatchable elements connected to the grid, such as loads, are not available; therefore d^a_k needs to be estimated. To this regard, it is supposed that the power flows through cluster interconnections, denoted by $p^p_{k,i}$ with $i \in \mathcal{I}^p_k$, are measured. Note that $p^p_{k,i}$ is considered positive when exported from area k. The net power of each area can be thus evaluated as

$$p^{a}_{k}(t) = \sum_{i\in\mathcal{I}^{p}_{k}} p^{p}_{k,i}(t)\,. \tag{13}$$

Before computing the optimal control action at time t, the local MPC computes the estimated disturbance $\tilde{d}^a_k(t)$ as the difference between the measured area net power (13), the programmed profile (10) and the active power variations committed at the previous control iteration

$$\begin{aligned} \tilde{d}^{a}_{k}(t) \approx \sum_{i\in\mathcal{I}^{p}_{k}} p^{p}_{k,i}(t) \;-\; \hat{p}^{a}_{k}(t) \;-\; \sum_{i\in\mathcal{N}_{Gk}} \Delta p^{g}_{i}(t-1) \;-\\ -\sum_{j\in\mathcal{N}_{Bk}} \Delta p^{b}_{j}(t-1). \end{aligned} \tag{14}$$

Finally, the total active power reserves are also computed, since this information will be needed by the supervisory layer, as described in Section 4. These are defined as

$$\begin{aligned} r^{a\uparrow}_{p,k}(t) &= \sum_{i\in\mathcal{N}_{Gk}} r^{g\uparrow}_{p,i}(t) \;+ \sum_{j\in\mathcal{N}_{Bk}} \min\left(r^{b\uparrow}_{p,j}(t),\; \frac{r^{b\downarrow}_{e,j}(t+1)}{\tau_s} \right),\\ r^{a\downarrow}_{p,k}(t) &= \sum_{i\in\mathcal{N}_{Gk}} r^{g\downarrow}_{p,i}(t) \;+ \sum_{j\in\mathcal{N}_{Bk}} \min\left(r^{b\downarrow}_{p,j}(t),\; \frac{r^{b\uparrow}_{e,j}(t+1)}{\tau_s} \right), \end{aligned} \tag{15}$$

where, for the BESSs, the minimum between the effective power margin and the deliverable/absorbable power based on the stored energy is considered. Moreover, the energy reserves are at the next time instant are taken into account given the state update (5).

3. LOCAL MPC FORMULATION

The local MPC problem is now formally stated for a generic time instant t. In the following, h denotes a generic time instant within the prediction horizon of length N, i.e. $h \in T_N = \{t, \dots, t+N-1\}$. The main objective of the local regulator is to counteract the power imbalances, tracking the expected power profile of area k, enforcing

$$p^{a}_{k}(h) = \hat{p}^{a}_{k}(h)\,, \quad \forall h \in T_N\,.$$

However, the power variability may be too large to be compensated with the power reserves of the local DERs, causing the unfeasibility of the constraint defined above. A slack variable Δp^{req}_k is hence included as follows

$$p^{a}_{k}(h) = \hat{p}^{a}_{k}(h) \;-\; \Delta p^{req}_{k}(h)\,, \quad \forall h \in T_N\,, \tag{16}$$

where $\Delta p^{req}_k(h)$ can become different from zero just in case it is not possible to track the expected net output power using local resources. Precisely, if the local MPC regulator selects $\Delta p^{req}_k(h) \ge 0$, it means that local power imbalances cause a power shortage in area k, as it follows that $p^a_k(h) \le \hat{p}^a_k(h)$. On the other hand, $\Delta p^{req}_k(h) < 0$ means that there

is an excess of power in area k that cannot be internally absorbed. The optimal value of the slack variable $\Delta p_k^{req*}(t)$ denotes the power support currently request by the local MPC of the k-th area, which is then transmitted to the corresponding supervisory agent.
If for any area $\Delta p_k^{req*}(t) \neq 0$, the upper layer is activated, and all the supervisory agents interact through a fully distributed consensus-based algorithm. The goal is to optimally vary the net power of each area so that the power requests are satisfied. To impose that the power variations committed by the supervisory layer are actually tracked, constraint (16) is replaced by the following

$$p_k^a(h) = \hat{p}_k^a(h) - \Delta p_k^{req}(h) + \Delta p_k^{sup*}(t-1) \qquad (17)$$

where the variable $\Delta p_k^{sup*}(t-1)$ denotes the optimal power variation requested by the supervisory layer, and it is considered a parameter during the solution of the local MPC problems. A time shift is introduced since this variation is supposed to be computed by the supervisory layer at the previous time instant with respect to the local MPC execution.
The local MPC problem formulation must include the models of local units, defined by (1)-(7). Furthermore, a zero terminal constraint is introduced to enforce the restoration of the scheduled stored energy in the BESSs at the end of the optimization horizon,

$$\Delta e_j^b(t+N) = 0\,, \qquad \forall j \in \mathcal{N}_{Bk}\,. \qquad (18)$$

The expression for total power reserves (15) and the net output power (12) in the area are also included in the problem formulation. Moreover, since the output power trajectory depends on the future realization of the disturbance, which is unknown, it is assumed that the disturbance remains constant for the whole prediction horizon and that it is equal to the value estimated by (14) at time t,

$$d_k^a(h) = \tilde{d}_k^a(t), \qquad \forall h \in T_N\,. \qquad (19)$$

The cost function of the local MPC regulator is formulated as follows

$$J_k^a(t) = \sum_{h=t}^{N} \Big\{ w_{req}\,(\Delta p_k^{req}(h))^2 + \sum_{\forall j \in \mathcal{N}_{Bk}} w_j^b\,(\Delta p_j^b(h))^2 + \\ + \sum_{\forall i \in \mathcal{N}_{Gk}} \Big[w_i^g\,(\Delta p_i^g(h))^2 + w_i^{\delta g}\,(\Delta p_i^g(h) - \Delta p_i^g(h-1))^2 \Big] \Big\},$$

where w_{req} , w_j^b, w_i^g, and $w_i^{\delta g}$ are positive terms.
The proposed cost function aims to minimize the the power variations committed to DERs, while fulfilling the prescribed constraints. Notice that the variation of mGENs active power adjustments between consecutive time instants is penalized, to avoid an unnecessary variability of mGENs' set-points, encouraging the exploitation of BESSs to compensate the fast component of power fluctuations.
In summary, the local MPC problem is stated as follows

$$\begin{aligned} &\min_{\Delta p^g_{\forall i}\ \Delta p^b_{\forall j}, \Delta p_k^{req}} \quad J_k^a(t) \\ &\text{subject to} \quad (1)\text{-}(8),\ \forall i \in \mathcal{N}_{Gk}, \forall j \in \mathcal{N}_{Bk}, \forall h \in T_N, \\ &\qquad (12), (15), (17),\ \forall h \in T_N, \\ &\qquad (18), (19). \end{aligned}$$

According to a standard receding horizon approach, the local MPC regulator implements the first step of the optimal input sequence, i.e. $\Delta p_i^{g*}(t), \Delta p_j^{b*}(t)$, $\forall i \in \mathcal{N}_{Gk}, \forall j \in \mathcal{N}_{Bk}$. At each time instant, the local MPC regulator of area k communicates to the supervisory agent the optimal value of power request $\Delta p_k^{req*}(t)$ and the effective available power reserves in the area, i.e. $r_{p,k}^{a\uparrow*}(t)$ and $r_{p,k}^{a\downarrow*}(t)$.

4. SUPERVISORY LAYER DESIGN

The supervisory layer is executed right after the local MPC regulator in case any area is not able to compensate the local power variability by means of the power reserves offered by local DERs. This layer is implemented through a fully distributed approach, where each agent directly interacts with few others. Before describing in detail the adopted distributed algorithm, the centralized formulation of the supervisory layer problem is presented.
As shown in the previous section, the local MPC regulator of area k computes the optimal power request $\Delta p_k^{req*}(t)$, taking values different from zero just in case of necessity. At this stage, the supervisory layer must commit the necessary power variations to other areas, selecting the variables $\Delta p_j^{sup}(t)$, $\forall j \in \{1, \ldots, M\} \setminus k$. It must hold that

$$\sum_{k=1}^{M} \Delta p_k^{sup}(t) = \sum_{k=1}^{M} \Delta p_k^{req*}(t) \qquad (20)$$

to ensure the balance between the power requests and the power variations commited by the supervisory layer. It should be noted that, in case two local MPC regulators send two equal but opposite requests, the overall request to the supervisory layer is null. This is a desired feature since it means that the overall distribution network is self-balanced, even though the single areas are not.
As previously described, a request $\Delta p_k^{req*}(t) > 0$ indicates that there is a power shortage in area k. Consequently, the supervisory layer cannot ask the area itself to supply additional power, and it must be enforced that $\Delta p_k^{sup}(t) \leq 0$. On the other hand, if $\Delta p_k^{req*}(t) < 0$, it must hold that $\Delta p_k^{sup}(t) \geq 0$. The following constraint is thus stated:

$$\operatorname{sign}(\Delta p_k^{req*}(t))\, \Delta p_k^{sup}(t) \leq 0\,. \qquad (21)$$

The committed power variations must not exceed the available power reserves in each area, i.e.

$$r_{p,k}^{a\downarrow*}(t) \leq \Delta p_k^{sup}(t) \leq r_{p,k}^{a\uparrow*}(t)\,. \qquad (22)$$

The supervisory layer is therefore designed to minimize the following cost function for each area k

$$J_k^{sup}(t) = c_k\,(\Delta p_k^{sup}(t))^2\,, \qquad (23)$$

where the coefficient c_k can be different among the areas. Here, it has been chosen to use time-varying costs defined as $c_k(t) = 1/(r_k^{a\uparrow*}(t) + r_k^{a\downarrow*}(t))$, such that the larger the power reserves that area k owns, the lower the cost associated to the committed power variation $\Delta p_k^{sup}(t)$.
The centralized problem formulation that should be solved at the supervisory layer is hence formalized as follows

$$\begin{aligned} &\min_{\Delta p_k^{sup}(t)} \quad \sum_{k=1}^{M} J_k^{sup}(t) \\ &\text{subject to} \quad (21)\text{-}(22), \quad \forall k = \{1, \ldots, M\}, \\ &\qquad (20), \end{aligned} \qquad (24)$$

which is a static optimization problem. As the optimal values $\Delta p_k^{sup*}(t)$ are computed for each area, these are sent to the local MPC regulators to be executed at the next time instant, see (17). The optimization problem (24) can be solved using a fully distributed approach, meaning that each agent directly interacts with the others, without the need of any central supervising entity. The implemented algorithm is based on the Dual Consensus ADMM (DC-ADMM) method described in Chang et al. (2014). To apply this algorithm, let us model a multi-agent communication network as an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, \ldots, M\}$ is the set of nodes (i.e. the agents) and $\mathcal{E}$ is the set of edges. Precisely, an edge $(i, j) \in \mathcal{E}$ if, and only if, agent i and agent j are neighbours, meaning that they can exchange messages between each other. Thus, it is possible to define the index subset of neighbours for each agent i as $\mathcal{N}_i = \{j \in \mathcal{V} \,|\, (i, j) \in \mathcal{E}\}$, and the number of neighbours for agent i as $n_i = |\mathcal{N}_i|$. Considering the application framework of this work, the agents are the supervisory agents implemented at the top of the local MPC regulators. The approach requires two assumptions: 1) the undirected communication graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ must be connected and, 2) the primal problem must be convex. If the mentioned assumptions hold, the DC-ADMM algorithm asymptotically converges to the optimal solution of (24), as reported in (Chang et al., 2014, Teorem 2).
The optimization problem (24) is convex, while the communication graph is connected by design. It is indeed assumed that each area can interact with all its direct neighboring areas and that there are not isolated areas, as they are all part of the same distribution network. The DC-ADMM algorithm involves a procedure where the supervisory agents iteratively solve local optimization problems and exchange precise local variables through the communication graph $\mathcal{G}$ until they converge to the optimal solution. As it will be shown in the numerical results, this algorithm allows to find the optimal solution of (24) in a very reduced number of iterations. This is also due to the fact that most of the model and problem complexity is addressed by the local MPC regulators, while the supervisory layer is formulated as a simple static problem, aiming to quickly define the optimal power exchanges among areas.

5. NUMERICAL RESULTS

The proposed control architecture has been tested on the benchmark grid depicted in Figure 2, obtained from the interconnection of IEEE 37-bus and 13-bus test feeders, where generators, batteries and solar panels have been deployed throughout the network. The benchmark grid has been partitioned in $M = 4$ areas according to a topological criterion, and it has been simulated in MATLAB, while CPLEX has been used to solve optimization problems. Both the power profiles and the forecasts of loads and non-dispatchable ERs were provided by RSE S.p.A and were measured from secondary substations and solar panels located in Milan, Italy. Since the forecasted profiles can be significantly inaccurate, see Figure 3, the power absorbed from the main utility may experience significant deviations from the contracted profile, as apparent in Figure 4(a).
The implementation of the proposed control system allows to effectively compensate the deviation and to fulfill the contracted power program, as shown in Figure 4(b).

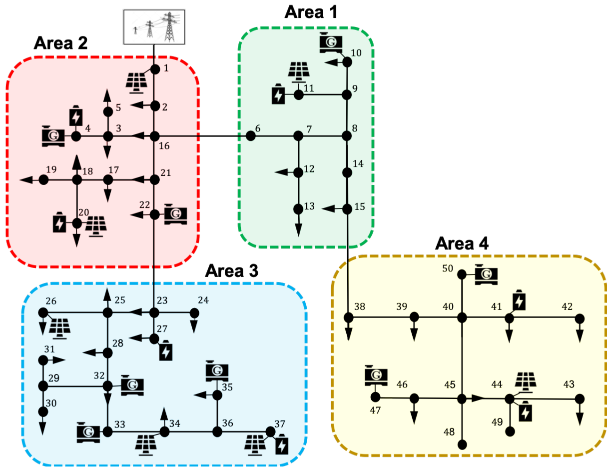
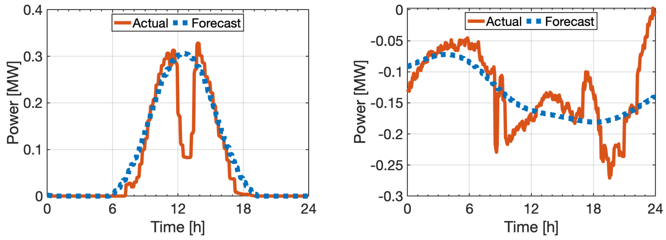


Fig. 2. Benchmark grid, based on IEEE 37-bus and 13-bus test feeders, with several ERs deployed.

To assess the performances of the proposed scheme, in Figure 5(a) the aggregated disturbance acting on area 1 is reported, limited to the interval 15:00 to 22:00 for convenience, while in Figure 5(b) the control action taken by the local MPC of the same area is depicted. It can be noticed that fast component of the disturbance is addressed by the local BESS, while the mGEN is used to compensate the slow trends. It should be noted that the generator power set-point is always maintained within the capability limits, see Figure 6(a). Similarly, as shown in Figure 6(b), owing to the zero-terminal constraint (18), the energy stored in the battery is always maintained close to the scheduled trajectory, and is restored to the nominal value within the end of the day. In the interval spanning from 18:30 to 21:00, the disturbance acting on area 1 becomes so large that the mGEN is saturated. Therefore, the local MPC regulator of this area requires additional power to the supervisory layer, which is addressed by decreasing the power set-points of the other areas by means of the variables Δp_2^{sup}, Δp_3^{sup}, $\Delta p_4^{sup} > 0$, as shown in Figure 7.
The supervisory layer has been implemented according to the DC-ADMM Algorithm, by allowing the communication only among neighboring areas. Figure 8 shows the effectiveness of the DC-ADMM algorithm, at time instant 20:00. Indeed, from Figure 8(a) it should be noted that the DC-ADMM achieves the same optimal cost of the centralized solution in about 15 iterations. Similar considerations hold for the power variations Δp_k^{sup} committed to each area k, as reported in Figure 8(c). The computational time required for the execution of local MPCs takes about 0.15 seconds, while the DC-ADMM algorithm - despite being executed sequentially - converges in about 3 seconds.

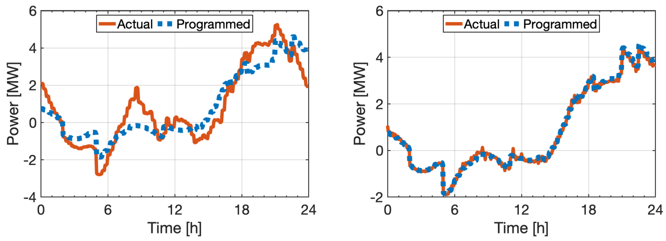
6. CONCLUSION

In this work a fully distributed control scheme has been proposed to perform power balancing in a distribution network partitioned in areas. The control scheme consists of low-level local MPC regulators to restore the power balance of each area. When any of these local controllers is not able to operate autonomously, a supervisory layer based on *Dual Consensus ADMM* is executed to coordinate the areas to support the needy one. Numerical results witness the effectivness of the approach.



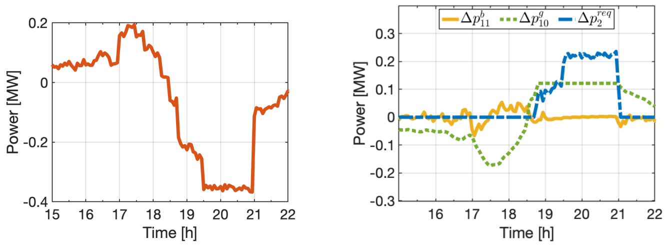
(a) (b)

Fig. 3. Power profiles of solar generator (a) and load (b) at node 26.



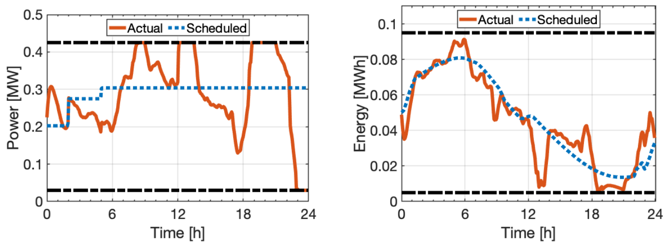
(a) (b)

Fig. 4. Power absorbed from the main utility without (a) and with (b) the proposed control scheme.



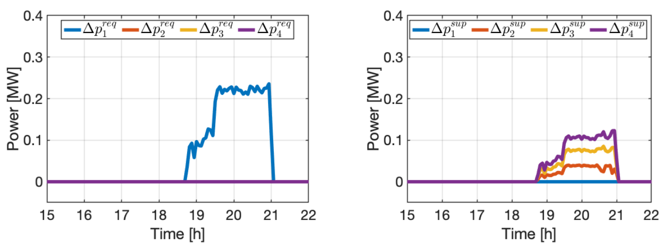
(a) (b)

Fig. 5. (a) Aggregated disturbance acting on area 1 and (b) local MPC control action between 15:00 and 22:00.



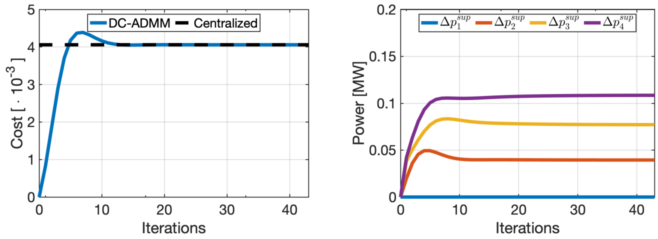
(a) (b)

Fig. 6. (a) Power profile of the mGEN at node 10, (b) energy stored in the BESS at node 11.



(a) (b)

Fig. 7. (a) Power requests issued to the supervisory layer, (b) actuation of the requests from 15:00 to 22:00.



(a) (b)

Fig. 8. DC-ADMM (a) trend of the overall cost function, (b) trend of the committed power variations.

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