

Distributed Stabilization of Interconnected Multiagent Systems using Structurally Nonsymmetric Control Layers

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Abstract: Recently, graph theoretic distributed protocols have been introduced for the stabilization of interconnected multiagent systems with separate agent and control layers. For the case of unstable local agent dynamics, the existing results focus on only the matched interconnections. Further, except for a multiagent system of first- and second-order agents, the existing results are limited to the structurally symmetric control layers based on the undirected communication among controllers. We aim to relax these restrictions for multiagent systems with partially known unmatched or matched interconnections. We propose two step-by-step procedures to design robust distributed stabilization gains for the candidate nonsymmetric control layers in the presence of agent- and multiagent system-level modeling uncertainties. Combined with an optimal control formulation, we develop a matrix algebraic approach for the unmatched scenario and a Lyapunov-based approach for the matched case. In each case, we prove that all state trajectories of the two-layer interconnected multiagent system exponentially converge to the origin. We examine the feasibility of the proposed ideas in simulation.

Keywords: Distributed control, cooperative control, multiagent systems.

1. INTRODUCTION

In response to the advances in embedded sensing, communication, and computation technologies, graph theoretic approaches have received significant attention for distributed consensus in multiagent systems (MASs). Initial research work was mainly centered around simple agent models such as single, double, and high-order integrators to understand and build a relationship between graph and control theories Olfati-Saber et al. (2007). More complicated modeling scenarios are attracting attention in recent years. For example, Ai et al. (2017) studied consensus in the presence of local (agent-level) modeling uncertainties where each individual agent's dynamics depend on its own variables, Oh et al. (2014) investigated the average consensus problem in an MAS of interconnected agents with linear time-invariant dynamics, and Tuna (2016) discussed a consensus problem for a completely known interconnected MAS using a matrix-weighted Laplacian viewpoint.

Parallel to the research on distributed consensus, Rieger et al. (2013) proposed the concept of multilayer control based on the graph theoretic approaches. Focusing on the physical and execution layers in the work of Rieger et al. (2013), Egerstedt (2015) discussed that the architectural aspect of cyber-physical systems could be captured using graph theoretic approaches. The idea, however, is limited to a completely known interconnected MAS of single integrator agents while the robustness with respect to the (modeling) uncertainties is a critical issue for multilayer control systems (Dahleh and Rinehart, 2011).

Rezaei and Stefanovic (2016) proposed a distributed decoupling protocol for interconnected MASs with unknown interconnection nonlinearities in the state space domain. The method, however, was limited to known interconnection topolo-

gies and locally stable agent dynamics (i.e., by ignoring the interconnection terms of the state space model). Since the control communication protocol is the same as the agent layer interconnection topology, it cannot fully capture the architectural aspect of cyber-physical systems as discussed above. Rezaei and Stefanovic (2017a) revised the approach such that the architectural aspects of cyber-physical systems could be captured; however, the idea is limited to a special interconnection topology. In another effort, Rezaei and Stefanovic (2019a) proposed a distributed stabilization protocol for interconnected MASs without those restrictions. It can be used to model a cyber-physical system with separate agent and control layer topologies; however, that solution approach is based on a structurally symmetric control layer topology and the matched interconnection scenario (defined in Sections 2 and 3, respectively). In the work of Rezaei and Stefanovic (2019b), the authors developed a structurally nonsymmetric control layer topology using a Lyapunov-based formulation. The result, however, is limited to the interconnected MASs of single and double integrator agents. (We refer to Rezaei and Stefanovic (2019a,b) for a more comprehensive literature survey on this topic.)

In this paper, we develop a two-layer (closed-loop) interconnected MAS with separate agent- and control-layer topologies. In a step-by-step manner, we propose two mixed graph and optimal control design procedures to design structurally nonsymmetric control layers and local stabilization gains for MASs with either unmatched and matched interconnections over the agent layer. In the unmatched scenario, we propose a matrix algebraic formulation which relies on the inherent properties of M -matrices and, in the matched scenario, a Lyapunov-based formulation which uses the existing flexibility in the Lyapunov stability analysis of the final design. These ideas are interchangeable between the unmatched and matched scenarios, and

we prove the proposed nonsymmetric distributed protocols are able to steer all state trajectories of the two-layer interconnected MAS to the origin despite the agent- and MAS-level modeling uncertainties.

We provide the required preliminary definitions in Section 2, the main results in Section 3, a discussion on the proposed ideas in Section 4, simulation studies in Section 5, and concluding remarks in Section 6.

2. PRELIMINARIES

We mainly follow standard notation. The symbol $\mathbf{0}$ refers to a matrix of all zeros with appropriate dimension, $\text{diag}\{\cdot\}$ a (block) diagonal matrix of the elements in $\{\cdot\}$, and $\text{col}\{x_i\}$ an aggregated column vector of x_i for all i belonging to a given set. $\|\cdot\|$ denotes the (induced) 2-norm of the (matrix) vector given as its input argument.

A directed graph or digraph \mathcal{G} is a collection of nodes \mathcal{V} and directed edges $\mathcal{E} : \mathcal{V} \times \mathcal{V}$ which can be characterized by its adjacency matrix \mathcal{A} or Laplacian matrix \mathcal{L} whose definitions can be found in the standard texts (Mesbahi and Egerstedt, 2010). We consider two graph topologies: \mathcal{G}_a to represent the physical interaction of agents' dynamics over the agent layer, and \mathcal{G}_c to model the communication (information flow) between controllers over the control layer. We allow the existence of selfloops over both layers where, by a selfloop, we refer to an edge outgoing from and returning to the same node without passing through any other nodes. Since the standard definitions of adjacency and Laplacian matrices do not admit selfloops, we redefine them in the rest of this section and use the results in the derivations of Section 3.

An agent layer digraph \mathcal{G}_a with N nodes is characterized by an adjacency matrix $\mathcal{A}_a = [a_{ij}^a] \in \mathbb{R}^{N \times N}$ where $a_{ij}^a \neq 0$ if the i^{th} agent is affected by the j^{th} agent's dynamics for $i, j \in \{1, 2, \dots, N\}$, and $a_{ij}^a = 0$ otherwise. Note that, different from the standard definitions, $j = i$ is acceptable and each a_{ij}^a is a real valued scalar with either positive or negative sign. The set \mathcal{N}_i^a indicates the i^{th} agent's neighbors over \mathcal{G}_a which may include the number i as well (selfloop).

A control layer digraph \mathcal{G}_c with N nodes is characterized by a modified Laplacian matrix $\mathcal{H}_c = \mathcal{L}_c + \mathcal{S}_c \in \mathbb{R}^{N \times N}$. $\mathcal{L}_c \in \mathbb{R}^{N \times N}$ is a standard Laplacian matrix of a digraph \mathcal{G}'_c obtained by removing all selfloops: $\mathcal{L}_{ij}^c = -a_{ij}^c$ and $\mathcal{L}_{ii}^c = \sum_{j \in \mathcal{N}_i^c} a_{ij}^c$ with positive edge weights a_{ij}^c (and no selfloop). The set \mathcal{N}_i^c characterizes the i^{th} agent's (controller) neighbors over \mathcal{G}_c , excluding the number i (selfloop). $\mathcal{S}_c = \text{diag}\{s_i^c\} \in \mathbb{R}^{N \times N}$ is a diagonal matrix representing the selfloops where $s_i^c > 0$ when there is a selfloop around the i^{th} controller, and $s_i^c = 0$ otherwise. These selfloops and directed one-way communications between the control nodes create a structurally nonsymmetric control layer to be discussed in Section 3, visually understood in the simulation setup of Section 5.

Both \mathcal{G}_a and \mathcal{G}_c can be disconnected; however, all control nodes in each connected component of \mathcal{G}_c must have access (direct path) to at least one node with a selfloop. For a structurally symmetric scenario, i.e., an undirected graph $\mathcal{G}_c^{\text{ss}}$ (instead of \mathcal{G}'_c) with selfloops, all eigenvalues are strictly positive real-valued scalars (Rezaei and Stefanovic, 2019a). In Section 3, we assume that the agent layer owner shares only the scalar $\|\mathcal{A}_a\|$ with the control layer designer such that, possibly by intention,

the detailed information of the agent layer interconnection topology is kept confidential. We use \mathcal{G}_c (and \mathcal{H}_c) as a design degree of freedom to be discussed in Section 3.

3. MAIN RESULTS

We consider the following interconnected MAS:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + \phi_i(\eta_i(t), t) \\ \eta_i(t) &= C_{\eta i} \sum_{j \in \mathcal{N}_i^a} a_{ij}^a x_j(t) \end{aligned} \quad (1)$$

where $i \in \{1, 2, \dots, N\}$ denotes the agent number, $x_i \in \mathbb{R}^{n_x}$ the i^{th} agent's state variable, $u_i \in \mathbb{R}^{n_u}$ control input, $\eta_i \in \mathbb{R}^{n_\eta}$ interconnection's state variable, and (A, B) a pair of stabilizable system matrices with compatible dimensions. As discussed at the end of Section 2, the positive scalar $\|\mathcal{A}_a\|$ is shared with the control layer design; however, the agent layer topology (both structure and edge weights) of \mathcal{G}_a is kept confidential. The nonlinear functions $\phi_i : \mathbb{R}^{n_\eta} \times \mathbb{R}^+ \rightarrow \mathbb{R}^{n_x}$ and interconnection allocation matrices $C_{\eta i}$ are discussed in the next two paragraphs.

In the *unmatched scenario*, the interconnections are not in the range space of the control input matrix, $\phi_i = B_g g_i(y_i, t)$, and the agent model (1) is rewritten as follows:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + B_g g_i(y_i(t), t) \\ y_i(t) &= C_{y i} \sum_{j \in \mathcal{N}_i^a} a_{ij}^a x_j \end{aligned} \quad (2)$$

which recovers the (non-specified) model (1) for $B_g = I_{n_x}$.

In the *matched scenario*, the interconnections are in the range space of the control input matrix, $\phi_i = B f_i(z_i, t)$, and the agent model (1) is rewritten as follows:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B(u_i(t) + f_i(z_i(t), t)) \\ z_i(t) &= C_{z i} \sum_{j \in \mathcal{N}_i^a} a_{ij}^a x_j \end{aligned} \quad (3)$$

The agent models (2) and (3) satisfy the following assumption.

Assumption 1. The functions f_i and g_i satisfy $f_i(\mathbf{0}, t) = \mathbf{0}$ and $g_i(\mathbf{0}, t) = \mathbf{0}$, are Lipschitz (with unknown Lipschitz constants), norm bounded $f_i^T(z_i, t)f_i(z_i, t) \leq \gamma_{fi} z_i^T z_i$ and $g_i^T(y_i, t)g_i(y_i, t) \leq \gamma_{gi} y_i^T y_i$ with scalars $\gamma_f = \max_i(\gamma_{fi})$ and $\gamma_g = \max_i(\gamma_{gi})$ known by the control designer. The interconnection allocation matrices $C_{z i}$ and $C_{y i}$ are unknown, and satisfy the norm conditions $\|C_{z i}\|^2 \leq \gamma_{cz i}$ and $\|C_{y i}\|^2 \leq \gamma_{cy i}$ with scalars $\gamma_{cz} = \max_i(\gamma_{cz i})$ and $\gamma_{cy} = \max_i(\gamma_{cy i})$ known by the control designer.

We note that the proposed formulation is capable of simultaneously modeling both agent- and MAS-level uncertainties. The former is characterized by selfloops of \mathcal{G}_a , i.e., when $i \in \mathcal{N}_i^a$, and the latter by the edges of \mathcal{G}_a , i.e., when $j \neq i$ and $j \in \mathcal{N}_i^a$.

We consider an exponential *stabilization objective* (convergence of all trajectories to the origin):

$$\lim_{t \rightarrow \infty} x_i(t) = \mathbf{0} \quad \forall i \in \{1, 2, \dots, N\} \quad (4)$$

to be achieved using only a few agents' absolute measurements, and despite the lack of detailed knowledge about the interconnection topology and type of nonlinearities. Note that neither the distributed decoupling idea of Rezaei and Stefanovic (2016) (limited to the Hurwitz A and known \mathcal{G}_a) nor the distributed stabilization idea of Rezaei and Stefanovic (2019a) (limited to the matched scenario) is applicable to the distributed stabilization problem (4) of this paper. This is because the control layer \mathcal{G}_c is structurally nonsymmetric. Similarly, the idea of Rezaei and Stefanovic (2019b) is focused on the design of \mathcal{G}_c for an MAS of only first- and second-order agents.

In Subsection 3.1, we lay a foundation for the distributed implementation of centralized, cooperative, and decentralized stabilization protocols using a highly flexible graph theoretic formulation. In Subsection 3.2, we propose a step-by-step matrix algebraic approach to obtain the stabilization gain for a candidate control layer topology in the unmatched scenario. In Subsection 3.3, we propose a step-by-step Lyapunov-based approach to obtain the stabilization gain for a candidate control layer topology in the matched scenario.

3.1 Foundation: A graph theoretic formulation

We propose a distributed stabilization protocol:

$$u_i(t) = K \left(\sum_{j \in \mathcal{N}_i^c} a_{ij}^c (x_i - x_j) + s_i^c x_i \right) \quad (5)$$

where the structurally nonsymmetric control layer topology \mathcal{G}_c (neighbor sets \mathcal{N}_i^c and weighting scalars $a_{ij}^c, s_i^c \geq 0$) and the stabilization gain $K \in \mathbb{R}^{n_u \times n_x}$ are two design degrees of freedom.

Relevant to the configurations in the literature of large-scale systems (e.g., Lunze (1992)), we use the control layer topology \mathcal{G}_c as an MAS-level design degree of freedom in order to customize the control configuration depending on the available type of measurements, in a distributed fashion:

- *Decentralized control*: $s_i^c > 0$ and $\mathcal{N}_i^c = \emptyset$ for all $i \in \{1, 2, \dots, N\}$.
- *Cooperative control*: $s_i^c > 0$ for a few i , and \mathcal{N}_i^c are assigned such that each control node has a direct path to a control node with a selfloop.
- *Centralized control*: $s_i^c > 0$ for all i and \mathcal{N}_i^c representing a complete digraph, i.e., $\mathcal{N}_i^c = \{1, 2, \dots, N\} \setminus \{i\}$.

In this paper, the “type” of measurements refers to the agent owner’s willingness to directly contribute toward the stabilization of the interconnected MAS. In particular, an $s_i^c > 0$ means the i^{th} agent shares its absolute state measurement with the control operator and an $s_i^c = 0$ the agent’s controller operates based only on the relative information with respect to its neighbors over \mathcal{G}_c . The relative measurement means the control layer operator does not have the permission to use the absolute measurement of the considered agent. This is different from the traditional applications of distributed consensus, e.g., unmanned vehicles, where the relative measurement is a consequence of using distance measurement sensors.

A challenge arises due to the fact that the same distributed stabilization protocol (5) must be used under both unmatched and matched scenarios (Equations (2) and (3), respectively). In Subsection 3.2, we propose a step-by-step procedure to design a candidate K and the associated \mathcal{G}_c under the unmatched scenario. In Subsection 3.3, we propose another step-by-step approach to design \mathcal{G}_c and K under the matched scenario. The procedures are interchangeable between the two scenarios.

3.2 Design and analysis: Unmatched interconnection

We start this subsection by aggregating the agent models (2), with unmatched interconnections:

$$\dot{x} = \bar{A}x + \bar{B}u + \bar{B}_g g(y) \quad \text{and} \quad y = C_y (\mathcal{A}_a \otimes I_{n_x}) x \quad (6)$$

where $x = \text{col}\{x_i\} \in \mathbb{R}^{Nn_x}$, $u = \text{col}\{u_i\} \in \mathbb{R}^{Nn_u}$, $y = \text{col}\{y_i\} \in \mathbb{R}^{Nn_y}$, $\bar{A} = I_N \otimes A$, $\bar{B} = I_N \otimes B$, $C_y = \text{diag}\{C_{y_i}\} \in \mathbb{R}^{Nn_y \times Nn_x}$, $\mathcal{A}_a \in \mathbb{R}^{N \times N}$ as defined in Section 2, and $g = \text{col}\{g_i\} : \mathbb{R}^{Nn_y} \times \mathbb{R}^+ \rightarrow \mathbb{R}^{Nn_g}$. We also obtain the following model of the control layer by aggregating the distributed stabilization protocol (5) for all $i \in \{1, 2, \dots, N\}$:

$$u = (\mathcal{H}_c \otimes K)x \quad (7)$$

where $\mathcal{H}_c \in \mathbb{R}^{N \times N}$ denotes the modified Laplacian matrix of the control layer topology \mathcal{G}_c (see Section 2).

Note that (6) and (7) model a two-layer interconnected MAS with separate agent and control layers, respectively. The agent layer topology is unknown and the control layer topology is a design degree of freedom. Therefore, the proposed graph theoretic framework not only provides a high-level flexibility to formulate various control configurations (see the gray box in Subsection 3.1), but also captures the architectural aspect of cyber-physical systems in a straightforward manner.

We will use the control layer topology (graph) \mathcal{G}_c as an MAS-level degree of freedom to obtain a candidate K in a step-by-step manner. Prior to the discussion on the design steps, we note that the following fact holds for any valid control layer topology \mathcal{G}_c and the associated \mathcal{H}_c , introduced in Section 2 (Zhang et al., 2015).

Fact 1. There exists a positive definite matrix $\Delta = \text{diag}\{\delta_i\} \in \mathbb{R}^{N \times N}$ with scalars $\delta_i > 0$ such that $\Delta \mathcal{H}_c + \mathcal{H}_c^T \Delta > \mathbf{0}$.

The matrix $\Delta = \text{diag}\{\delta_i\}$ is obtained according to $\delta_i = \frac{\delta_i^n}{\delta_i^d}$ where the numerators satisfy $\text{col}\{\delta_i^n\} = (\mathcal{H}_c^{-1})^T \mathbf{1}_N$ and the denominators satisfy $\text{col}\{\delta_i^d\} = \mathcal{H}_c^{-1} \mathbf{1}_N$. We observe that $\text{col}\{\delta_i^n\}$ gives the column sums and $\text{col}\{\delta_i^d\}$ the row sums of \mathcal{H}_c^{-1} . Since \mathcal{H}_c is a non-singular M-matrix, we know \mathcal{H}_c^{-1} exists and all of its elements are non-negative real-valued scalars (Alefeld and Schneider, 1982). Thus, the positive definiteness of Δ can be understood from this fundamental property of M-matrices and non-singularity of \mathcal{H}_c^{-1} .

In the next design procedure, we formulate a step-by-step approach in order to obtain a candidate stabilization gain K associated to a candidate control layer topology \mathcal{G}_c .

Design Procedure 1. Under the unmatched scenario, the candidate \mathcal{G}_c and K of the distributed stabilization protocol (5) are designed as follows:

- (1) Choose a structurally nonsymmetric control layer \mathcal{G}_c to formulate a control configuration according to the gray box in Subsection 3.1, and let $\kappa > 0$ be a real-valued scalar such that the inequality $\Delta \mathcal{H}_c + \mathcal{H}_c^T \Delta \succ 2\kappa \Delta$ is satisfied. Let the state weighting matrix $Q \in \mathbb{R}^{n_x \times n_x}$ and the control input weighting matrices $R_v \in \mathbb{R}^{n_u \times n_u}$ and $R_w \in \mathbb{R}^{n_g \times n_g}$ be three positive definite design matrices. Introduce a modified state weighting matrix $Q_m = Q + R_{wm}$ where $R_{wm} = \gamma_g \gamma_{cy} \lambda_{\max}(R_w) \|\mathcal{A}_a\|^2 I_{n_x}$.
- (2) Find the solutions $v_i' = Kx_i' \in \mathbb{R}^{n_u}$ and $w_i' = Gx_i' \in \mathbb{R}^{n_g}$ of the following modified LQR problem:

$$\begin{aligned} & \min_{v_i', w_i' \in \mathcal{C}_i} \int_0^\infty (x_i'^T Q_m x_i' + v_i'^T R_v v_i' + w_i'^T R_w w_i') dt \\ & \text{subject to} \quad \dot{x}_i' = Ax_i' + \kappa Bv_i' + B_g w_i' \end{aligned}$$

where \mathcal{C}_i is the set of all admissible (static linear state feedback) stabilizing control signals v_i' and fictitious con-

trol signals w_i' for the auxiliary networked nominal dynamics $\dot{x}'_i = Ax'_i + \kappa Bv'_i + B_g w'_i$.

(3) \mathcal{G}_c represents a valid control layer topology and K a valid stabilization gain if the following condition is satisfied:

$$\bar{Q}_{ss}^{um} \succ \mathbf{0} \quad (8)$$

where, using the operator $[\Delta \mathcal{H}_c]_{sym} = \frac{1}{2}(\Delta \mathcal{H}_c + \mathcal{H}_c^T \Delta)$, we have defined $\bar{Q}_{ss}^{um} = (2(\frac{[\Delta \mathcal{H}_c]_{sym}}{\kappa} - \Delta) \otimes K^T R_v K) + (\Delta \otimes (Q + K^T R_v K - 2G^T R_w G))$.

In Step (1), we define all required parameters and matrices, and specify the control configuration according to the gray box in Subsection 3.1. In Step (2), we note that the modified LQR problem is subject to an auxiliary networked nominal dynamics designed for the selected control layer topology (using κ). Also, the word ‘‘modified’’ refers to the use of a modified state weighting matrix Q_m instead of the arbitrary positive definite matrix Q , ‘‘networked’’ to the presence of κ which represents the effect of the control layer topology (vs. the nominal part $\dot{x}'_i = Ax'_i + Bv'_i$ of agents (2)), and ‘‘auxiliary’’ to the auxiliary (or fictitious) control input w_i . This fictitious signal is added for the analysis purpose (in Step (3)) and, otherwise, will never be implemented. In Step (3), we need to verify an $Nn_x \times Nn_x$ matrix inequality. This is a feasible idea noting that \mathcal{G}_c (consequently \mathcal{H}_c) is a design degree of freedom. However, we further know $K^T R_v K \succ \mathbf{0}$ and $\frac{[\Delta \mathcal{H}_c]_{sym}}{\kappa} - \Delta \succ \mathbf{0}$ (see Step (1)). This means we can replace the high-dimension condition (8) with the following low-dimension one (Rezaei and Stefanovic, 2016):

$$Q + K^T R_v K - 2G^T R_w G \succ \mathbf{0} \quad (9)$$

The use of (8) and (9) will be further clarified in the proof of Theorem 1.

Following the above design procedure, we know that the candidate stabilization gain K and the fictitious gain G are characterized as follows (Dorato et al., 1995):

$$K = -\kappa R_v^{-1} B^T P \quad \text{and} \quad G = -R_w^{-1} B_g^T P \quad (10)$$

in which the positive definite matrix $P \in \mathbb{R}^{Nn_x \times Nn_x}$ is the unique stabilizing solution of the following ARE:

$$A^T P + PA + Q_m - \kappa^2 P B R_v^{-1} B^T P - P B_g^T R_w^{-1} B_g P = \mathbf{0} \quad (11)$$

The existence and uniqueness of P are guaranteed based on the stabilizability and observability of the triple $(Q_m^{1/2}, A, [\kappa B, B_g])$ in which $Q_m^{T/2} Q_m^{1/2} = Q_m$. Note that the pair $(A, [\kappa B, B_g])$ is stabilizable because (A, B) characterizes a stabilizable system and $\kappa > 0$. Also, since \mathcal{G}_c (or \mathcal{H}_c) is a design degree of freedom, the scalar κ can (and must) be appropriately adjusted to avoid any poor controllability issues in the modified LQR problem of Design Procedure 1 or singularity issues in ARE (11).

Based on (10) and (11), it is straightforward to find three aggregated equalities: $I_N \otimes K = -I_N \otimes \kappa R_v^{-1} B^T P$, $I_N \otimes G = -I_N \otimes R_w^{-1} B_g^T P$, and $I_N \otimes (A^T P + PA + Q_m - \kappa^2 P B R_v^{-1} B^T P - P B_g^T R_w^{-1} B_g P) = \mathbf{0}$. Thus, the followings hold as well:

$$\begin{aligned} \Delta \otimes (K + \kappa R_v^{-1} B^T P) &= \mathbf{0} & \Delta \otimes (G + R_w^{-1} B_g^T P) &= \mathbf{0} \\ \Delta \otimes (A^T P + PA + Q_m - \kappa^2 P B R_v^{-1} B^T P - P B_g^T R_w^{-1} B_g P) &= \mathbf{0} \end{aligned}$$

We premultiply the first equality with x^T (defined after the aggregated model (6)) and postmultiply all of these equalities with x . After a few manipulations, we conclude that the following fact holds when K and G are obtained according to Design Procedure 1 (Lin, 2007).

Fact 2. The following MAS-level equalities hold in an MAS of auxiliary networked nominal dynamics and the candidate gains K and G of Design Procedure 1:

$$\begin{aligned} 2v^T \bar{R}_v + \kappa \bar{V}_x^T \bar{B} &= \mathbf{0} & 2w^T \bar{R}_w + \bar{V}_x^T \bar{B}_g &= \mathbf{0} \\ x^T \bar{Q}_m x + v^T \bar{R}_v v + w^T \bar{R}_w w + \bar{V}_x^T (\bar{A}x + \kappa \bar{B}v + \bar{B}_g w) &= \mathbf{0} \end{aligned}$$

where $v = \text{col}\{v_i\} = \bar{K}x = (I_N \otimes K)x$, $w = \text{col}\{w_i\} = \bar{G}x = (I_N \otimes G)x$, $V_x^T = \frac{\partial \bar{V}}{\partial x}$, $\bar{V} = x^T (\Delta \otimes P)x$, $\bar{Q}_m = \Delta \otimes Q_m$, $\bar{R}_v = \Delta \otimes R_v$, and $\bar{R}_w = \Delta \otimes R_w$.

Now we are ready to propose the main result of this subsection in the next theorem.

Theorem 1. (Robust stability, unmatched scenario) Let the candidate \mathcal{G}_c and K be designed using Design Problem 1. In a partially unknown MAS of agents (2) with unmatched interconnections, the linear stabilization protocol (5) exponentially steers all state trajectories of the two-layer structurally nonsymmetric interconnected MAS to the origin.

Proof: To facilitate the analysis, we rewrite (and decompose) the two-layer MAS of (6) and (7) as follows:

$$\underbrace{\dot{x} = \bar{A}x + \kappa \bar{B}v}_{\text{Networked nominal dynamics}} + \underbrace{\bar{B}_g g + \kappa \bar{B} \bar{E}_c v}_{\text{Uncertainties over } \mathcal{G}_a \text{ and } \mathcal{G}_c} \quad (12)$$

in which $v = (I_N \otimes K)x$, and $\bar{E}_c = (\frac{\mathcal{H}_c}{\kappa} - I_N) \otimes I_{n_u}$ is treated as the source of a fictitious modeling uncertainty (to be able to propose a low dimension formulation in Step (2) of Design Procedure 1). We introduce the following candidate Lyapunov function:

$$\bar{V}(x) = x^T (\Delta \otimes P)x \succ \mathbf{0}$$

Along the uncertain trajectories of the two-layer MAS (12), we find:

$$\begin{aligned} \dot{\bar{V}} &= \bar{V}_x^T \dot{x} = \bar{V}_x^T (\bar{A}x + \kappa \bar{B}v + \bar{B}_g g + \kappa \bar{B} \bar{E}_c v) \\ &= \bar{V}_x^T (\bar{A}x + \kappa \bar{B}v + \bar{B}_g w) - \bar{V}_x^T \bar{B}_g w + \kappa \bar{V}_x^T \bar{B} \bar{E}_c v + \bar{V}_x^T \bar{B}_g g \end{aligned}$$

which, using Fact 2, results in the followings:

$$\begin{aligned} \dot{\bar{V}} &= -x^T \bar{Q}_m x - v^T \bar{R}_v v - w^T \bar{R}_w w + 2w^T \bar{R}_w w - 2v^T \bar{R}_v \bar{E}_c v - 2w^T \bar{R}_w g \\ &\leq -x^T (\Delta \otimes (Q + K^T R_v K - 2G^T R_w G))x - 2v^T \bar{R}_v \bar{E}_c v \\ &\leq -x^T \bar{Q}_{ss}^{um} x \prec \mathbf{0} \end{aligned}$$

where the negative definiteness is guaranteed by design (see Step (3) of Design Procedure 1). The negative definiteness remains valid if we use the low-dimension condition (9). This is because $-2v^T \bar{R}_v \bar{E}_c v \leq 0$ and $\Delta \succ \mathbf{0}$. Moreover, using the Rayleigh-Ritz inequality, we find $\lambda_{\min}(\Delta \otimes P) \|x\|^2 \leq \bar{V} \leq \lambda_{\max}(\Delta \otimes P) \|x\|^2$ and $\dot{\bar{V}} \leq -\lambda_{\min}(\bar{Q}_{ss}^{um}) \|x\|^2$ which indicate the exponential convergence of all trajectories to the origin with a bound $\|x(t)\| \leq \alpha \exp^{-\beta t} \|x(0)\|$ characterized by $\alpha = \sqrt{\frac{\lambda_{\max}(\Delta) \lambda_{\max}(P)}{\lambda_{\min}(\Delta) \lambda_{\min}(P)}}$ and $\beta = \frac{\lambda_{\min}(\bar{Q}_{ss}^{um})}{2\lambda_{\max}(\Delta) \lambda_{\max}(P)}$ (Khalil, 2002). ■

3.3 Design and analysis: Matched interconnection

We start this subsection by aggregating the agent models (3), subject to the matched interconnections, and finding the following agent layer dynamics:

$$\dot{x} = \bar{A}x + \bar{B}(u + f(z)) \quad \text{and} \quad z = C_z (\mathcal{A}_a \otimes I_{n_x})x \quad (13)$$

where $z = \text{col}\{z_i\} \in \mathbb{R}^{Nn_z}$, $C_z = \text{diag}\{C_{zi}\} \in \mathbb{R}^{Nn_z \times Nn_x}$, and $f = \text{col}\{f_i\} : \mathbb{R}^{Nn_z} \times \mathbb{R}^+ \rightarrow \mathbb{R}^{Nn_u}$. Since we use the same distributed stabilization protocol (5) for both matched and unmatched scenarios, the control layer is still modeled by (7) as in the previous subsection.

In the next design procedure, we formulate a step-by-step approach in order to design a candidate control layer topology \mathcal{G}_c and a candidate stabilization gain K .

Design Procedure 2. Under the matched scenario, the candidate \mathcal{G}_c and K of distributed stabilization protocol (5) are designed as follows:

- (1) Choose a structurally non-symmetric control layer \mathcal{G}_c to formulate a control configuration according to the gray box in Subsection 3.1. Let $\mathcal{H}_c \in \mathbb{R}^{N \times N}$ be its modified Laplacian matrix.
- (2) Find a (non-unique) correction matrix $\mathcal{H}_c^{cor} \in \mathbb{R}^{N \times N}$ such that $\mathcal{H}_c^{ss} = \mathcal{H}_c + \mathcal{H}_c^{cor} \in \mathbb{R}^{N \times N}$ represents a valid modified Laplacian matrix associated to a structurally symmetric control layer topology \mathcal{G}_c^{ss} (see Section 2). Let $\sigma > 0$ be the smallest (real-valued positive) eigenvalue of \mathcal{H}_c^{ss} . Let the state weighting matrix $Q \in \mathbb{R}^{n_x \times n_x}$ and input weighting matrix $R_v \in \mathbb{R}^{n_u \times n_u}$ be two positive definite design matrices, and introduce a modified state weighting matrix $Q_m = Q + R_{vm}$ where $R_{vm} = \frac{1}{\sigma^2} \gamma_g \gamma_{cz} \lambda_{\max}(R_v) \|\mathcal{A}_a\|^2 I_{n_x}$.
- (3) Find the solution $v'_i = Kx'_i$ of the following modified LQR problem:

$$\begin{aligned} \min_{v'_i \in \mathcal{C}_i} \quad & \int_0^\infty (x'_i(\tau)^T Q_m x'_i(\tau) + v'_i(\tau)^T R_v v'_i(\tau)) d\tau \\ \text{subject to} \quad & \dot{x}'_i = Ax'_i + \sigma Bv'_i \end{aligned}$$

where \mathcal{C}_i is the set of all admissible (static linear state feedback) stabilizing signals for $\dot{x}'_i = Ax'_i + \sigma Bv'_i$.

- (4) \mathcal{G}_c represents a valid control layer topology and K a valid stabilization gain if the following condition is satisfied:

$$\bar{Q}_{ss}^m \succ \mathbf{0} \quad (14)$$

where $\bar{Q}_{ss}^m := (I_N \otimes Q) + \frac{1}{\sigma} ((\mathcal{H}_c^{ss} - 2[\mathcal{H}_c^{cor}]_{sym} - \sigma I_N) \otimes K^T R_v K)$.

We can discuss the above steps similar to Design Procedure 1. Further, in Step (4), we directly use the existing flexibility in the Lyapunov-based proof of stability (see Theorem 2). This is unlike Design Procedure 1 which relies on Fact 1 as a property of the M-matrix \mathcal{H}_c .

We know the candidate control gain is characterized by:

$$K = -\sigma R_v^{-1} B^T P \quad (15)$$

and the minimum cost of the modified LQR formulation in Step (3) of Design Procedure 2 is given by $V(x'_i(0)) = x'^T_i(t) P x'_i(t)|_{t=0}$ (Dorato et al., 1995). The positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ is the unique stabilizing solution of the following ARE:

$$A^T P + PA + Q_m - \sigma^2 P B R_v^{-1} B^T P = \mathbf{0} \quad (16)$$

and the existence and uniqueness of $P \succ \mathbf{0}$ are guaranteed based on the stabilizability and observability of the triple $(Q_m^{1/2}, A, \sigma B)$ in which $Q_m^{T/2} Q_m^{1/2} = Q_m$. The above networked nominal dynamics, represented by $(A, \sigma B)$, are stabilizable. This is because the pair (A, B) is stabilizable by assumption, and $\sigma > 0$ by design. As discussed after Design Procedure 1 for the unmatched scenario, the designer can use the freedom in \mathcal{G}_c , Q , and R_v in order to obtain a sufficiently large $\sigma > 0$ and avoid any potential poor controllability or singularity issues in solving the above ARE.

Based on (15) and (16), it is straightforward to find $I_N \otimes (K + \sigma R_v^{-1} B^T P) = \mathbf{0}$ and $I_N \otimes (A^T P + PA + Q_m - \sigma^2 P B R_v^{-1} B^T P) = \mathbf{0}$. After a few manipulations, we conclude the following fact if the candidate K is obtained according to Design Procedure 2.

Fact 3. The following MAS-level equalities hold in an MAS of networked nominal dynamics and the candidate gain K of Design Procedure 2:

$$\begin{aligned} 2v^T \bar{R}_v + \sigma \bar{V}_x^T \bar{B} &= \mathbf{0} \\ x^T \bar{Q}_m x + v^T \bar{R}_v v + \bar{V}_x^T (\bar{A}x + \sigma \bar{B}v) &= \mathbf{0} \end{aligned}$$

where $\bar{V} = x^T (I_N \otimes P)x$, $\bar{Q}_m = I_N \otimes Q_m$, and $\bar{R}_v = I_N \otimes R_v$.

Note that \bar{V} , \bar{Q}_m , and \bar{R}_v are different from those of Fact 2 for the unmatched scenario. We provide the main result of this subsection in the next theorem.

Theorem 2. (Robust stability, matched scenario) Let the candidate \mathcal{G}_c and K be designed based on Design Procedure 2. In a partially unknown MAS of agents (3) with matched interconnections, the linear stabilization protocol (5) exponentially steers all state trajectories of the two-layer structurally nonsymmetric interconnected MAS to the origin.

Proof: Based on the proposed approach in Design Procedure 2, we first note that the aggregate distributed protocol (5) can be written as follows:

$$u = (\mathcal{H}_c^{ss} \otimes K)x - (\mathcal{H}_c^{cor} \otimes K)x$$

We rewrite the two-layer interconnected MAS (13) as follows:

$$\underbrace{\dot{x} = \bar{A}x + \sigma \bar{B}v}_{\text{Networked nom. dyn.}} + \underbrace{\bar{B}f(z) + \sigma \bar{B} \bar{E}_c v}_{\text{Uncert. over } \mathcal{G}_a \text{ and } \mathcal{G}_c} - \underbrace{\bar{B}(\mathcal{H}_c^{cor} \otimes K)x}_{\text{Design Proc. 2}} \quad (17)$$

where $\bar{E}_c = (\frac{\mathcal{H}_c^{ss}}{\sigma} - I_N) \otimes I_{n_u} \succ \mathbf{0}$ is treated as a source of fictitious modeling uncertainties (to obtain K using a low-dimension formulation in Step (3) of Design Procedure 2).

We propose a candidate Lyapunov function:

$$\bar{V}(x) = x^T (I_N \otimes P)x \succ \mathbf{0}$$

and find $\dot{\bar{V}} = \bar{V}_x^T (\bar{A}x + \sigma \bar{B}v + \sigma \bar{B} \bar{E}_c v + \bar{B}f - \bar{B}(\mathcal{H}_c^{cor} \otimes I_{n_u})v)$ along the unknown trajectories of the two-layer MAS (17) with a structurally nonsymmetric control layer topology \mathcal{G}_c . This can be grouped as $\dot{\bar{V}} = \bar{V}_x^T (\bar{A}x + \sigma \bar{B}v) + \sigma \bar{V}_x^T \bar{B} \bar{E}_c v - \sigma \bar{V}_x^T \bar{B}(\mathcal{H}_c^{cor} \otimes I_{n_u})v + \sigma \bar{V}_x^T \bar{B}f$ where $f_\sigma = \frac{1}{\sigma} f$ is an aggregated vector of scaled nonlinearities. Ultimately, we reach to:

$$\begin{aligned} \dot{\bar{V}} &= -x^T \bar{Q}_m x - v^T \bar{R}_v v - 2v^T \bar{R}_v \bar{E}_c v + \frac{2}{\sigma} v^T \bar{R}_v (\mathcal{H}_c^{cor} \otimes I_{n_u})v - 2v^T \bar{R}_v f_\sigma \\ &\leq -x^T \bar{Q}_{ss}^m x \prec \mathbf{0} \end{aligned}$$

based on Fact 3 and decomposition $\bar{Q}_m = \bar{Q} + \bar{R}_{vm}$. Note that the negative definiteness is guaranteed by Step (4) of Design Procedure 2. We further find $\lambda_{\min}(P)\|x\|^2 \leq \bar{V} \leq \lambda_{\max}(P)\|x\|^2$ and $\dot{\bar{V}} \leq -\lambda_{\min}(\bar{Q}_{ss}^m)\|x\|^2$, based on the Rayleigh-Ritz inequality. This indicates the robust exponential convergence of all state trajectories to the origin, i.e., $\|x(t)\| \leq \alpha \exp^{-\beta t} \|x(0)\|$ with $\alpha = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ and $\beta = \frac{\lambda_{\min}(\bar{Q}_{ss}^m)}{2\lambda_{\max}(P)}$ (Khalil, 2002). ■

4. DISCUSSION

The problem considered in this paper can be discussed based on the literature of networked control systems. In particular, the pioneering work of Wang and Lemmon (2011) (subsection IV.B) proposed a design formulation to obtain the local stabilization gain K for a completely known linear time-invariant system. Their (non-graph theoretic) design formulation requires to simultaneously solve a set of linear matrix inequalities (LMIs) in order to obtain K , the dimension of these LMIs are varying depending on the completely known interconnection topology, it does not capture the architectural aspect of cyber-physical systems because the networked control topology is identical to the physical interconnection topology, and all agents must contribute toward the stabilization by sharing their absolute measurements with the control layer operator.

We have proposed a mixed graph and optimal control framework to deal with a scenario where the detailed interconnection topology is unknown, there exist unmatched or matched agent- and MAS-level nonlinear modeling uncertainties, and only a few agents might be willing to share their absolute information with the control layer operator. In this way, the problem foundation is markedly broadened. Furthermore, unlike Wang and Lemmon (2011) and Rezaei and Stefanovic (2016), we are able to systematically capture the architectural aspect of cyber-physical systems because the control layer (a design degree of freedom) is completely different from the unknown agent layer interconnection topology.

The modified LQR formulations of this paper may end in a conservative solution approach. But this is well justified, because the agent layer owner shares neither the type of nonlinearities nor detailed information about the interconnection topology and allocation matrices, which creates both agent- and MAS-level modeling uncertainties. Recently, assuming a known interconnection topology \mathcal{G}_a (Wang and Lemmon, 2011), we have proved that the distributed stabilization problem can be addressed using a non-modified LQR formulation and an adaptive control idea with a control cooperation layer whose topology is different from that of agent (and decoupling) layer (Rezaei and Stefanovic, 2020). Also, as a key trade-off, the need for the known upper-bounds on the unknown interconnection terms in Assumption 1 is relaxed. We are currently working to extend that result to unmatched and nonlinear interconnection scenarios over the agent layer, and to structurally nonsymmetric control layers. An extension of these ideas to dynamic (Rezaei and Stefanovic, 2017b) or static output feedback would be interesting as well.

5. SIMULATION VERIFICATION

Now we verify the feasibility of the proposed ideas in Section 3. We first introduce the agent layer topology \mathcal{G}_a and control layer topology \mathcal{G}_c as depicted in Fig. 1. We examine the unmatched scenario in Subsection 5.1 and validate the matched scenario in Subsection 5.2.

The agent layer topology \mathcal{G}_a is chosen such that the agents 1 and 2 build a sub-MAS which is totally decoupled from other agents. Also, we note that the agent 5 is not affected by any agents, and agent 8 is affected by agent 7. Over this layer, each dashed black arrow represents an edge weight of -1 and solid gray arrow an edge weight of 1. Following the definition of \mathcal{A}_a in Section 2, the (unknown) adjacency matrix \mathcal{A}_a can be obtained using the agent layer graph in Fig. 1.

The structurally nonsymmetric control layer topology \mathcal{G}_c is chosen to be different from the unknown \mathcal{G}_a , and to satisfy all requirements of Section 2. Control nodes 1 and 2 show a distributed implementation of a centralized controller in the sense that the control action of each node is determined based on all information in the sub-MAS of agents 1 and 2 (we have limited this case to two agents in order to avoid visualization problem). Also, the control nodes 3 to 7 represent a cooperative control protocol in which only agents 4 and 6 share their absolute information with the control layer operator. The control node 8 represents a decentralized control configuration in the sense that it relies on only its own absolute measurement for the stabilization purpose, despite the fact that it is affected by agent 7 over \mathcal{G}_a . Over this graph, the green arrow shows an edge weight of 1.5, black a weight of 1, blue a weight of 0.5, and

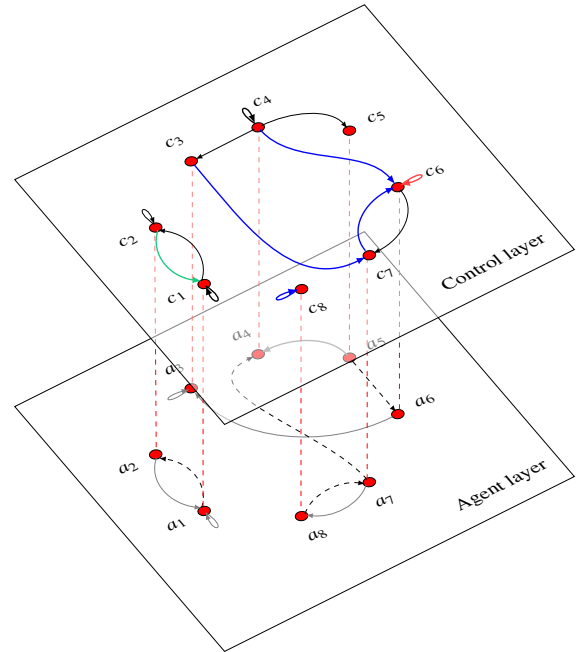


Fig. 1. Two-layer (closed-loop) interconnected MAS with separate control and agent layers. c_i and a_i denote i^{th} controller and agent, respectively. Due to the space limitation, we use the same topology for both unmatched and matched scenarios. (See Section 5 for a description of the setup.)

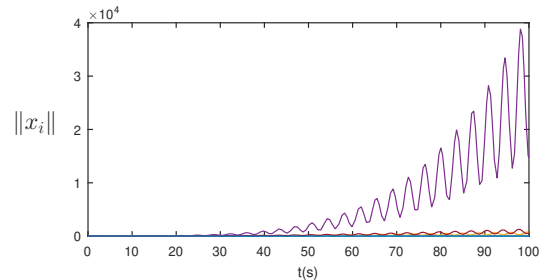


Fig. 2. Diverging norms $\|x_i\|$ of (open-loop) agent layer dynamics in Fig. 1 for $i \in \{1, 2, \dots, 8\}$.

pink a weight of 0.25. We follow the definition of the modified Laplacian matrix (Section 2) to obtain \mathcal{H}_c using the control layer graph in Fig. 1.

5.1 Unmatched interconnection

We characterize an MAS of 8 interconnected agents (2) using the following system matrices:

$$A = \begin{bmatrix} 0 & 1.5 \\ -0.5 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad B_g = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

The (unknown) coupling matrices and nonlinearities are $C_{y_i} = [0, 1]$ for all $i \in \{1, \dots, 8\}$, $g_1(y_1) = 0.5 \sin(y_1)$, $g_2(y_2) = -0.4y_2$, $g_3(y_3, t) = 0.5 \tanh(t) \tanh(y_3)$, $g_4(y_4) = -0.4y_4$, $g_5(y_5) = -0.5 \sin(y_5)$, $g_6(y_6, t) = 0.4 \sin(t) \sin(y_6)$, $g_7(y_7) = 0.5y_7$, and $g_8(y_8) = 0.4 \sin(y_8)$. As depicted in Fig. 2, we note that the above interconnected MAS (shown over the agent layer) results in unstable open-loop dynamics with diverging behavior.

We follow the steps of Design Procedure 1 to find $K = [-2.2738, -4.3828]$, $G = [1.8281, -1.5016]$ (will not be implemented, as discussed in Subsection 3.2), and the control layer \mathcal{G}_c of Fig. 1 which characterizes a mixed centralized,

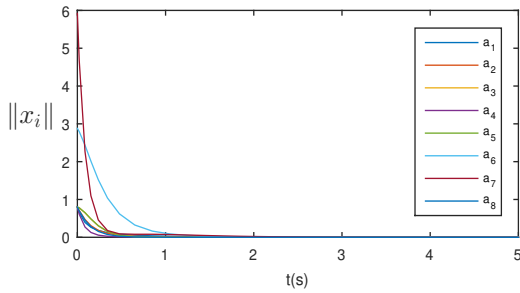


Fig. 3. Unmatched scenario: Converging norms $\|x_i\|$ of the two-layer interconnected MAS in Fig. 1 for $i \in \{1, 2, \dots, 8\}$

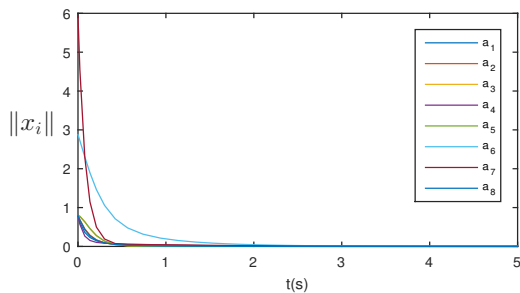


Fig. 4. Matched scenario: Converging norms $\|x_i\|$ of the two-layer interconnected MAS in Fig. 1 for $i \in \{1, 2, \dots, 8\}$.

cooperative, and decentralized configuration. Figure 3 depicts the converging response of all state trajectories to the origin.

5.2 Matched interconnection

We assume that the interconnected MAS in the matched scenario is characterized similar to the unmatched one in Subsection 5.1 replacing g_i by f_i , y_i by z_i , and C_{y_i} by C_{z_i} .

Following the steps of Design Procedure 1, we find $K = [-1.7388, -4.9160]$ and the control layer \mathcal{G}_c of Fig. 1 (without loss of generality, we indeed fix it in advance to avoid the need for a new figure for the two-layer MAS). Figure 4 depicts the converging response of all state trajectories to the origin.

6. SUMMARY

We consider the distributed stabilization problem in interconnected multiagent systems, where the underlying dynamics are subject to both agent- and multiagent system-level modeling uncertainties in either unmatched or matched scenarios, the interconnection topology is only partially known to the control designer, and only a few agents may provide their absolute measurements to the control operator. We propose two mixed graph and optimal control formulations which are capable of handling the aforementioned challenges. The proposed graph theoretic framework not only captures the architectural aspect of cyber-physical systems, but also makes it possible to simultaneously implement all centralized, cooperative, and decentralized control configurations in a distributed fashion. We comment on the trade-offs in the assumptions and solution approaches of this paper compared to a few other references, and validate the feasibility of the proposed ideas in simulation.

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