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Control of Fully Actuated Mechanical Systems via Super-twisting Based Lyapunov Redesign

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Abstract: The problem control of fully actuated of mechanical systems under uncertainties is considered. With this aim a concept of Lyapunov redesign is revisited. The derivative of the Lyapunov function for a nominal model of the robot is used for the sliding surface design. This surface permits to design a super-twisting controller allowing to compensate the Lipschitz uncertainties, providing theoretically exact convergence of the states of uncertain system to the origin by means of a continuous control signal. The proposed result is illustrated for simulation example controlling an uncertain planar robot.

Keywords: Sliding mode control, Stability of nonlinear systems, Robust control.

1. INTRODUCTION

Rely on a mathematical model is a common practice in control engineering, however such models are different from the real systems due to uncertainties. Even when the model is close enough to the real system, another problem is to design a control law that may accomplish the task when there are external changes in the process. Consider for example a robot manipulator, a common task is to move objects from one point to another, and those objects may have different masses. Then, it is convenient to design a controller that solves the problem of uncertainty in the process, without designing a whole new controller. This result in the same problem of control of uncertain systems.

With uncertain systems in mind, Gutman (1979) presents his approach to robustification based on a known Lyapunov function (LF) for the system without uncertainties. This approach requires to add a discontinuous unit controller to the nominal one to robustly compensate the uncertainties. Simultaneosly, Leitmann (1979) presents the same idea for stabilization of uncertain linear systems, proving that the system with the robustifying control designed is asymptotically stable despite bounded uncertainties. This robustifying term is discontinuous on a surface with relative degree one, defined by the derivative of the nominal Lyapunov function. This approach became a classical technique for robustification of nonlinear uncertain systems, called later Lyapunov Redesign (Khalil, 2002, Ch. 14.2).

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However, the control law presented in (Gutman, 1979) and (Leitmann, 1979) results in a discontinuous control signal, which causes undesired *Chattering*. Substituting discontinuous control law with a continuous one, (Barmish et al., 1983; Ryan and Corless, 1984; Leitmann, 1993; Corless, 1993) began to put an extra term on the control, such that it becomes continuous. In this case they can only prove practical stability of the uncertain system.

Nowadays, to substitute the discontinuous controllers by continuous ones, The Super-Twisting Controller (STA) (Levant, 1993; Moreno and Osorio, 2012; Seeber and Horn, 2018) has been introduced. This algorithm can compensate Lipschitz perturbations theoretically exactly, ensuring the finite time converge of the sliding variable and its derivative to zero.Ventura and Fridman (2019) have shown that at least for systems with fast actuators, STA provide better *Chattering* attenuation.

Moreno and Osorio (2012) proposed a Lyapunov based analysis of stability of STA, but STA gains proposed by Levant (1998) are not covered. Recently, Seeber and Horn (2018) presented necessary and sufficient conditions for convergence of the STA. The Extensions for the MIMO case of the STA are presented in Nagesh and Edwards (2014) and Lopez-Caamal and Moreno (2018).

However to apply STA for systems with a relative degree greater than one, it is necessary to design a sliding surface with relative degree one with respect to the input. This has been done for SISO *linear* systems in Gonzalez et al. (2012). This result is generalized to the MIMO case in Vidal et al. (2017). However, to apply this approach it is necessary to

- transform the system to the regular form, which is complicated in the nonlinear case, and

- design the sliding surface, that maybe is naturally unrelated with the system's nominal model.

This paper proposes a STA based Lyapunov Redesign strategy for *nonlinear* systems with Lipschitz matched uncertainties. It takes the concept of the classical Lyapunov Redesign compensating the perturbation in the derivative of the LF, but in this case, using the STA. Proposed approach can compensate theoretically only Lipschitz perturbations, and it has two principal advantages,

- transformation to the regular form is not needed, which is very important for the *nonlinear* case,
- the sliding variable and its *derivative* converge to zero in finite time.

Notation Let $\mathbb{R}_+$ be the set of all positive real numbers. For any real value $w \in \mathbb{R}$ and vector $x \in \mathbb{R}^n$, $\|x\|$, is the euclidean norm of x. Let $\mathbb{I}$ be the identity matrix.

2. CLASSICAL LYAPUNOV REDESIGN AND PROBLEM STATEMENT

Following Khalil (2002), consider the uncertain system

$$\dot{x} = f(x) + g(x)\left[u + \delta(t,x)\right], \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$ and $\delta(t,x)$ is an uncertain term.

Assumption 1. Suppose that, for the nominal system corresponding to system (1) ($\delta(t,x) = 0$), there exist a control law $u = \psi(x)$ and a Lyapunov function $V(x)$, satisfying

$$c_1(\|x\|) \le V(x) \le c_2(\|x\|), \tag{2a}$$

$$\frac{\partial V(x)}{\partial x}\left[f(x) + g(x)\psi(x)\right] \le -c_3(\|x\|), \tag{2b}$$

with class $\mathscr{K}$ functions c_i, for $i = 1,2,3$.

To compensate $\delta(t,x) \neq 0$, a new term v should be added to the control law, then the control law will have the form $u = \psi(x) + v$. Differentiating the Lyapunov function V along the trajectories of uncertain systems (1), results in

$$\begin{aligned}\dot{V} =& \frac{\partial V(x)}{\partial x}\left[f(x) + g(x)\left(\psi(x) + v + \delta(t,x)\right)\right] \\ =& \frac{\partial V(x)}{\partial x}\left[f(x) + g(x)\psi(x)\right] + \frac{\partial V(x)}{\partial x} g(x)\left(v + \delta(t,x)\right) \\ \le& -c_3(\|x\|) + \frac{\partial V(x)}{\partial x} g(x)\left(v + \delta(t,x)\right),\end{aligned} \tag{3}$$

Define the variable $w = \frac{\partial V(x)}{\partial x} g(x) = L_g V(x)$, where $L_g V(x)$ is the Lie derivative of $V(x)$ along $g(x)$. We can rewrite the derivative as

$$\dot{V} = \frac{\partial V(x)}{\partial x}\left[f(x) + g(x)\psi(x)\right] + w\left(v + \delta(t,x)\right). \tag{4}$$

It is easy to see that the effect of the uncertainty term is in the same channel that the robustifying control v.

Assumption 2. Assume that $\delta(t,x)$ bounded as

$$\|\delta(t,x)\| \le \Delta(t,x). \tag{5}$$

To compensate uncertainty term δ, (Leitmann, 1979) and (Gutman, 1979) suggested a discontinuous unit control law

$$v = -\eta(t,x)\frac{w}{\|w\|}, \tag{6}$$

understanding solution of system (1) in Filippov (1988) sense. Now

$$\begin{aligned}\dot{V} &\le -c_3(\|x\|) + wv + \|w\|\|\delta\| \\ &\le -c_3(\|x\|) - \eta(t,x)\|w\| + \Delta(t,x)\|w\|.\end{aligned} \tag{7}$$

Choosing $\eta(t,x) = \Delta(t,x)$, it is easy to see that the stability of the origin of system (1) is kept despite of the uncertainty $\delta(t,x)$. Moreover, $\dot{V}$ satisfies (2b), which is independent of the perturbation. However, (6) produces a discontinuous control signal, causing undesirable *chattering effect*.

Problem statement

Our aim in this paper is to derive a *continuous control law* to compensate the effect of the perturbation in the derivative of the Lyapunov Function in (4). Substituting the discontinuous control (6) by a continuous control signal, and using it for control of fully actuated mechanical systems.

3. CONTINUOUS LYAPUNOV REDESIGN

It is possible to eliminate the effect of the perturbation δ in (4) by driving the variable w to zero in finite time, so that we recover (2b). To achieve this, consider the time derivative of w

$$\dot{w} = L_{\bar{f}(x)} w(x) + L_{g(x)} w(x)\left(v + \delta(t,x)\right), \tag{8}$$

with $\bar{f}(x) = f(x) + g(x)\psi(x)$. We will therefore attain the goal of this paper by designing a STA control law, enforcing the trajectories to the second order sliding mode set $\dot{w} = w = 0$, and producing a continuous control signal.

As a solution, consider Variable Gain Generalized Super-Twisting (VGSTA)(Gonzalez et al., 2012), (Vidal et al., 2017),

$$\begin{aligned}u_{st} &= -k_1(t,x)\phi_1(w) + \rho, \\ \dot{\rho} &= -k_2(t,x)\phi_2(w),\end{aligned} \tag{9}$$

where the functions $\phi_1(w) = \frac{w}{\|w\|^{1/2}} + \beta w$ and $\phi_2(w) = \frac{w}{2\|w\|} + \frac{2\beta}{2}\frac{w}{\|w\|^{1/2}} + \beta^2 w$. Note that $\phi_2 = \phi_1'\phi_1$, with $\phi_1' = \left[\frac{1}{\|w\|^{1/2}}\left(\mathbb{I} - \frac{ww^T}{2\|w\|}\right) + \beta\mathbb{I}\right]$.

Assumption 3. Assume that the perturbation term in (8) can be separated as

$$L_g w(x)\delta(t,x) = d_1(t,x) + \delta_z(t,x), \tag{10}$$

where δ_z is chosen such that $g^\perp(x)\frac{\partial \delta_z}{\partial x} = 0$, being $g^\perp$ an orthogonal vector to $g(x)$. There exist known non-negative functions $\rho_1(t,x) \ge 0$ and $\rho_2(t,x) \ge 0$ such that

$$\begin{aligned}\|d_1(t,x)\| &\le \rho_1(t,x)\|\phi_1(w)\| \\ \|d_2(t,x)\| &\le \rho_2(t,x)\|\phi_2(w)\|.\end{aligned} \tag{11}$$

with $d_2(t,x) = \frac{\partial \delta_z(t,x)}{\partial x}\bar{f}(x) + \frac{\partial \delta_z}{\partial t}$.

Remark 1. Assumption 3 is restrictive (Castillo et al., 2018) but there exists some class of systems satisfying this asumption, for example the mechanical systems considered in this work.

Assumption 4. The determinant of the square Matrix $L_{g(x)}w(x)$ from (8) is bounded away from zero for all x.

Remark 2. Assumption 4 is satisfied in the case that V is strictly convex (i.e. $\left(\partial^2 V/\partial x^2\right) > 0$ for every x) and $g(x)$ is constant. In particular, for linear systems with a quadratic Lyapunov Function (see Example 1). Furthermore, if (1) is a passive system nondegenerate at $x = 0$ with a storage function $V \in C^2$, this assumption is satisfied locally at $x = 0$ (Byrnes et al., 1991, see Proposition 4.5).

Proposition 1. Consider the perturbed system (1) and suppose that the assumptions 3 and 4 are satisfied. Set the control v

$$v = -\left[L_{g(x)}w(x)\right]^{-1}\left(L_{\bar{f}(x)}w(x) - u_{st}\right) \tag{12}$$

with u_{st} as in (9), and the functions $k_1(t,x)$ and $k_2(t,x)$

$$
\begin{aligned}
k_1(t,x) =& c_1 + \frac{1}{\theta}\left\{\frac{1}{4\varepsilon}\left[2\varepsilon\rho_1+\rho_2\right]^2 + 2\varepsilon\rho_2 + \varepsilon \right. \\
&\left. + \left[2\varepsilon+\rho_1\right]\left(\theta+4\varepsilon^2\right)\right\}, \\
k_2(t,x) =& \theta + 4\varepsilon^2 + 2\varepsilon k_1(t,x).
\end{aligned} \tag{13}
$$

with $\beta > 0$, $\varepsilon > 0$, $\theta > 0$, $c_1 > 0$.

Then trajectories of (1) reach the set $w = \dot{w} = 0$ in finite time. After that, (2b) will be satisfied, and consequently, the origin of (1) will be asymptotically stable.

4. CASE OF STUDY: MECHANICAL SYSTEMS

4.1 Fully actuated mechanical systems

Consider the mechanical system of n degrees of freedom (Corless, 1989).

$$J(t,q)\ddot{q} = U(t,q,\dot{q}) + W(t,q,\dot{q})\hat{u} \tag{14}$$

with $q \in \mathbb{R}^n$, $\hat{u} \in \mathbb{R}^n$, $J : \mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^{n\times n}$, $U : \mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^{n\times n}$ and $W : \mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^{n\times n}$. where $J(t,q)$ is assumed to be symmetric, $W(t,q,\dot{q})$ is non singular and there exist Lipschitz positive functions $\hat{\beta}_i$ for $i = 1,3$. such that $\|U(t,q,\dot{q})\| \leq \hat{\beta}_0(t,q,\dot{q})$, $\lambda_{max}[J(t,q)] \leq \hat{\beta}_1(t,q)$ and $\lambda_{min}[J(t,q)] \geq \hat{\beta}_2(t,q)$. For a desired tracking trajectory $\bar{q} \in C^2$,

$$x = \begin{bmatrix} q(t) - \bar{q}(t) \\ \dot{q}(t) - \dot{\bar{q}}(t) \end{bmatrix}$$

choosing a nonsingular matrix $T \in \mathbb{R}^{n\times n}$, it is posible to define a new control input

$$\hat{u} = \left[T^T W(t,q,\dot{q})\right] u,$$

such that we can rewrite the system as,

$$\dot{x} = Ax + B\left[h(t,x) + G(t,x)u\right]$$

with

$$
\begin{aligned}
A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \quad B &= \begin{bmatrix} 0 \\ T \end{bmatrix}, \\
h(t,x) &= T^{-1}J^{-1}U - T^{-1}\ddot{\bar{q}}, \\
G(t,x) &= \tilde{J}^{-1}, \ \ \tilde{J} = T^T J T.
\end{aligned}
$$

Design the control as $u = u_{nom}(t,x) + v(t,x)$, let us rewrite the dynamics of the closed loop as,

$$\dot{x} = \bar{f}(t,x) + B\left[h(t,x) + G(t,x)u\right],$$

with $\bar{f}(t,x) = Ax + BG(t,x)u_{nom}(t,x)$. Nominal closed loop system has the form, $\dot{x} = Ax + BG(t,x)u_{nom}(t,x)$. Choosing any matrix $Q^T = Q > 0$, $\sigma \in \mathbb{R}_+$ and $\gamma(t,x) \in \mathbb{R}$ as scalars, such that,

$$\gamma(t,x) \geq \sigma\lambda_{max}\left[\tilde{J}(t,x)\right], \quad \forall (t,x),$$

then, Corless (1989) propose the control of the form

$$u_{nom}(t,x) = -\gamma(t,x)B^T P x$$

where $P = P^T > 0$ solution of the algebraic Ricatti equation

$$PA + A^T P - \sigma P B B^T P + 2Q = 0.$$

Taking $V = \frac{1}{2}x^T P x$ as the Lyapunov function for the nominal system, $\dot{V}$ takes the form $\dot{V} \leq -x^T Q x$.

Now, $w = B^T P x$, its time derivative is

$$
\begin{aligned}
\dot{w} =& B^T P\dot{x}, \\
=& B^T P[\bar{f}(t,x) + B\left[h(t,x) + G(t,x)u\right], \\
=& B^T P\bar{f}(t,x) + B^T PBG(t,x)v(t,x) + \delta(t,x)
\end{aligned}
$$

with $\delta(t,x) = B^T PBh(t,x)$. Then, we can design

$$v(t,x) = -\left[B^T PBG(t,x)\right]^{-1}(B^T P\bar{f}(t,x) - u_{st})$$

that leads the dynamics of w,

$$\dot{w} = u_{st} + \delta(t,x).$$

4.2 2-DoF robot

Consider the 2-Degrees of Freedom robot with the model presented in (Slotine and Li, 1991, Ch. 6, p. 211),

$$J(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau, \tag{15}$$

where $q = [q_1 \ \ q_2]$, $\tau = [\tau_1 \ \ \tau_2]$. The matrices J (Inertia Matrix),

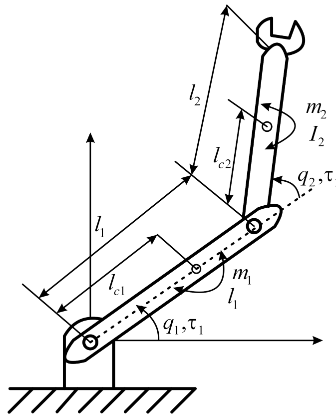

Fig. 1. Schematic of the robot.

C (Coriolis terms) y g (gravitational terms) , are as follows

$$J(q) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \ \ g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{16}$$

$$C(q,\dot{q}) = \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix} \tag{17}$$

taking $U(t,q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q)$ and $W(t,q,\dot{q}) = T = I_2$, with

$$
\begin{aligned}
J_{11} =& m_1 l_{c1}^2 + I_1 + m_2[l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)] + I_2, \\
J_{22} =& m_2 l_{c2}^2 + I_2, \\
J_{12} =& J_{21} = m_2 l_1 l_{c2}\cos(q_2) + m_2 l_{c2}^2 + I_2, \\
h =& m_2 l_1 l_{c2}\sin(q_2), \\
g_1 =& m1 l_{c1} g\cos(q_1) + m_2 g[l_{c2}\cos(q_1+q_2) + l_1\cos(q_1)], \\
g_2 =& m_2 l_{c2} g\cos(q_1+q_2).
\end{aligned}
$$

Then, with $B = [0 \ \mathbb{I}_2]$, the system can be rewritten as

$$
\begin{aligned}
\dot{x}_1 =& x_2, \\
\dot{x}_2 =& \left[J(x_1)\right]^{-1}\left(C(x)x_2 - g(x_1) + \tau\right),
\end{aligned}
$$

one can take $\sigma = 1$, and the control then is $u_{nom}(t,x) = -B^T P x$ with P solution of the Ricatti equation setting $Q = \mathbb{I}_4$. Then,

$$P = \begin{bmatrix} 3.1075 & 0 & 1.4142 & 0 \\ 0 & 3.1075 & 0 & 1.4142 \\ 1.4142 & 0 & 2.1974 & 0 \\ 0 & 1.4142 & 0 & 2.1974 \end{bmatrix}$$

Also, the trajectories requested $(q(t))^T = [\sin(t) \ \cos(t)]$. In simulation the following parameters are used. If we take the perturbations as $d_1 = -C(x_2)x_2$ and $\delta_z = -g(x_1) + \varphi(t)$, with $\varphi(t) = 0.1\sin(5t) + 0.5\sin\left(\frac{t}{4}\right) + 1$. We can bound them as

Table 1. Parameters

Parameter	**value**
l_1	0.5
l_2	0.2
l_{c1}	0.32
l_{c2}	0.08
m_1	1
m_2	0.5
g	9.81
I_1	1e-2
I_2	0.5e-2

$$
\begin{aligned}
\|d_1\| &\leq \rho_1(t,x)\|\phi_1(w)\|, \\
&= \left(\Gamma_1\|x_2\|^2+\gamma_1\right)\|\phi_1(w)\|, \\
\|d_2\| &\leq \rho_2(t,x)\|\phi_2(w)\| \\
&= \left(\Gamma_2\|x_2\|+\gamma_2\right)\|\phi_2(w)\|.
\end{aligned}
$$

Then, the gains are as follows

$$
\begin{aligned}
k_1(t,x) =& 12+\frac{1}{4}\left[\left(\Gamma_1\|x_2\|^2+\gamma_1\right)+\left(\Gamma_2\|x_2\|+\gamma_2\right)\right]^2 \\
&+2\left(\Gamma_2\|x_2\|+\gamma_2\right)+5\left(\Gamma_2\|x_2\|+\gamma_2\right) \\
k_2(t,x) =& 5+2k_1(t,x).
\end{aligned}
$$

Simulations were made using the constants $\beta=\delta=\varepsilon=1$, and the values $\Gamma_1=m_2 l_1 l_{c2}+1$, $\gamma_1=1$, $\Gamma_2=m_1 l_{c1} g+m_2(l_{c2}+l_1)+m_2 l_{c2} g$, $\gamma_2=1.1$.

4.3 Analysis of the results

In order to compare the behavior of the proposed approach, simulations with the classical Lyapunov redesign are presented. Gains calculations are calculated according to Corless (1989).

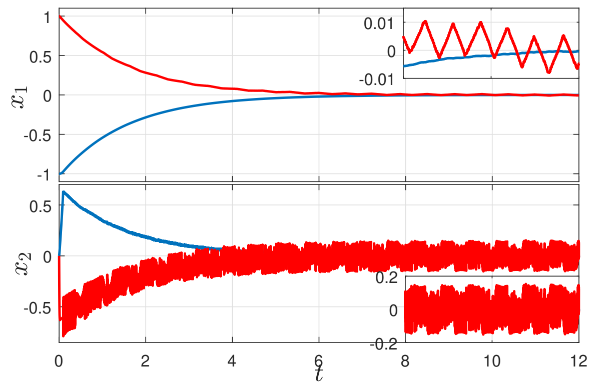

Fig. 2. States of the closed loop with the unit control based Lyapunov redesign.

It can be clearly seen in Figure 2 that the system states asymptotically converges to the origin, however, in the velocity coordinates the discontinuous control causes *Chattering*. In Figure 3, shows the discontinuous control signal. In Figure 4 illustrates the convergence of the states to the origin without visible *chattering*. The attenuation of *Chattering* can be observed in Figure 5 drawing the control signal, and also the sliding variable converges with a smaller chattering amplitude.

5. DISCUSSION

As a matter of comparison, recall the two main methodologies of sliding mode controller design for uncertainties compensation: Conventional (Utkin, 1992; Edwards and Spurgeon,

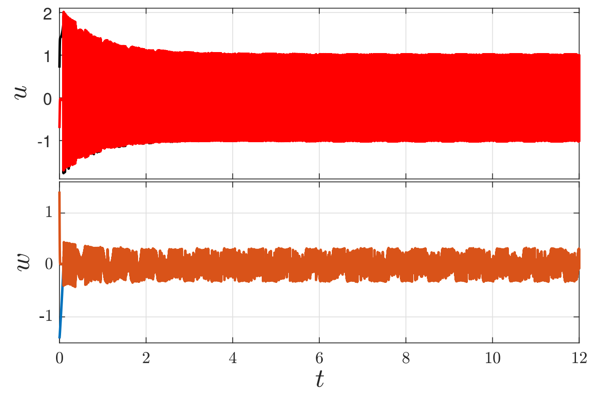

Fig. 3. Control u and sliding variable w with the unit control based Lyapunov redesign.

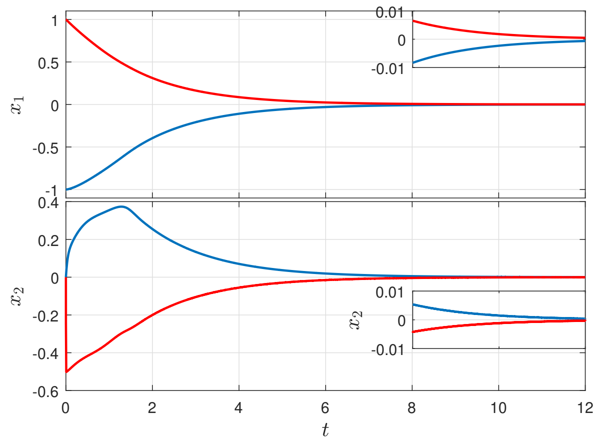

Fig. 4. States of the closed loop with the STA based Lyapunov redesign.

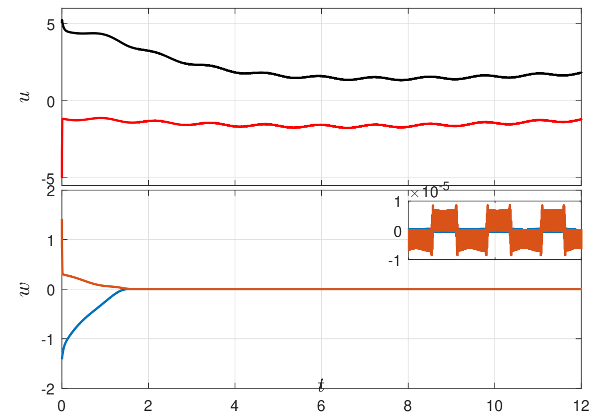

Fig. 5. Control u and sliding variable w with the STA Lyapunov redesign.

1998), and Integral Sliding Mode design (Matthews and DeCarlo, 1988; Utkin and Shi, 1996). Conventional sliding mode design consists in two steps, a sliding surface design with the desired dynamics and a control law that ensures finite-time and theoretically exact stabilization to the reduced order dynamics. However, it is not clear how to design such surface so that one can have a desired performance of the system in the sliding mode. In contrast to the conventional sliding mode design, the sliding mode based Lyapunov redesign, provides a sliding surface with desired properties. Meanwhile, integral Sliding Modes (ISM) design compensates matched disturbances/uncertainties while keeping the trajectories of the nominal system from the initial time moment, for the case when the initial conditions are known. Nonetheless, it mantains the order of the sliding mode dynamics as the same of the system, that means there is not reduced order dynamics.

On the other hand, in this works conservative assumptions were made in order to apply the VGSTA (Gonzalez et al., 2012; Vidal et al., 2017). First, the control coefficient must be know, which is in practice a clear disadvantage. Also, exist the problem of *algebraic loop* mentioned in Castillo et al. (2018) in the case of time and state dependent perturbation. This problem is avoided in Gonzalez et al. (2012) assuming that the gradient of the perturbation is bounded by some constant, that implies that u_{st} should be also bounded. This assumption recalls the so call *algebraic loop*, since VGSTA is not necessarily bounded, and it has a direct effect on the states of the system. Then, a future work is clear, obtain a Super-twisting based Lyapunov redesign when there are uncertainties in the control matrix and state dependent perturbations, dealing with the algebraic loop.

6. CONCLUSIONS

In the presented paper, Super-twisting based Lyapunov redesign is perform for nonlinear MIMO systems. The problem has been solved producing a continuous control signal, allowing to compensate Lipschitz uncertainties providing theoretically exact convergence of the states. Moreover, for the proposed approach a transformation into the regular form is not necessary, and unlike integral sliding modes, reduces the order of the sliding mode dynamics ensuring the stability. The sliding surface is induce by LF, that is why is not needed to be designed, and not only the sliding variable, but its derivative also converges to zero in finite time. The feasibility of the proposed design, is shown in a example robotic manipulator considered in Corless (1989).

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