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Fixed-Time Control for a Class of Unknown Nonlinear Affine Systems and Its applications to a Lithography Machine

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Abstract: The fixed-time control problems of a class of unknown nonlinear affine systems subject to external disturbances, unknown input dead zone and output constraints are considered in this paper. The fixed-time state feedback control strategy with adaptive neural networks (NNs) is designed. In the control design, the log-type barrier Lyapunov function (BLF) is chosen to handle the system output constraint. Then, neural networks(NNs) are applied to compensate for the adverse impact of unknown input dead zone and deal with system uncertainties. The novel virtual controllers and novel online updating laws of neural network weights are proposed to fulfill the fixed-time stability of closed-loop systems. The boundednesses of all the signals in closed-loop system are demonstrated via Lyapunov stability theory. Eventually, the experiment performed on the lithography machine is served to demonstrate good performance.

Keywords: fixed-time control, output constraints, neural networks, input dead zone, lithography machine

1. INTRODUCTION

In most of the practical systems, since the initial conditions are unknown and constantly changing, the finite-time control cannot guarantee convergence performance of the closed-loop system. By way of solving this problem, the theory of fixed-time control is first investigated in Polyakov (2012). Fixed-time control can well estimate the the upper bound of convergence time, which is independent of the initial conditions and only related to the control parameters.

Input dead zone and output constraints exist in many practical systems, which will have adverse effects on control performance if overlook these nonlinearities. As for output constraint, the control method based on Barrier Lyapunov Function (BLF) has been proved to be valid in dealing with a large class of constraint problems. Ngo et al. (2005) employed a log-type BLF to prevent constraint violation but only for systems whose system information were known. Due to the general uncertainty in the actual system and the existence of uncertainty will reduce the control performance of the system. NNs were applied to approximate uncertainty of system and the log-type BLF was combined with adaptive neural network control to accomplish output constraints for nonlinear systems with uncertain in Huang et al. (2019); Jia and Song (2017). On the other hand, to deal with the input dead zone, dead zone inverse method was used to solve the problem of dead zone nonlinearity early in Zhou et al. (2006). However, dead zone inverse scheme needs the condition for linear parameterization which is not satisfied with the complex nonlinear dead zone in engineering. To overcome this problem, NNs were also used to approximate the input dead zone. In Liu and Zhou (2010), the unknown nonlinear function and dead zone input were approximated by the fuzzy-neural networks. Ulteriorly, Radial Basis Function Neural Networks (RBFNNs)-based control consumes less calculation resources than other structures which leads to faster convergence of errors. Hence, in Yu and Du (2011), a novel adaptive neural network control scheme was developed to tackle the stabilization problem with RBFNNs approximating the dead zone input. The above literatures, however, did not take both input dead zone and output constraint into account. To address the system control problem with multiple constraints, recently, He et al. (2015a,b); Li et al. (2019) have proposed adaptive learning control for uncertain nonlinear systems with input dead zone and output constraint. However, these controllers only achieved asymptotic stability or finite-time stability of closed-loop systems.

Since the establishment of fixed-time stability requires more rigorous conditions, the existing methods in solving input dead zone and output constraints He et al. (2015a,b); Li et al. (2019), which cannot be directly extended to fixed-time control. In addition, the singularity problem caused by the use of BLF in the fixed-time control design Jin (2019) and the unknown nonlinearity of the input dead

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zone in the actual system, which makes the design of a fixed-time controller extremely difficult and complicated. Therefore, there were few results on how to solve the problems of both input dead zone and output constraint in fixed-time control. Ni et al. (2019) firstly considered the fixed-time control problem of nonstrict-feedback nonlinear system with input dead zone and output constraint. But the design of the controller need to know the relevant information of input dead zone, which is extremely difficult to obtain in practical systems. Besides, it did not consider the external disturbances. These deficiencies may make the control performance of the actual systems worse or even unstable.

Moreover, nonlinear affine systems are a common class of nonlinear systems, and many practical systems can be written as them such as power systems, robot arms, induction motor and linear motor etc. However, there are no research results on the fixed-time control of unknown nonlinear affine systems under the conditions of unknown input dead zone, output constraints and external disturbances. Motivated by aforementioned observations, this paper aims to tackle the fixed-time control problem for unknown nonlinear affine systems subject to unknown input dead zone, output constraints and external disturbances. RBFNNs are respectively utilized to approximate system uncertainties and compensate for the effect of input dead zone. The novel virtual controllers and the novel online updating laws of neural network weights are designed to accomplish the fixed-time stability of closed-loop systems.

Compared with the prior results, the contributions of this paper are summarized as follows:

(1) Compared with He et al. (2015b), the novel virtual controllers are designed by backstepping method, and the novel neural network weight updating laws are designed to achieve the fixed-time stability of unknown nonlinear affine systems with output constraints, unknown input dead zone and external disturbances.
(2) Compared with Ni et al. (2019), the fixed-time controller designed in this paper does not need any information about input dead zone and considers the external disturbance of the system, which improves the control performance of the actual system.
(3) In this paper, the fixed-time control is first applied to the motion control of lithography machine and achieve high precision tracking.

Notation 1. In the later sections, $\lambda(\cdot)$ is the eigenvalue of the matrix and $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of the matrix. For two $m \times n$-dimensional matrices $A = [a_{ij}], B = [b_{ij}], i = 1, \cdots, m, j = 1, \cdots, n$, we define $(A \circ B)_{ij} = (a)_{ij}(b)_{ij}$ as hadamard product of matrices A and B.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Problem Formulation

Consider a class of nonlinear affine systems in the following form He et al. (2015b)

$$\begin{aligned} \dot{x}_j &= x_{j+1} \quad (j = 1, 2, \ldots, n-1) \\ \dot{x}_n &= f(x) + \mathrm{g}(x)u + d(t) \\ \mathrm{y} &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is system state vector, $d(t)$ is the unknown external disturbance. $f(x) \in \mathbb{R}, \mathrm{g}(x) \in \mathbb{R}$ are unknown system functions related to the states of system (1). u is the control input of system (1) and input dead zone is defined as:

$$u = D(v) = \begin{cases} D_r(v) & \text{if } v \geq b_r \\ 0 & \text{if } b_l < v < b_r \\ D_l(v) & \text{Otherwise} \end{cases} \tag{2}$$

where v is the dead zone input, b_r and b_l are unknown breakpoints of the dead zone and $D_r(\cdot)$ and $D_l(\cdot)$ are unknown smooth functions of the dead zone.
To illustrate some results and analysis in this paper, the following assumptions are presented

Assumption 1. The desired trajectory $\mathrm{y}_d(t)$ is known and bounded by a positive constant y_{db} and its derivatives are all bounded.

Assumption 2. In system (1), the unknown system function $f(x)$ and $\mathrm{g}(x)$ are all bounded, and the derivative of $\mathrm{g}(x)$ is continuous and bounded.

Assumption 3. Ge et al. (2001) The unknown system function $\mathrm{g}(x)$ is strictly either positive or negative and two positive constants $\overline{\mathrm{g}}, \underline{\mathrm{g}}$ can be find such that $\overline{\mathrm{g}} \geq |\mathrm{g}(x)| \geq \underline{\mathrm{g}} > 0$, in this paper, without losing generality, we assume $\mathrm{g}(x) > 0$.

Assumption 4. As for the external disturbance $d(t)$, there exists a positive constant $\overline{d}$ such that $|d(t)| \leq \overline{d}$.

Remark 1. Assumptions 1 - 4 are common assumptions for theoretical analysis of nonlinear systems in plenty of literatures. Assumption 1 is the basic requirement in the literatures involving the backstepping scheme. Assumption 2 is practically satisfied in most engineering applications. Assumption 3 implies that the control gain of the system cannot be increased to infinity and ensures the controllability of the system, moreover, the bounds of $\mathrm{g}(x)$ are unknown in this paper and only used for theoretical analysis. In Assumption 4, bounded external distance is a common precondition in many literatures on system robustness.

2.2 Definitions and useful Lemmas

Definition 1. For the vector $x = [x_1, x_2, \cdots, x_m]^T \in \mathbb{R}^m$, define $x^n = [x_1^n, x_2^n, \cdots, x_m^n]^T \in \mathbb{R}^m$ and $x^{nT} = [x_1^n, x_2^n, \cdots, x_m^n] \in \mathbb{R}^m$, where $m, n \in \mathbb{N}^*$.

Lemma 1. Zuo (2015) For any $x_i \in \mathbb{R}^+, i = 1, \cdots, n$ and $a \in (0, 1]$, we have

$$\left(\sum_{i=1}^{n} x_i\right)^a \leq \sum_{i=1}^{n} x_i^a \tag{3}$$

Lemma 2. Zuo (2015) For any $x_i \in \mathbb{R}^+, i = 1, \cdots, n$ and $a \in (1, \infty)$, we have

$$\left(\sum_{i=1}^{n} x_i\right)^a \leq n^{a-1} \sum_{i=1}^{n} x_i^a \tag{4}$$

Lemma 3. Jin (2019) For any $x, y \in \mathrm{R}$ and $\epsilon > 0, a > 1, b > 1, (a-1)(b-1) = 1$, then

$$xy \leq \frac{\epsilon^a}{a}|x|^a + \frac{1}{b\epsilon^b}|y|^b \tag{5}$$

Lemma 4. He et al. (2015b) For any $\mu \in \mathbb{R}^+$, the following inequality holds for $z \in \mathbb{R}$ in the interval $|z| < |\mu|$:

$$\ln \frac{\mu^2}{\mu^2 - z^2} \leq \frac{z^2}{\mu^2 - z^2} \tag{6}$$

Lemma 5. Jin (2019) For any $x, y, z \in \mathbb{R}^m$, the following equality holds

$$x^T(y \circ z) = (x \circ y)^T z \tag{7}$$

Lemma 6. For any $x, y \in \mathbb{R}^m$ and $\delta > 0$, the following equalities holds

$$-x^{3T}y \leq \frac{3\delta^{\frac{4}{3}}}{4}x^{3T}x + \frac{1}{4\delta^4}y^{2T}y^2 \tag{8}$$

$$-x^T y^3 \leq 3x^{2T}y^2 + \frac{1}{12}y^{2T}y^2 \tag{9}$$

Proof. See the Appendix A. ■

Lemma 7. Jin (2019) Consider the nonlinear system shown as $\dot{z}(t) = f(z(t))$, where $z \in \mathbb{R}^n$ is the system state and $f(\cdot) : \mathbb{R}^+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a nonlinear function.and the origin of it is presumed an equilibrium point. If a continuous and differentiable positive definite function V satisfies

$$\dot{V}(z) \leq -aV^p(z) - bV^q(z) \tag{10}$$

where $a, b > 0$ and $p > 1, 0 < q < 1$. Then, the system's origin is said to be fixed-time stable with the setting time T_{fd} estimated by

$$T_{fd} \leq T_{max} = \frac{1}{a(p-1)} + \frac{1}{b(1-q)} \tag{11}$$

The purpose of this paper is to design a fixed-time controller for system (1) subject to unknown input dead zone, unknown external distances and output constraints. And the controller can ensure the fixed-time convergence of output tracking error while not violate the constraint.

3. FIXED-TIME STATE FEEDBACK CONTROL BASED NEURAL NETWORK OF SYSTEM

In this part, the backstepping scheme is adopted in the design of controller.

Step 1.

Letting $e_1 = x_1 - y_d$ and making its time derivative, we have

$$\dot{e}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d \tag{12}$$

Then, the virtual controller α_1 is designed as

$$\alpha_1 = -k_{1,1}(\mu^2 - e_1^2)^{\frac{1}{4}}\frac{1}{e_1}\xi_{e_1} - k_{1,2}(\mu^2 - e_1^2)^{-1}e_1^3 + \dot{y}_d \tag{13}$$

and piecewise smooth function ξ_{e_1} is defined as

$$\xi_{e_1} = \begin{cases} (e_1^2)^{\frac{3}{4}} & \text{if } |e_1| \geq \gamma_1 \\ e_1^2(\gamma_1^2)^{-\frac{1}{4}} & \text{Otherwise} \end{cases} \tag{14}$$

where $k_{1,1}, k_{1,2} > 0$ and $0 < \gamma_1 < \mu$.
considering the following barrier Lyapunov function candidate as

$$V_1 = \frac{1}{2}\ln\frac{\mu^2}{\mu^2 - e_1^2} \tag{15}$$

Letting $\alpha_1 = x_2$ and substituting it into (12), we get $\dot{e}_1 = -k_{1,1}(\mu^2 - e_1^2)^{\frac{1}{4}}\frac{1}{e_1}\xi_{e_1} - k_{1,2}(\mu^2 - e_1^2)^{-1}e_1^3$. Then, taking $\dot{e}_1$ into the time derivative of V_1, by Lemma 1 - Lemma 4, we obtain two cases:

Case 1) ($|e_1| \geq \gamma_1$):

$$\dot{V}_1 \leq -k'_{1,1}(V_1)^{\frac{3}{4}} - k'_{1,2}(V_1)^2 \tag{16}$$

Case 2) ($|e_1| < \gamma_1$):

$$\dot{V}_1 \leq -k'_{1,1}(V_1)^{\frac{3}{4}} - k'_{1,2}(V_1)^2 + a_1 \tag{17}$$

where $k'_{1,1} = 2^{\frac{3}{4}}k_{1,1}, k'_{1,2} = 4k_{1,2}, a_1 = k_{1,1}(\frac{\gamma_1^2}{\mu^2-\gamma_1^2})^{\frac{3}{4}}$, since $0 < \gamma_1 < \mu$, a_1 is a bounded positive number.
Combining with (16) and (17), we can write the above two cases ($|e_1| \geq \gamma_1$ and $|e_1| < \gamma_1$) in the same form as

$$\dot{V}_1 \leq -k'_{1,1}(V_1)^{\frac{3}{4}} - k'_{1,2}(V_1)^2 + a_1 \tag{18}$$

Remark 2. For virtual controller in (13), the piecewise smooth function ξ_{e_1} designed in (14) is used to avert singularity caused by the term $\frac{1}{e_1}\xi_{e_1}$. Due to (13), we can get $\lim_{e_1 \to 0}\frac{1}{e_1}\xi_{e_1} = \lim_{e_1 \to 0} e_1(\gamma_1^2)^{-\frac{1}{4}} = 0$. In addition, a very small value of γ_1 will lead to a large value of the virtual controller, which will lead to the failure of the control algorithm. So we need to choose a suitable γ_1 to meet the actual requirements. It is worth mentioning that, compared with Jin (2019), we use inequality scaling to discuss Lyapunov functions segmented by singularity under the same structure, which makes our following conclusions more complete.

Step j ($2 \leq j \leq n-1$).

Letting $e_j = x_j - \alpha_{j-1}$, and its time derivative yields

$$\dot{e}_j = \dot{x}_j - \dot{\alpha}_{j-1} = x_{j+1} - \dot{\alpha}_{j-1} \tag{19}$$

The virtual controller α_j is chosen as

$$\alpha_j = -k_{j,1}\frac{1}{e_j}\xi_{e_j} - k_{j,2}e_j^3 + \dot{\alpha}_{j-1} \tag{20}$$

and piecewise smooth function ξ_{e_j} is defined as

$$\xi_{e_j} = \begin{cases} (e_j^2)^{\frac{3}{4}} & \text{if } |e_j| \geq \gamma_j \\ e_j^2(\gamma_j^2)^{-\frac{1}{4}} & \text{Otherwise} \end{cases} \tag{21}$$

where $0 < \gamma_j, k_{j,1}, k_{j,2} > 0$
Considering the Lyapunov function candidate as

$$V_j = V_{j-1} + \frac{1}{2}e_j^2 \tag{22}$$

and making time derivative of V_j and taking (19) - (21) in it, by lemma 1-lemma 4, we also obtain two cases

Case 1) ($|e_j| \geq \gamma_j$)

$$\dot{V}_j \leq -k'_{j,1}(V_j)^{\frac{3}{4}} - k'_{j,2}(V_j)^2 + \sum_{i=1}^{j-1}a_i \tag{23}$$

Case 2) ($|e_j| < \gamma_j$):

$$\dot{V}_j \leq -k'_{j,1}(V_j)^{\frac{3}{4}} - k'_{j,2}(V_j)^2 + \sum_{i=1}^{j}a_i \tag{24}$$

where

$$k'_{j,1} = \min(2^{\frac{3}{4}}k_{1,1}, 2^{\frac{3}{4}}k_{2,1}, \cdots, 2^{\frac{3}{4}}k_{j,1}) \tag{25}$$

$$k'_{j,2} = \min(\frac{4}{j}k_{1,2}, \frac{4}{j}k_{2,2}, \cdots, \frac{4}{j}k_{j,2}) \tag{26}$$

and

$$a_i = k_{i,1}(\gamma_i^2)^{\frac{3}{4}} \quad i = 2, \cdots, n-1 \tag{27}$$

α_i is a bounded positive number, and we can write the above two cases ($|e_j| \geq \gamma_j$ and $|e_j| < \gamma_j$) in the same form as:

$$\dot{V}_j \leq -\acute{k}_{j,1}(V_j)^{\frac{3}{4}} - \acute{k}_{j,2}(V_j)^2 + \sum_{i=1}^{j} a_i \tag{28}$$

Step n.

Letting $e_n = x_n - \alpha_{n-1}$, and its time derivative yields

$$\dot{e}_n = \dot{x}_n - \dot{x}_{nd} = f(x) + \mathrm{g}(x)u - \dot{\alpha}_{n-1} + d(t) \tag{29}$$

Defining the difference between the dead zone input and output as $\triangle u$, i.e., $\triangle u = v - u$ and substituting $\triangle u$ into $\dot{e}_n$, we obtain

$$\dot{e}_n = f(x) + \mathrm{g}(x)(v - \triangle u) - \dot{\alpha}_{n-1} + d(t) \tag{30}$$

Considering the Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2}\frac{e_n^2}{\mathrm{g}(x)} \tag{31}$$

Substituting $\dot{e}_n$ into the time derivative of the Lyapunov function (31), we have

$$\dot{V}_n = \dot{V}_{n-1} + \frac{fe_n}{\mathrm{g}} + (v - \triangle u)e_n + \frac{e_n d(t)}{\mathrm{g}} - \frac{\dot{\alpha}_{n-1}e_n}{\mathrm{g}} - \frac{1}{2}\frac{\dot{\mathrm{g}}e_n^2}{\mathrm{g}^2} \tag{32}$$

Consider system (1) under Assumptions 1-5 and the dead zone input v is designed as

$$v = -k_{n,1}\frac{1}{e_n}\xi_{e_n} \cdot \mathrm{g}^{-\frac{3}{4}} - k_{n,2}e_n^3 \cdot \mathrm{g}^{-2} + \frac{\dot{\alpha}_{n-1} - f}{\mathrm{g}} + \frac{1}{2}\frac{\dot{\mathrm{g}}e_n}{\mathrm{g}^2} - \frac{e_n}{\mathrm{g}^2} - ke_n + \triangle u \tag{33}$$

and piecewise smooth function ξ_{e_n} is defined as

$$\xi_{e_n} = \begin{cases} (e_n^2)^{\frac{3}{4}} & \text{if } |e_n| \geq \gamma_n \\ e_n^2(\gamma_n^2)^{-\frac{1}{4}} & \text{Otherwise} \end{cases} \tag{34}$$

where $0 < \gamma_n, 1 < k$ and $k_{n,1}, k_{n,2} > 0$.

Substituting (33) and (34) into (32), due to $\frac{e_n d(t)}{g} - \frac{e_n^2}{g^2} \leq \frac{1}{4}|d(t)|^2 \leq \frac{1}{4}\overline{d}^2$, by lemmas 1-4, we obtain two cases

Case 1) ($|e_n| \geq \gamma_n$):

$$\dot{V}_n \leq -\acute{k}_{n,1}(V_n)^{\frac{3}{4}} - \acute{k}_{n,2}(V_n)^2 + \sum_{i=1}^{n-1} a_i + \frac{1}{4}\overline{d}^2 \tag{35}$$

Case 2) ($|e_n| < \gamma_n$):

$$\dot{V}_n \leq -\acute{k}_{n,1}(V_n)^{\frac{3}{4}} - \acute{k}_{n,2}(V_n)^2 + \sum_{i=1}^{n} a_i + \frac{1}{4}\overline{d}^2 \tag{36}$$

where

$$\acute{k}_{n,1} = \min(2^{\frac{3}{4}}k_{2,1}, 2^{\frac{3}{4}}k_{3,1}, \cdots, 2^{\frac{3}{4}}k_{n,1})$$
$$\acute{k}_{n,2} = \min(\frac{4}{n}k_{2,2}, \frac{4}{n}k_{3,2}, \cdots, \frac{4}{n}k_{n,2}) \tag{37}$$

and

$$a_n = k_{n,1}(\gamma_n^2)^{\frac{3}{4}}\mathrm{g}^{-\frac{3}{4}} \tag{38}$$

As to Assumption 2, a_n is a bounded positive number, and we can write the above two cases ($|e_n| \geq \gamma_n$ and $|e_n| < \gamma_n$) in the same form as:

$$\dot{V}_n \leq -\acute{k}_{n,1}(V_n)^{\frac{3}{4}} - \acute{k}_{n,2}(V_n)^2 + \sum_{i=1}^{n} a_i + \frac{1}{4}\overline{d}^2 \tag{39}$$

since $a_i, (i = 1, \cdots, n)$ are bounded positive constants and the orders of almost all actual systems are finite, the term $\sum_{i=1}^{n} \alpha_i$ is bounded by a positive constant.

However, the dead zone input v in (33) contains the terms of unknown system functions $f(x), \mathrm{g}(x)$, and unknown input deadzone effect Δu. In practice, these unknown terms make the control input unavailable for the system (1). Hence, we use neural networks to approximate the above unknown terms as follows:

$$\theta^T\psi(S) = -k_{n,1}\frac{1}{e_n}\xi_{e_n} \cdot \mathrm{g}^{-\frac{3}{4}} - k_{n,2}e_n^3 \cdot \mathrm{g}^{-2} + \frac{\dot{\alpha}_{n-1} - f}{\mathrm{g}} + \frac{1}{2}\frac{\dot{\mathrm{g}}e_n}{\mathrm{g}^2} - \frac{e_n}{\mathrm{g}^2} - \varepsilon_1 \tag{40}$$

$$\theta_d^T\psi(S_d) = \triangle u - \varepsilon_2 \tag{41}$$

where $\theta^T \in \mathbb{R}^l$ and $\theta_d^T \in \mathbb{R}^l$ are the ideal weights and ε_1 and ε_2 are approximation errors of the neural networks, l is the number of neural nodes. ε_1 and ε_2 satisfy $|\varepsilon_1| < \overline{\varepsilon}_1$ and $|\varepsilon_2| < \overline{\varepsilon}_2$, $\overline{\varepsilon}_1, \overline{\varepsilon}_2 > 0$ with $S, S_d \in \Omega_S$, where Ω_S is a compact set. $S = [x_1^T, x_2^T, \cdots, x_j^T, e_n], S_d = [x_1^T, x_2^T, \cdots, x_j^T, v]$ are the input of the neural networks. The basis function of the Radial Basis Function (RBF) $\psi(S) = [\psi_1(S), \cdots, \psi_l(S)] \in \mathbb{R}^l, \psi(S_d) = [\psi_1(S_d), \cdots, \psi_l(S_d)] \in \mathbb{R}^l$ are chosen as the Gaussian function of the form $\psi_i(S) = \exp(\frac{-(S-\varsigma_i)^T(S-\varsigma_i)}{\iota_i^2})$, $\psi_i(S_d) = \exp(\frac{-(S_d-\varsigma_i)^T(S_d-\varsigma_i)}{\iota_i^2}), i = 1, \cdots, l$, where $\varsigma_i = [\varsigma_{i1}, \cdots, \varsigma_{il}] \in \mathbb{R}^l$ is the center of the receptive field and ι_i is the width of the Gaussian function. In addition, from Ge et al. (2001), the norm of the vector function $\psi(\cdot)$ are bounded by the number of neural nodes, i.e., $\|\psi(\cdot)\|^2 < l$.

Then, dead zone input v in (33) is rewritten as:

$$v = -ke_n + \hat{\theta}^T\psi(S) + \hat{\theta}_d^T\psi(S_d) \tag{42}$$

where $\hat{\theta}^T\psi(S)$ and $\hat{\theta}_d^T\psi(S_d)$ approximate to the $\theta^T\psi(S)$ and $\theta_d^T\psi(S_d)$. And the approximation error of weights are defined as $\tilde{\theta} = \hat{\theta} - \theta, \tilde{\theta}_d = \hat{\theta}_d - \theta_d$.

Substituting (42) into (30), we have

$$\dot{e}_n = \mathrm{g}(\tilde{\theta}^T\psi(S) + \tilde{\theta}_d^T\psi(S_d) - k_{n,1}\frac{1}{e_n}\xi_{e_n} \cdot \mathrm{g}^{-\frac{3}{4}} - k_{n,2}e_n^3 \cdot g^{-2} + \frac{1}{2}\frac{\dot{\mathrm{g}}e_n}{\mathrm{g}^2} - \frac{e_n}{\mathrm{g}^2} + \frac{d(t)}{\mathrm{g}} - ke_n - \varepsilon_1 - \varepsilon_2) \tag{43}$$

Then, we reselect the Lyapunov function V_n of the form

$$V_n = V_{n-1} + \frac{1}{2}\frac{e_n^2}{\mathrm{g}} + \frac{1}{2}\tilde{\theta}^T\Phi^{-1}\tilde{\theta} + \frac{1}{2}\tilde{\theta}_d^T\Phi_d^{-1}\tilde{\theta}_d^T \tag{44}$$

and the novel online updating laws of the NN weights are designed as

$$\dot{\hat{\theta}} = -\Phi(\psi(S)e_n + \sigma_1\hat{\theta} \circ \hat{\theta} \circ \hat{\theta} + \sigma_2\hat{\theta}) \tag{45}$$

$$\dot{\hat{\theta}}_d = -\Phi_d(\psi(S_d)e_n + \sigma_{d1}\hat{\theta}_d \circ \hat{\theta}_d \circ \hat{\theta}_d + \sigma_{d2}\hat{\theta}_d) \tag{46}$$

where $\sigma_1, \sigma_2, \sigma_{d1}, \sigma_{d2} > 0$ and Φ, Φ_d are positive definite matrices. The symbol '$\circ$' is hadamard product operator.

Making the time derivative of the Lyapunov function (44) and substituting (43) and (45) - (46)into it, we can obtain

$$\begin{aligned}\dot{V}_n \leq &- \acute{k}_{n-1,1}(V_{n-1})^{\frac{3}{4}} - \acute{k}_{n-1,2}(V_{n-1})^2 - k_{n,1}(e_n^2)^{\frac{3}{4}}\mathrm{g}^{-\frac{3}{4}} \\ &- k_{n,2}e_n^4\mathrm{g}^{-2} - \sigma_1\tilde{\theta}^T(\hat{\theta} \circ \hat{\theta} \circ \hat{\theta}) - \sigma_2\tilde{\theta}^T\hat{\theta} \\ &- \sigma_{d1}\tilde{\theta}_d^T(\hat{\theta}_d \circ \hat{\theta}_d \circ \hat{\theta}_d) - \sigma_{d2}\tilde{\theta}_d^T\hat{\theta}_d \\ &- e_n(\varepsilon_1 + \varepsilon_2) + \frac{e_n d(t)}{\mathrm{g}} - \frac{e_n^2}{\mathrm{g}^2} - ke_n^2 + \sum_{i=1}^{n} a_i \end{aligned} \tag{47}$$

Next, we will further process the terms that with the weight of neural networks in (47).
For the term $-\sigma_1\tilde{\theta}^T(\hat{\theta}\circ\hat{\theta}\circ\hat{\theta})$ in (47), by Lemma 5 we have

$$-\sigma_1\tilde{\theta}^T(\hat{\theta}\circ\hat{\theta}\circ\hat{\theta}) = -\sigma_1\tilde{\theta}^{3T}\tilde{\theta} - 3\sigma_1\tilde{\theta}^{3T}\theta - 3\sigma_1\tilde{\theta}^{2T}\theta^2 - \sigma_1\tilde{\theta}^T\theta^3 \quad (48)$$

For $-3\sigma_1\tilde{\theta}^{3T}\theta$ and $-\sigma_1\tilde{\theta}^T\theta^3$in (48), using Lemma 6, we have

$$-3\sigma_1\tilde{\theta}^{3T}\theta \leq \frac{9\sigma_1\delta^{\frac{4}{3}}}{4}\tilde{\theta}^{3T}\tilde{\theta} + \frac{3\sigma_1}{4\delta^4}(\theta^2)^T\theta^2 \quad (49)$$

$$-\sigma_1\tilde{\theta}^T\theta^3 \leq 3\sigma_1\tilde{\theta}^{2T}\theta^2 + \frac{\sigma_1}{12}(\theta^2)^T\theta^2 \quad (50)$$

For $-\sigma_1\tilde{\theta}^{3T}\tilde{\theta}$ in (48), using Lemma 2, we have

$$-\sigma_1\tilde{\theta}^{3T}\tilde{\theta} \leq -\frac{\sigma_1}{l\lambda^2_{\max}(\Phi^{-1})}(\tilde{\theta}^T\Phi^{-1}\tilde{\theta})^2 \quad (51)$$

Then, substituting (49)-(51) into (48) and choosing $0 < \delta < (\frac{2}{3})^{\frac{3}{2}}$, we can transform (48) into

$$-\sigma_1\tilde{\theta}^T(\hat{\theta}\circ\hat{\theta}\circ\hat{\theta}) \leq \frac{4\sigma_1 - 9\sigma_1\delta^{\frac{4}{3}}}{l\lambda^2_{\max}(\Phi^{-1})}(\frac{1}{2}\tilde{\theta}^T\Phi^{-1}\tilde{\theta})^2 + (\frac{3\sigma_1}{4\delta^4} + \frac{\sigma_1}{12})(\theta^2)^T\theta^2 \quad (52)$$

Note that the term $-\sigma_{d1}\tilde{\theta}_d^T(\hat{\theta}_d\circ\hat{\theta}_d\circ\hat{\theta}_d)$ in (47) has the same structure as $-\sigma_1\tilde{\theta}^T(\hat{\theta}\circ\hat{\theta}\circ\hat{\theta})$, therefore, we can also get

$$-\sigma_{d1}\tilde{\theta}_d^T(\hat{\theta}_d\circ\hat{\theta}_d\circ\hat{\theta}_d) \leq \frac{4\sigma_{d1} - 9\sigma_{d1}\delta_d^{\frac{4}{3}}}{l\lambda^2_{\max}(\Phi_d^{-1})}(\frac{1}{2}\tilde{\theta}_d^{\ T}\Phi_d^{-1}\tilde{\theta}_d)^2 + (\frac{3\sigma_{d1}}{4\delta_d^4} + \frac{\sigma_{d1}}{12})(\theta_d^2)^T\theta_d^2 \quad (53)$$

For the term $-\sigma_2\tilde{\theta}^T\hat{\theta}$ in (47), we can get

$$-\sigma_2\tilde{\theta}^T\hat{\theta} \leq -\frac{1}{2}\sigma_2\tilde{\theta}^T\tilde{\theta} + \frac{1}{2}\sigma_2\theta^T\theta \quad (54)$$

Furthermore, for $-\frac{1}{2}\sigma_2\tilde{\theta}^T\tilde{\theta}$ in (54), we have

$$-\frac{1}{2}\sigma_2\tilde{\theta}^T\tilde{\theta} = -\frac{1}{4}\sigma_2\tilde{\theta}^T\tilde{\theta} - \sigma_2(\tilde{\theta}^T\tilde{\theta})^{\frac{3}{4}} + \sigma_2(\tilde{\theta}^T\tilde{\theta})^{\frac{1}{2}} - \frac{1}{4}\sigma_2\left[(\tilde{\theta}^T\tilde{\theta})^{\frac{1}{2}} - 2(\tilde{\theta}^T\tilde{\theta})^{\frac{1}{4}}\right]^2 \quad (55)$$

and

$$\sigma_2(\tilde{\theta}^T\tilde{\theta})^{\frac{1}{2}} \leq \frac{\sigma_2}{8}\tilde{\theta}^T\tilde{\theta} + 2\sigma_2 \quad (56)$$

Then, substituting (55) and (56) into (54), we can transform (54) into

$$-\sigma_2\tilde{\theta}^T\hat{\theta} \leq -\sigma_2(\frac{2}{\lambda_{\max}(\Phi^{-1})})^{\frac{3}{4}}(\frac{1}{2}\tilde{\theta}^T\Phi^{-1}\tilde{\theta})^{\frac{3}{4}} + \frac{1}{2}\sigma_2\theta^T\theta + 2\sigma_2 \quad (57)$$

Note that the term $-\sigma_{d2}\tilde{\theta}_d^T\hat{\theta}_d$ (47) has the same structure as $-\sigma_2\tilde{\theta}^T\hat{\theta}$, therefore, we can also get

$$-\sigma_{d2}\tilde{\theta}_d^T\hat{\theta}_d \leq -\sigma_{d2}(\frac{2}{\lambda_{\max}(\Phi_d^{-1})})^{\frac{3}{4}}(\frac{1}{2}\tilde{\theta}_d^T\Phi_d^{-1}\tilde{\theta}_d)^{\frac{3}{4}} + \frac{1}{2}\sigma_{d2}\theta_d^T\theta_d + 2\sigma_{d2} \quad (58)$$

For $-e_n(\varepsilon_1+\varepsilon_2) + \frac{e_n d(t)}{g} - \frac{e_n^2}{g^2}$ in (47), it is easy to obtain $-e_n(\varepsilon_1+\varepsilon_2) \leq e_n^2 + \frac{1}{2}(\varepsilon_1^2+\varepsilon_2^2)$ and $\frac{e_n d(t)}{g} - \frac{e_n^2}{g^2} \leq \frac{1}{4}|d(t)|^2 \leq \frac{1}{4}\overline{d}^2$, therefore, we have

$$-e_n(\varepsilon_1+\varepsilon_2) + \frac{e_n d(t)}{g} - \frac{e_n^2}{g^2} \leq e_n^2 + \frac{1}{2}(\overline{\varepsilon}_1^2 + \overline{\varepsilon}_2^2) + \frac{1}{4}\overline{d}^2 \quad (59)$$

Substituting (52)-(53), (57)-(58) and (59) into (47) yields

$$\dot{V}_n = -k'_{n-1,1}(V_{n-1})^{\frac{3}{4}} - k'_{n-1,2}(V_{n-1})^2 - k_{n,1}(e_n^2)^{\frac{3}{4}}\mathrm{g}^{-\frac{3}{4}} - k_{n,2}e_n^4\mathrm{g}^{-2} - \kappa_1(\frac{1}{2}\tilde{\theta}^T\Phi^{-1}\tilde{\theta})^2 - \rho_1(\frac{1}{2}\tilde{\theta}^T\Phi^{-1}\tilde{\theta})^{\frac{3}{4}} - \kappa_{d1}(\frac{1}{2}\tilde{\theta}_d^T\Phi_d^{-1}\tilde{\theta}_d^T)^2 - \rho_{d1}(\frac{1}{2}\tilde{\theta}_d^T\Phi_d^{-1}\tilde{\theta}_d)^{\frac{3}{4}} + C \quad (60)$$

where

$$C = \kappa_2(\theta^2)^T\theta^2 + \kappa_{d2}(\theta_d^2)^T\theta_d^2 + \frac{1}{2}\rho_2\theta^T\theta + \frac{1}{2}\rho_{d2}(\theta_d)^T\theta_d + \frac{1}{2}(\overline{\varepsilon}_1^2 + \overline{\varepsilon}_2^2) + \frac{1}{4}\overline{d}^2 + \sum_{i=1}^{n} a_i \quad (61)$$

Using lemma 1 and Lemma 2, we have

$$\dot{V}_n \leq -\zeta_1(V_n)^{\frac{3}{4}} - \zeta_2(V_n)^2 + C \quad (62)$$

where

$$\zeta_1 = \min(2^{\frac{3}{4}}k_{1,1}, 2^{\frac{3}{4}}k_{2,1}, \cdots, 2^{\frac{3}{4}}k_{n,1}, \rho_1, \rho_{d1}) \quad (63)$$

$$\zeta_2 = \min(\frac{4}{n+2}k_{1,2}, \frac{4}{n+2}k_{2,2}, \cdots, \frac{4}{n+2}k_{n,2}, \frac{1}{n+2}\kappa_1, \frac{1}{n+2}\kappa_{d1}) \quad (64)$$

and $\kappa_1 = \frac{4\sigma_1 - 9\sigma_1\delta^{\frac{4}{3}}}{l\lambda^2_{\max}(\Phi^{-1})}$, $\kappa_2 = \frac{3\sigma_1}{4\delta^4} + \frac{\sigma_1}{12}$, $\kappa_{d1} = \frac{4\sigma_{d1} - 9\sigma_{d1}\delta_d^{\frac{4}{3}}}{l\lambda^2_{\max}(\Phi_d^{-1})}$, $\kappa_{d2} = \frac{3\sigma_{d1}}{4\delta_d^4} + \frac{\sigma_{d1}}{12}$, $0 < \delta < (\frac{2}{3})^{\frac{3}{2}}$, $0 < \delta_d < (\frac{2}{3})^{\frac{3}{2}}$, $\rho_1 = \sigma_2(\frac{2}{\lambda_{\max}(\Phi^{-1})})^{\frac{3}{4}}$, $\rho_2 = 2\sigma_2 + \frac{1}{2}\theta^T\theta$, $\rho_{d1} = \sigma_{d2}(\frac{2}{\lambda_{\max}(\Phi_d^{-1})})^{\frac{3}{4}}$, $\rho_{d2} = 2\sigma_{d2} + \frac{1}{2}\theta_d^T\theta_d$.

Theorem 1. Under Assumptions 1-4, with the virtual controllers (13) and (20), dead zone input (42), and the online updating laws of neural network weights (45) and (46) which are applied in the nonlinear affine system (1), the following results hold.. (a)The output of system will not exceed preset output constraints. (b) All the signals of the closed-loop system are bounded.(c) The output tracking error e_1 will converge into the sets Θ_{e_1}, defined by

$$\Theta_{e_1} : \left\{ e_1 \in \mathbb{R} \big| |e_1| \leq \sqrt{\frac{\mu^2(e^{2\bar{C}} - 1)}{e^{2\bar{C}}}} \right\} \quad (65)$$

in fixed time T_{fd}

$$T_{fd} \leq T_{\max} = \frac{4}{\zeta_1} + \frac{1}{\zeta_2(1-\varpi)} \quad (66)$$

where $\bar{C} = \sqrt{\frac{C}{\varpi\zeta_2}}$ and $0 < \varpi < 1$.

Proof. (a) For inequality (62), if $V_n^2 \geq \frac{C}{\zeta_2}$, we have $\dot{V}_n \leq -\zeta_1(V_n)^{\frac{3}{4}} < 0$. Therefore, the Lyapunov function V_n is bounded. This makes the barrier Lyapunov function (15) bounded, thus output tracking error e_1 is bounded by constraint μ. By Assumption 1, system output is bouned by $\mu + \mathrm{y}_{db}$.
(b) From (a), we know $V_i (i = 1\cdots n-1)$ are bounded due to the boundedness of V_n, therefore the error e_i are bounded. The piecewise smooth function ξ_{e_i} are continuous, which lead the virtual controllers α_i to be continuous

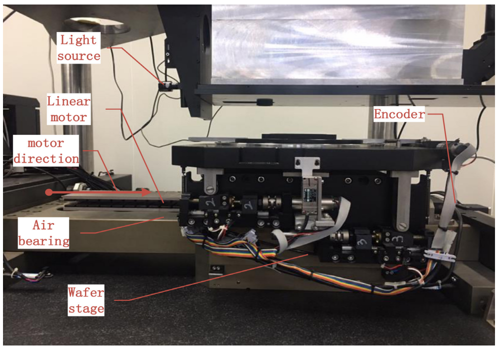

Fig. 1. The lithography machine system.

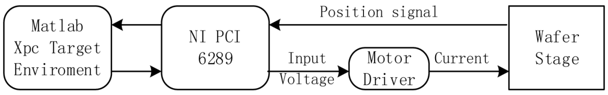

Fig. 2. Schematic diagram of experiment setup.

and bounded. Moreover, according to (44), the weights of neural network $\hat{\theta}, \hat{\theta}_d$ are also bounded due to the boundedness of V_n. The boundedness of input of the dead zone is guaranteed by the boundedness of the weights of neural network and the Gaussian function $\psi(S), \psi(S_d)$. Further, it can easily explain that all the other signals in the closed-loop system are also bounded.
(c)When there exist a positive constant $0 < \varpi < 1$ which satisfying $C \leq \varpi V_n^2 \zeta_2$. From (62), we have

$$\dot{V}_n \leq -\zeta_1(V_n)^{\frac{3}{4}} - (1-\varpi)\zeta_2(V_n)^2 \quad (67)$$

Based on Lemma 7, V_n will converge to the set $\{V_n \leq \sqrt{\frac{C}{\varpi\zeta_2}}\}$ within the setting time estimated in (66). This guarantees that $V_1 = \frac{1}{2}\ln\frac{\mu^2}{\mu^2-e_1^2} \leq \frac{C}{\varpi\zeta_2}$. Then, tracking error e_1 will converge into the set Θ_{e_1} in fixed time T_{fd}. ■

4. EXPERIMENT

The good performance of the proposed methods will be shown by the lithography machine system which driven by the linear motor in this section.

4.1 Nonlinear model of linear motor

A nonlinear affine system model in Tan et al. (2001) can be developed to describe the dynamics of the linear motor.

$$\begin{aligned}\dot{x} &= v\\ \dot{v} &= \frac{u - f_{friction} - f_{ripple}}{m} + \omega(t)\\ \mathrm{y} &= x \end{aligned}\quad (68)$$

where $f_{friction}$ is the friction force, f_{ripple} is the ripple force, m is the mass of load, u is the developed force, and $\omega(t)$ represents unknown external disturbances. It is worth noting that the motor has the unknown input dead zone which means that the motor cannot work when the input is within the breakpoints of the dead zone.

4.2 Experiment Setup

The lithography machine system is displayed in Fig. 1. The wafer stage driven by the linear motor (UM, Tecnotion) is assembled on the air bearing. Position of the wafer stage is acquired by the NI data acquisition system (NI PCI 6289), which measured by the grating ruler (Mercury II 5000, Micro-E) with the resolution of $5\mu m$. The control input is generated by the MATLAB-XPC-Target system in Fig. 2, which transmitted to drive the liner motor through the data acquisition system and motor driver. The sampling frequency is selected as 1000Hz.

4.3 Fixed-time State Feedback Control Based RBFNN Experiment Implementation

The effectiveness of state feedback control based RBFNN with output constraint will be verified by linear motor experiment implementation. Before experiment, the relevant parameters selection are set as follows, the number of neural nodes is set as $l = 2^4$, the center of activation function $\psi(\cdot)$ is selected in the area of $[-1,1]\times[-1,1]\times[-1,1]\times[-1,1]$. Initial weights of the neural network are set as $\hat{\theta}(0) = \hat{\theta}_d(0) = [0,\ldots,0]^T \in \mathbb{R}^{2^4}$. Then, the simulation time is set as $t = 20s$, and the controller parameters are chosen as $k_{1,1} = 20, k_{1,2} = 5, k = 10, \gamma_1 = 0.01, \Phi = 100I_{16\times16}, \Phi_d = I_{16\times16}, \sigma_2 = \sigma_{d2} = \sigma_1 = \sigma_{d1} = 0.001$. The constraint selected as $\mu = 0.1$. Based on the above parameters selection, we set desired trajectory and initial values as: $\mathrm{y}_d = \sin(\frac{t}{2}) + 0.05$, $x(0) = [0.01, 0]^T$.

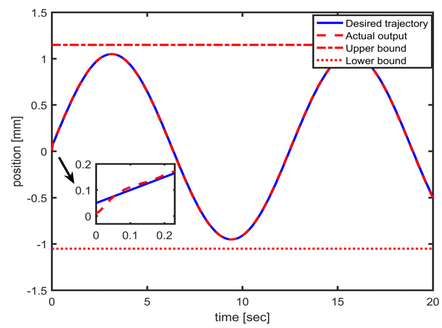

Fig. 3. Actual outputs and reference trajectory under fixed-time state feedback control based RBFNN.

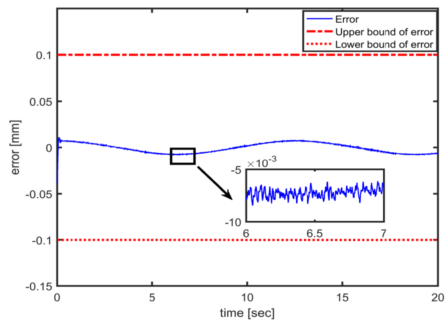

Fig. 4. Tracking errors $e_1 = x_1 - \mathrm{y}_d$ under fixed-time state feedback control based RBFNN.

The detailed experiment results are given in Fig. 3 - Fig. 6. In Fig. 3, the wafer stage can track the desired trajectory well without exceeding the preset output constraint through the fixed-time adaptive neural network controller designed in this paper. Fig. 4 shows that the tracking error $e_1 = x_1 - \mathrm{y}_d$ converging into a small neighborhood of desired trajectory y_d within the setting time. The variation of

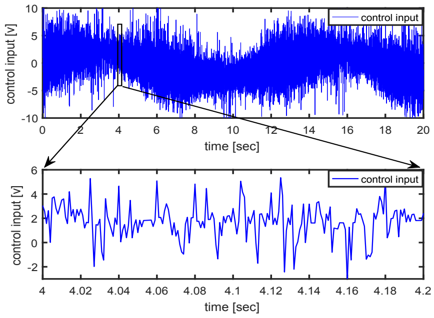

Fig. 5. Control input under fixed-time state feedback control based RBFNN.

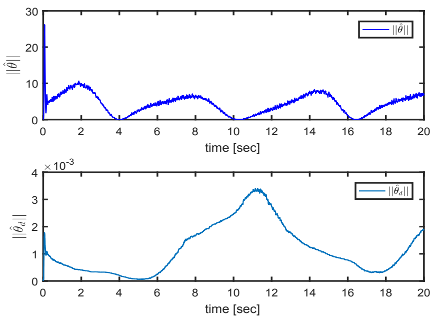

Fig. 6. Euclidean norm $\|\hat{\theta}\|, \|\hat{\theta}_d\|$ under fixed-time state feedback control based RBFNN.

control input and Euclidean norm of the RBFNN weights $\hat{\theta}, \hat{\theta}_d$ are shown in Fig. 5 - Fig. 6.

5. CONCLUSION

In this work, the fixed-time control problem for an unknown nonlinear affine system with unknown input dead zone, external disturbance and output constraints has been addressed. A novel fixed-time adaptive neural network control scheme has been proposed, which combines a log-type BLF with backstepping design method to obtain virtual controllers and dead zone input. Then, RBFNNs are utilized to compensate for unknown dead zone effect and deal with system uncertainties. On this basis, a new online weight updating algorithm of NNs has been designed. At last, the experiment of lithography machine has shown the proposed method can ensure a good performance of the system under the constraints of output and unknown input dead zone.

Appendix A. PROOF OF LEMMA 6

Before the proof, we define $x = [\varphi_1, \varphi_2, \cdots, \varphi_m]^T \in \mathbb{R}^m$ and $y = [\nu_1, \nu_2, \cdots, \nu_m]^T \in \mathbb{R}^m$.
Then, for the term $-x^{3T}y$, using Lemma 3, we can obtain

$$-x^{3T}y \leq \frac{3\delta^{\frac{4}{3}}}{4}\sum_{i=1}^{m}\varphi_i^4 + \frac{1}{4\delta^4}\sum_{i=1}^{m}\nu_i^4 \tag{A.1}$$

For the term $-x^T y^3$, we can get

$$-x^T y^3 \leq 3\sum_{i=1}^{m}\varphi_i^2\nu_i^2 + \frac{1}{12}\sum_{i=1}^{m}\nu_i^4 \tag{A.2}$$

Then, by (A.1) and (A.2), one can get Lemma 6.

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