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Parameter Estimation in Input Matrix Under Gain Constraints in Specified Frequency Ranges

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Abstract: This paper addresses parameter estimation problem of Continuous-/Discrete-Time (CT/DT) Linear Time-Invariant (LTI) systems, whose gain properties should satisfy given constraints in *a priori* specified frequencies, using measured data. The following are supposed in our problem: i) only input matrix has parameters to be estimated; ii) the state and the input are both measured, and the derivative of the state is also measured in CT case, and iii) the gain constraints in specified frequency ranges are given beforehand. Under these suppositions, a formulation to minimize the difference between the measured state derivative and the expected state derivative (in CT case) or the difference between the measured one-step-ahead state and the expected one-step-ahead state (in DT case) in Euclidean norm with the supposed gain constraints satisfied is given in terms of Linear Matrix Inequality (LMI). The effectiveness of the proposed method is demonstrated by an academic example in DT case as well as flight data obtained by JAXA's airplane in CT case.

Keywords: Parameter estimation, Gain constraints, Generalized KYP (GKYP) lemma.

1. INTRODUCTION

Identification of plant systems is the fundamental first step to control practical systems, and there are many reports and publications on "system identification", e.g. (Ljung, 1998). Though, in many mechanical systems, the modeling is not usually conducted in "black-box modeling" (the systems' structure is completely unknown and thus the systems' degrees are first to be identified), but mostly conducted in "gray-box modeling" (the systems' structure is known and some parameters in systems are known but the values of some other parameters are unknown and they must be identified). In the latter case, the problem to be tackled is referred to *parameter estimation problem.*

However, it is often said that there is no systematic approach to parameter estimation problem, and heuristic approaches are often taken. One of the simplest methods is to use the least square method (Ljung, 1998; Jategaonkar, 2006) in which the difference between the measured state derivative and the expected state derivative (Continuous-Time case; CT case) or the difference between the measured one-step-ahead state and the expected one-step-ahead state (Discrete-Time case; DT case) is minimized in Euclidean norm. In this way, it is possible to estimate systems' parameters which match the supposed systems' time responses with the models' time responses. This approach is simple and effective; however, it sometimes occurs that some frequency-domain constraints should be satisfied. For example, if we would like to focus only on some specific frequency properties due to the unavoidable noise effect, it is necessary to apply appropriate filters to the measured signals in order to pick up the specific frequency properties; however, the filtered data cannot escape from the adopted filter characteristics. Rather than using filters, it is preferable to estimate systems' parameters directly without using any filters.

Based on the research background above, we tackle parameter estimation problem in Linear Time-Invariant (LIT) systems which minimizes the difference between the measured state derivative and the expected state derivative (CT) or the difference between the measured one-step-ahead state and the expected one-step-ahead state (DT) in Euclidean norm under some gain constraints in *a priori* specified frequency ranges. We will use Generalized KYP (GKYP) lemma (Iwasaki and Hara, 2003, 2005, 2007) to impose some gain constraints in *a priori* specified frequency ranges.

As a first step of our approach to parameter estimation problem under gain constraints, we suppose that parameters to be estimated are included only in input matrix in this paper. This restriction limits the applicability of our proposed method; however, we will show a practical application of our method, i.e. engine torque effect estimation for asymmetric torque input to twin-turbo-prop airplane MuPAL-α (Masui and Tsukano, 2000).

This paper is organized as follows: Section 2 defines our problem after the introduction of our supposed LTI plant, measured data and gain constraints, and briefly reviews GKYP lemma to impose gain constraints in specified frequency ranges; then Section 3 shows our proposed method, and two applications of our method are shown; Finally, Section 4 gives concluding remarks.

Notations: e_i denotes a column vector with its i-th element being set as 1 and others being set as 0, i.e. $e_i = [0 \ldots 0\,1\,0 \ldots 0]^T$, $\mathbf{0}$ and $\mathbf{I}$ respectively denote a zero matrix and an identity matrix with appropriate dimensions (if necessary, the dimension is denoted by the subscript), $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$, $\mathbb{C}^{n\times m}$ and $\mathbb{H}^n$ respectively denote the set of n-dimensional real vectors, the set of $n \times m$-dimensional real matrices, the set of $n \times m$-dimensional complex matrices and the set of $n \times n$-dimensional Hermitian matrices, and $\mathsf{He}\{X\}$ for a square matrix X denotes a short-hand notation of $X + X^T$. For a state vector x, $\delta[x]$ denotes $\dot{x} := \frac{d}{dt}x$ and $x^+ := x(k+1)$ in CT and DT cases respectively. For system P, its transfer function is represented by $P(\lambda)$ with its frequency variable λ. $\sigma(G, \Theta)$ denotes $[G^* \ \mathbf{I}]\,\Theta\,[G^* \ \mathbf{I}]^*$ for appropriately defined matrices G and Θ.

2. PRELIMINARIES

2.1 System Definition

This note considers an LTI system with the following state-space representation.

$$G(\theta) : \begin{cases} \delta[x] = Ax + B(\theta)u \\ \ \ y \ \ = \mathbf{I}_n x \end{cases}, \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^{n_u}$ respectively denote the state and the input, and matrices $A \in \mathbb{R}^{n\times n}$ and $B(\theta) \in \mathbb{R}^{n\times n_u}$ are state-space matrices. It is supposed that A is completely known and that its eigenvalues do not exist on the imaginary axis (CT) or on the unit circle (DT); however, $B(\theta)$ has *a priori* known structural constraints and contains unknown parameters $\theta = [\theta_1 \ \cdots \ \theta_r]^T$ which should be estimated using experimental data.

The following is supposed in this note.

Assumption 1. The vectors $\delta[x]$, x and u are all measurable, and they are obtained with a sufficiently small sampling period for a sufficiently long duration. □

This assumption seems to be restrictive from the viewpoint of applicability, particularly in the CT case. However, in mechanical systems, up-to-date measurement equipment can be used to measure the derivative of the state. Furthermore, it may be possible to use numerical derivatives. Therefore, in this note, we made Assumption 1.

The transfer function of $G(\theta)$ is denoted by $G(\lambda;\theta)$ with its frequency variable λ. Similarly, the transfer function of G with the estimated $\hat{\theta}$ is denoted by $G(\lambda;\hat{\theta})$; that is, $G(\lambda;\hat{\theta})$ denotes $G(\lambda;\hat{\theta}) = [\lambda \mathbf{I}_n - A]^{-1} B(\hat{\theta})$. The (i,j) entries of $G(\lambda;\theta)$ and $G(\lambda;\hat{\theta})$ are respectively denoted by $G_{i,j}(\lambda;\theta)$ and $G_{i,j}(\lambda;\hat{\theta})$.

2.2 Measured Data

It is supposed that l_q-step data ($q = 1, \ldots, \bar{q}$) for $\delta[x]$, x and u are given as follows.

$$\mathcal{X}_q = \left[x(0)^T \ x(1)^T \ \cdots \ x(l_q-1)^T\right]^T \in \mathbb{R}^{nl_q} \tag{2}$$

$$\delta[\mathcal{X}]_q = \left[\delta[x(0)]^T \ \delta[x(2)]^T \ \cdots \ \delta[x(l_q-1)]^T\right]^T \in \mathbb{R}^{nl_q} \tag{3}$$

$$\mathcal{U}_q = \left[u(0)^T \ u(1)^T \ \cdots \ u(l_q-1)^T\right]^T \in \mathbb{R}^{n_u l_q} \tag{4}$$

That is, $\bar{q}$-sets of $(\mathcal{X}, \delta[\mathcal{X}], \mathcal{U})$ are given.

2.3 Gain Constraints

In this note, the following gain constraints are *a priori* given.

Assumption 2. For each $G_{i,j}(\lambda;\theta)$, we have $\bar{l}_{i,j}$ sets of $(\lambda_{l_{\min}}, \lambda_{l_{\max}}, \bar{g}_l)$ ($l = 1, \ldots, \bar{l}_{i,j}$) indicating its gain upper bound constraints with *a priori* given positive scalars $\bar{g}_l$, and $m_{i,j}$ sets of $(\lambda_{m_{\min}}, \lambda_{m_{\max}}, g_m, \varepsilon_m)$ ($m = 1, \ldots, \bar{m}_{i,j}$) indicating its gain interval constraints with *a priori* given positive scalars g_m and ε_m.

$$|G_{i,j}(\lambda;\theta)| < \bar{g}_l, \ \forall \lambda \in [\lambda_{l_{\min}}, \lambda_{l_{\max}}], \tag{5}$$

$$|G_{i,j}(\lambda;\theta) - g_m| < \varepsilon_m, \ \forall \lambda \in [\lambda_{m_{\min}}, \lambda_{m_{\max}}], \tag{6}$$

where frequency bounds $\lambda_{l_{\min}}, \lambda_{l_{\max}}$ and $\lambda_{m_{\min}}, \lambda_{m_{\max}}$ are supposed to satisfy the following.

$$\begin{cases} 0 \le \lambda_{l_{\min}} < \lambda_{l_{\max}} \in \mathbb{R}_+ \cup \infty \ \ \text{(CT)} \\ 0 \le \lambda_{l_{\min}} < \lambda_{l_{\max}} \le \pi \ \ \text{(DT)} \end{cases}$$

$$\begin{cases} 0 \le \lambda_{m_{\min}} < \lambda_{m_{\max}} \in \mathbb{R}_+ \cup \infty \ \ \text{(CT)} \\ 0 \le \lambda_{m_{\min}} < \lambda_{m_{\max}} \le \pi \ \ \text{(DT)} \end{cases}$$

□

This assumption means that we have $\bar{l}_{i,j}$ sets which specify the admissible gain upper bounds and $\bar{m}_{i,j}$ sets which specify the admissible gain intervals for the (i,j) entry of $G(\lambda;\theta)$, i.e. $|G_{i,j}(\lambda;\theta)| < \bar{g}_l$ for *a priori* designated frequency ranges $[\lambda_{l_{\min}}, \lambda_{l_{\max}}]$, and $g_m - \varepsilon_m < |G_{i,j}(\lambda;\theta)| < g_m + \varepsilon_m$, for *a priori* designated frequency ranges $[\lambda_{m_{\min}}, \lambda_{m_{\max}}]$.

2.4 Problem Definition

Under assumptions 1 and 2, we address the following problem.

Problem 1. With given constraint sets of $(\lambda_{l_{\min}}, \lambda_{l_{\max}}, \bar{g}_l)$ ($l = 1, \ldots, \bar{l}_{i,j}$) and constraint sets of $(\lambda_{m_{\min}}, \lambda_{m_{\max}}, g_m, \varepsilon_m)$ ($m = 1, \ldots, \bar{m}_{i,j}$), and $\bar{q}$ measured data set $(\mathcal{X}_q, \delta[\mathcal{X}]_q, \mathcal{U}_q)$ ($q = 1, \ldots, \bar{q}$), find θ satisfying the following:

$$\min_{\eta = [\eta_1 \ldots \eta_{\bar{q}}]^T, \theta} \ \sum_{q=1}^{\bar{q}} \eta_q \ \text{s.t. } \mathbf{a}, \mathbf{b}, \text{and } \mathbf{c} \tag{7}$$

$$\mathbf{a:} \ \begin{cases} \eta_q > \left\| \delta[\mathcal{X}]_q - \left[(\mathbf{I}_l \otimes A)\,\mathcal{X}_q + (\mathbf{I}_l \otimes B(\theta))\,\mathcal{U}_q\right]\right\|, \\ \qquad\qquad\qquad\qquad\qquad\qquad q = \{1, \ldots, \bar{q}\} \end{cases}$$

$$\mathbf{b:} \ \begin{cases} |G_{i,j}(\lambda;\theta)| < \bar{g}_l, \ \forall \lambda \in [\lambda_{l_{\min}}, \lambda_{l_{\max}}], \\ i = \{1, \ldots, n\}, \ j = \{1, \ldots, n_u\}, \ l = \{1, \ldots, \bar{l}_{i,j}\} \end{cases}$$

$$\mathbf{c:} \ \begin{cases} |G_{i,j}(\lambda;\theta) - g_m| < \varepsilon_m, \ \forall \lambda \in [\lambda_{m_{\min}}, \lambda_{m_{\max}}], \\ i = \{1, \ldots, n\}, \ j = \{1, \ldots, n_u\}, \ m = \{1, \ldots, \bar{m}_{i,j}\} \end{cases}$$

This problem means that we look for θ which minimizes the sum of the errors between $\delta[\mathcal{X}]_q$ and $(\mathbf{I}_l \otimes A)\,\mathcal{X}_q + (\mathbf{I}_l \otimes B(\theta))\,\mathcal{U}_q$ under the gain constraints in Assumption 2.

Remark 1. As is obvious, it is also possible to exchange the imposed constraints and the cost function each other, *viz.*, if the upper bounds η_q for the errors between $\delta[\mathcal{X}]_q$ and $(\mathbf{I}_l \otimes A)\,\mathcal{X}_q + (\mathbf{I}_l \otimes B(\theta))\,\mathcal{U}_q$ are given, then it is possible to minimize the linear combination of the gain upper bounds $\bar{g}_l$ and the errors ε_m using the to-be-estimated parameter θ. In this note, we focus on Problem 1; however, similar discussions hold for the above problem setup as well. □

Table 1. Setting of Φ_σ and Ψ_σ in CT

Φ_σ	Ψ_σ		
	low frequency ($\lambda = j\omega, \lvert\omega\rvert \le \varpi_l$)	middle frequency ($\lambda = j\omega, \varpi_1 \le \omega \le \varpi_2$)	high frequency ($\lambda = j\omega, \lvert\omega\rvert \ge \varpi_h$)
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & \varpi_l^2 \end{bmatrix}$	$\begin{bmatrix} -1 & j\frac{\varpi_1+\varpi_2}{2} \\ -j\frac{\varpi_1+\varpi_2}{2} & -\varpi_1\varpi_2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -\varpi_h^2 \end{bmatrix}$

Table 2. Setting of Φ_σ and Ψ_σ in DT

Φ_σ	Ψ_σ		
	low frequency ($\lambda = e^{j\theta}, \lvert\theta\rvert \le \vartheta_l$)	middle frequency ($\lambda = e^{j\theta}, \vartheta_1 \le \theta \le \vartheta_2$)	high frequency ($\lambda = e^{j\theta}, \lvert\theta\rvert \ge \vartheta_h$)
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & -2\cos\vartheta_l \end{bmatrix}$	$\begin{bmatrix} 0 & \exp\left(j\frac{\vartheta_1+\vartheta_2}{2}\right) \\ \exp\left(-j\frac{\vartheta_1+\vartheta_2}{2}\right) & -2\cos\frac{\vartheta_2-\vartheta_1}{2} \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ -1 & 2\cos\vartheta_h \end{bmatrix}$

2.5 GKYP Lemma

The gain constraints in Assumption 2 are handled by the dual form of GKYP lemma (Iwasaki and Hara, 2003, 2007), because GKYP lemma (Iwasaki and Hara, 2005) has an ability to specify the maximum gains for *a priori* specified frequency ranges. Thus, we briefly review GKYP lemma and its dual form below.

In this subsection, the following LTI system G is considered.

$$\mathsf{G} : \begin{cases} \delta[\mathsf{x}] = \mathsf{Ax} + \mathsf{Bu} \\ \mathsf{y} \;\;\;= \mathsf{Cx} + \mathsf{Du} \end{cases}, \tag{8}$$

where $\mathsf{x} \in \mathbb{C}^n$, $\mathsf{u} \in \mathbb{C}^{n_u}$ and $\mathsf{y} \in \mathbb{C}^{n_y}$ are respectively the state, the input and the output, and matrices in (8) are supposed to be complex matrices with compatible dimensions.

Matrix Π_σ is defined as $\Pi_\sigma = \mathsf{diag}\left(\mathbf{I}_{n_y}, -\gamma^2\mathbf{I}_{n_u}\right)$ with a positive real scalar γ. The target frequency range for frequency variable λ is defined as follows:

$$\Lambda_\sigma(\Phi_\sigma, \Psi_\sigma) = \{\lambda \in \mathbb{C} : \sigma(\lambda, \Phi_\sigma) = 0,\ \sigma(\lambda, \Psi_\sigma) \ge 0\}, \tag{9}$$

where matrices $\Phi_\sigma, \Psi_\sigma \in \mathbb{H}^2$ are defined in Table 1 in CT case and Table2 in DT case at the top of this page.

With these preliminaries, GKYP lemma for the gain bound is given below.

Lemma 1. (Iwasaki and Hara, 2005) Suppose that Λ_σ represents curves[1] on the complex plane, and that $\det(\lambda\mathbf{I} - \mathsf{A}) \neq 0,\ \forall\lambda \in \Lambda_\sigma(\Phi_\sigma, \Psi_\sigma)$ holds. Then, the following two statements are equivalent.

- $\mathsf{G}(\lambda) = \mathsf{C}(\lambda\mathbf{I} - \mathsf{A})^{-1}\mathsf{B} + \mathsf{D}$ satisfies $\sigma\left(G(\lambda), \Pi_\sigma\right) \prec 0,\ \forall\lambda \in \Lambda_\sigma(\Phi_\sigma, \Psi_\sigma)$, *viz.*, the following holds:
$$\mathsf{G}(\lambda)^*\mathsf{G}(\lambda) \prec \gamma^2\mathbf{I}_{n_u},\ \forall\lambda \in \Lambda_\sigma(\Phi_\sigma, \Psi_\sigma). \tag{10}$$
- There exist matrices $P \in \mathbb{H}^n$ and $0 \prec Q \in \mathbb{H}^n$ satisfying the following inequality.
$$\begin{bmatrix} \mathsf{A} & \mathsf{B} \\ \mathbf{I} & \mathbf{0} \\ \mathsf{C} & \mathsf{D} \\ \mathbf{0} & \mathbf{I}_{n_u} \end{bmatrix}^* \begin{bmatrix} \begin{pmatrix} \Phi_\sigma \otimes P \\ + \Psi_\sigma \otimes Q \end{pmatrix} & \mathbf{0} \\ \mathbf{0} & \Pi_\sigma \end{bmatrix} \begin{bmatrix} \mathsf{A} & \mathsf{B} \\ \mathbf{I} & \mathbf{0} \\ \mathsf{C} & \mathsf{D} \\ \mathbf{0} & \mathbf{I}_{n_u} \end{bmatrix} \prec 0 \tag{11}$$

The dual formulation of GKYP lemma is given by using the following relation with $\Pi_\sigma^d = \mathsf{diag}\left(\mathbf{I}_{n_u}, -\gamma^2\mathbf{I}_{n_y}\right)$

$$\begin{aligned} &\mathsf{G}(\lambda^*)\mathsf{G}(\lambda^*)^* \prec \gamma^2\mathbf{I}_{n_y},\ \forall\lambda \in \Lambda_\sigma\left(\Phi_\sigma, \Psi_\sigma\right) \\ \Leftrightarrow\ &\mathsf{G}(\lambda^*)\mathsf{G}(\lambda^*)^* \prec \gamma^2\mathbf{I}_{n_y},\ \forall\lambda^* \in \Lambda_\sigma\left(\Phi_\sigma^T, \Psi_\sigma^T\right) \\ \Leftrightarrow\ &\sigma\left(\mathsf{G}(\lambda^*)^*, \Pi_\sigma^d\right) \prec 0,\ \forall\lambda^* \in \Lambda_\sigma\left(\Phi_\sigma^T, \Psi_\sigma^T\right) \end{aligned}$$

[1] For its definition, please see the reference.

Lemma 2. (Iwasaki and Hara, 2003) Suppose that Λ_σ represents curves on the complex plane, and that $\det(\lambda\mathbf{I} - \mathsf{A}^*) \neq 0,\ \forall\lambda \in \Lambda_\sigma(\Phi_\sigma, \Psi_\sigma)$ holds. Then, the following two statements are equivalent.

- $\mathsf{G}(\lambda^*)^* = \mathsf{B}^*(\lambda\mathbf{I} - \mathsf{A}^*)^{-1}\mathsf{C}^* + \mathsf{D}^*$ satisfies the following inequality.
$$\mathsf{G}(\lambda^*)\mathsf{G}(\lambda^*)^* \prec \gamma^2\mathbf{I}_{n_y},\ \forall\lambda \in \Lambda_\sigma(\Phi_\sigma, \Psi_\sigma) \tag{12}$$
- There exist matrices $P \in \mathbb{H}^n$ and $0 \prec Q \in \mathbb{H}^n$ satisfying the following inequality.
$$\begin{bmatrix} \mathsf{A}^* & \mathsf{C}^* \\ \mathbf{I} & \mathbf{0} \\ \mathsf{B}^* & \mathsf{D}^* \\ \mathbf{0} & \mathbf{I}_{n_y} \end{bmatrix}^* \begin{bmatrix} \begin{pmatrix} \Phi_\sigma^T \otimes P \\ + \Psi_\sigma^T \otimes Q \end{pmatrix} & \mathbf{0} \\ \mathbf{0} & \Pi_\sigma^d \end{bmatrix} \begin{bmatrix} \mathsf{A}^* & \mathsf{C}^* \\ \mathbf{I} & \mathbf{0} \\ \mathsf{B}^* & \mathsf{D}^* \\ \mathbf{0} & \mathbf{I}_{n_y} \end{bmatrix} \prec 0. \tag{13}$$

The structural constraint of Π_σ^d equivalently makes (13) transformed to the following inequality:

$$\begin{bmatrix} \left(\begin{bmatrix} \mathsf{A} & \mathbf{I} \\ \mathsf{C} & \mathbf{0} \end{bmatrix} \begin{pmatrix} \Phi_\sigma^T \otimes P \\ + \Psi_\sigma^T \otimes Q \end{pmatrix} \begin{bmatrix} \mathsf{A}^* & \mathsf{C}^* \\ \mathbf{I} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\gamma^2\mathbf{I}_{n_y} \end{bmatrix} \right) & \begin{bmatrix} \mathsf{B} \\ \mathsf{D} \end{bmatrix} \\ \begin{bmatrix} \mathsf{B}^* & \mathsf{D}^* \end{bmatrix} & -\mathbf{I}_{n_u} \end{bmatrix} \prec 0. \tag{14}$$

Matrix B has no multiplications with decision matrices P and Q, which is exploited in addressing Problem 1.

3. MAIN RESULT

3.1 Proposed Method

Using the formulation in (14), i.e. the dual formulation of GKYP lemma, we characterize the gain constraints in Assumption 2. Then, the following is proposed for Problem 1.

Theorem 1. Solve the following problem.

$$\min_{\substack{\eta,\,\theta,\,P_{ijl},P_{ijm} \in \mathbb{H}^n, \\ 0 \prec Q_{ijl},Q_{ijm} \in \mathbb{H}^n}} \sum_{q=1}^{\bar{q}} \eta_q^2 \text{ s.t. (16), (17), and (18)} \tag{15}$$

$$\begin{bmatrix} \eta_q^2 & * \\ \delta[\mathcal{X}]_q - \left[(\mathbf{I}_l \otimes A)\,\mathcal{X}_q + (\mathbf{I}_l \otimes B(\theta))\,\mathcal{U}_q\right] & \mathbf{I}_{nl_q} \end{bmatrix} \succ 0, \quad q = \{1, \ldots, \bar{q}\} \tag{16}$$

$$\begin{bmatrix} \left(\begin{bmatrix} A & \mathbf{I} \\ e_i^T & \mathbf{0} \end{bmatrix} \begin{pmatrix} \Phi_\sigma^T \otimes P_{ijl} \\ + \Psi_\sigma^T \otimes Q_{ijl} \end{pmatrix} \begin{bmatrix} A^T & e_i \\ \mathbf{I} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\bar{g}_l^2 \end{bmatrix} \right) & \begin{bmatrix} B(\theta)e_j \\ 0 \end{bmatrix} \\ \begin{bmatrix} e_j^T B(\theta)^T & 0 \end{bmatrix} & -1 \end{bmatrix} \prec 0,$$
$$i = \{1, \ldots, n\},\ j = \{1, \ldots, n_u\},\ l = \{1, \ldots, \bar{l}_{i,j}\} \tag{17}$$

$$\begin{bmatrix} \left(\begin{bmatrix} A & \mathbf{I} \\ e_i^T & \mathbf{0} \end{bmatrix} \begin{pmatrix} \Phi_\sigma^T \otimes P_{ijm} \\ +\Psi_\sigma^T \otimes Q_{ijm} \end{pmatrix} \begin{bmatrix} A^T & e_i \\ \mathbf{I} & \mathbf{0}\end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\varepsilon_m^2 \end{bmatrix} \right) & \begin{bmatrix} B(\theta)e_j \\ -g_m \end{bmatrix} \\ \begin{bmatrix} e_j^T B(\theta)^T & -g_m \end{bmatrix} & -1 \end{bmatrix} \prec 0,$$
$$i = \{1, \ldots, n\},\ j = \{1, \ldots, n_u\},\ m = \{1, \ldots, \bar{m}_{i,j}\} \tag{18}$$

If the problem is feasible and the optimal η and θ are given as $\eta_{\min} = [\eta_{1_{\min}}, \ldots, \eta_{\bar{q}_{\min}}]^T$ and $\hat{\theta}$ respectively, then $\hat{\theta}$ satisfies $\eta_{q_{\min}} > \left\| \delta[\mathcal{X}]_q - \left[(\mathbf{I}_l \otimes A)\, \mathcal{X}_q + \left(\mathbf{I}_l \otimes B(\hat{\theta}) \right) \mathcal{U}_q \right] \right\|$, $(q = 1, \ldots, \bar{q})$ and the constraints in Assumption 2.

Proof. The cost function in Problem 1 is directly transformed to (16). The gain constraints in Assumption 2 are also directly described as (17) and (18) using Lemma 2 and the formulation (14). □

Remark 2. As to-be-estimated parameter θ exists in $B(\theta)$ without multiplications with other decision variables, the bounds of each element in $B(\theta)$ can be also imposed; that is, $\underline{b}_{ij} \le e_i^T B(\theta) e_j \le \bar{b}_{ij}$ $(i = 1, \ldots, n,\ j = 1, \ldots, n_u)$ can be imposed with appropriate scalars $\underline{b}_{ij}$ and $\bar{b}_{ij}$. □

3.2 Application Examples

An Academic Example in DT Case An unstable LTI system $G(\theta)$ in (1) with the following state-space matrices is considered as an example of DT case.

$$[A \,|\, B] = \left[\begin{array}{cccc|cc} 0.95 & -0.002 & 0.2 & 0 & 0.5 & 0 \\ 0.95 & 1.1 & 0 & 0.1 & 0 & 0.5 \\ 0 & -0.1 & 0.3 & -0.4 & 0.2 & -0.3 \\ 0.05 & -0.05 & 0.5 & 0.6 & -0.5 & 0.2 \end{array}\right] \tag{19}$$

In this example, we do not suppose any structural constraints on B matrix; that is, all elements in B matrix can have non-zero elements in parameter estimation problem. The sampling period is set as 0.05 [s].

For this example, we first conducted simulations to obtain measured data in an ideal situation, i.e. noise-free for measured data, and they are shown as dotted lines in Fig. 1. The input matrix $B(\theta)$ is estimated as (20) via the least square method, *viz.* $\min_{\eta,\theta} \sum_{q=1}^{3} \eta_q^2$ s.t. (16), using the noise-free data.

$$B(\hat{\theta}) = \begin{bmatrix} 0.5 & -5.3 \times 10^{-10} \\ -5.6 \times 10^{-10} & 0.5 \\ 0.2 & -0.3 \\ -0.5 & 0.2 \end{bmatrix} \tag{20}$$

As is obvious, the estimation is almost perfect, and the errors η_q between $\delta[\mathcal{X}]_q$ and $\left[(\mathbf{I}_l \otimes A)\, \mathcal{X}_q + \left(\mathbf{I}_l \otimes B(\hat{\theta}) \right) \mathcal{U}_q \right]$ are given as 1.355×10^{-6} (1st data), 1.354×10^{-6} (2nd data), and 1.354×10^{-6} (3rd data). Due to space problem, we omit to show the Bode plots with the real parameters and the estimated ones. Though, it is confirmed that the frequency properties of the system using the estimated input matrix, i.e. $G(z, \hat{\theta})$, are almost identical to those of the actual system, i.e. $G(z, \theta)$.

We next consider the situation that all the measured data are corrupted by the uniform noise; the 1st and 2nd input data are both corrupted by the uniform noise ranging $[-0.5, 0.5]$; the 1st, 2nd, 3rd and 4th state data are corrupted by the uniform noise ranging $[-2.5, 2.5]$, $[-50, 50]$, $[-1, 1]$ and $[-5, 5]$ respectively. They are shown as solid lines in Fig. 1.

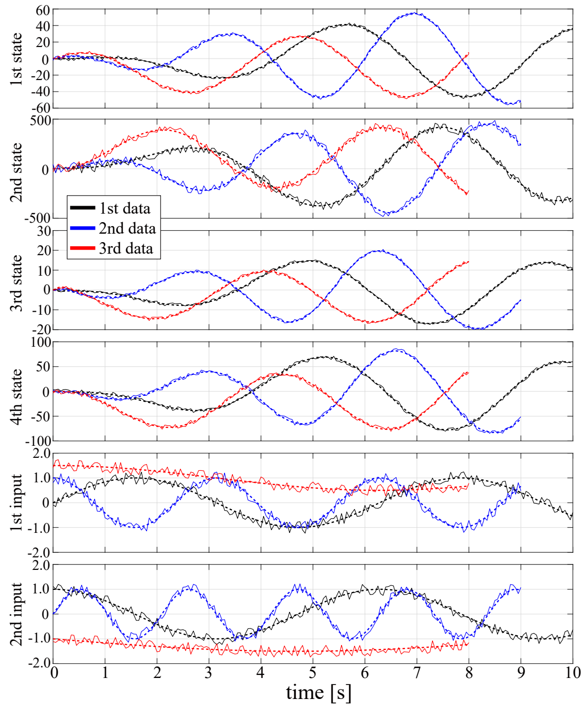

Fig. 1. Simulation data (dotted lines represent noise-free data, solid lines represent noise corrupted data)

Firstly, the input matrix is estimated via the least square method using the noisy data, and the following is obtained.

$$B(\hat{\theta}) = \begin{bmatrix} 0.490 & -0.006 \\ 0.071 & -0.025 \\ -0.147 & -0.383 \\ -0.603 & 0.177 \end{bmatrix} \tag{21}$$

The corresponding Bode plots are depicted in Fig. 2. It is confirmed that the peak gains for the 2nd input are estimated higher than actual.

Then, we impose the following gain constraints.

$$\begin{cases} |G_{12}(e^{j\theta}, \hat{\theta})| < 12,\ 0 \le \theta \le 10 \\ |G_{22}(e^{j\theta}, \hat{\theta})| < 110,\ 0 \le \theta \le 10 \\ |G_{32}(e^{j\theta}, \hat{\theta})| < 5,\ 0 \le \theta \le 10 \\ |G_{42}(e^{j\theta}, \hat{\theta})| < 20,\ 0 \le \theta \le 10 \end{cases} \tag{22}$$

The input matrix is then estimated as follows by using Theorem 1.

$$B(\hat{\theta}) = \begin{bmatrix} 0.505 & 0.025 \\ 0.072 & -0.024 \\ -0.144 & -0.377 \\ -0.606 & 0.172 \end{bmatrix} \tag{23}$$

The corresponding Bode plots are depicted in Fig. 3. The values of $B(\hat{\theta})$ in (23) are similar to those of $B(\hat{\theta})$ in (21); however, the peak gains of $G(z, \hat{\theta})$ become lower than in Fig. 2 to satisfy the constraints in (22). Thus, it has been demonstrated that our method has an ability to impose gain constraints in parameter estimation problem.

Torque Effect Parameter Estimation in CT Case We consider the parameter estimation problem for torque

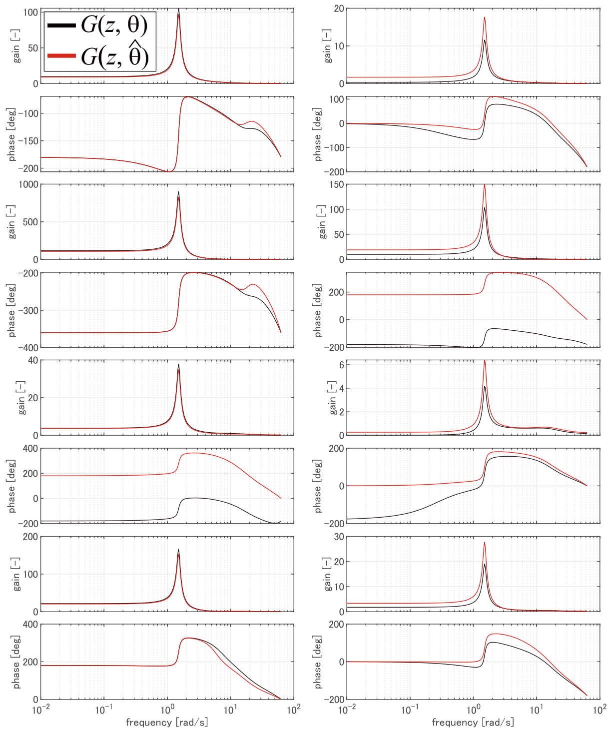

Fig. 2. Bode plots of $G(z, \theta)$ with actual B (black lines) and $G(z, \hat{\theta})$ with the estimated $B(\hat{\theta})$ in (21) (red lines)

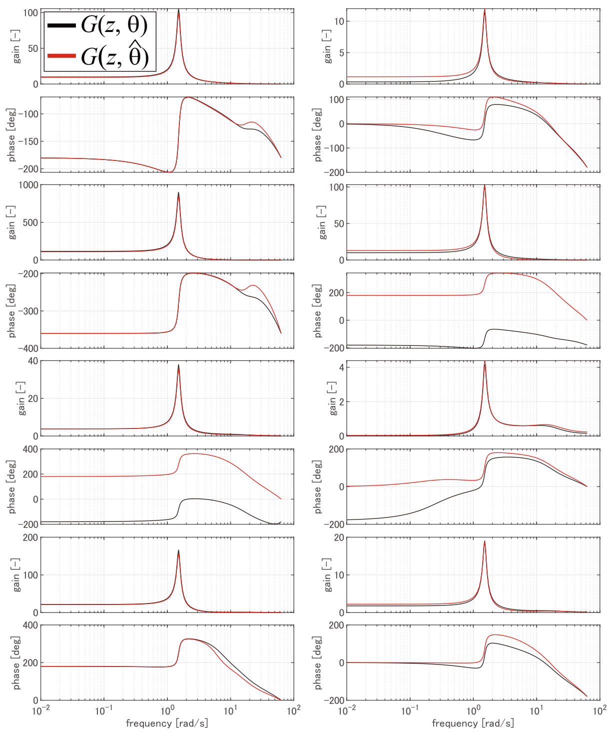

Fig. 3. Bode plots of $G(z, \theta)$ with actual B (black lines) and $G(z, \hat{\theta})$ with the estimated $B(\hat{\theta})$ in (23) (red lines)

effect in asymmetric power lever deviations for MuPAL-α based on Do228-202 (Masui and Tsukano, 2000).

In the standard linearized lateral-directional motions as in (Stevens and Lewis, 1992; Stengel, 2004), roll angle is almost identical to the integral of roll rate, and the roll angle equation can thus be removed in modeling problem. Then, the following state-space model is considered here with the gravity constant g, the initial pitch angle Θ_0 and the initial forward speed U_0:

$$\begin{bmatrix} \dot{v} - g\phi\cos\Theta_0 \\ \dot{p} \\ \dot{r} \end{bmatrix} - \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L'_{\delta_a} & L'_{\delta_r} \\ N'_{\delta_a} & N'_{\delta_r} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} Y_v & Y_p + W & Y_r - U_0 \\ L'_v & L'_p & L'_r \\ N'_v & N'_p & N'_r \end{bmatrix} \begin{bmatrix} v \\ p \\ r \end{bmatrix} + \begin{bmatrix} Y_{\Delta\tau} \\ L_{\Delta\tau} \\ N_{\Delta\tau} \end{bmatrix} \Delta\tau, \tag{24}$$

where v [m/s], p [rad/s], r [rad/s], ϕ [rad], δ_a [rad], δ_r [rad] and $\Delta\tau$ [%] respectively denote lateral airspeed, roll rate, yaw rate, roll angle, aileron deflection angle, rudder deflection angle and the differential torque, i.e. $\tau_l - \tau_r$ with left torque τ_l and right torque τ_r. The roles of the coefficients in the state-space matrices, i.e. Y_v, Y_p, etc., are well established and given in many books, e.g. (Stevens and Lewis, 1992; Stengel, 2004), thus the detailed descriptions are omitted here. On the other hand, the coefficients $Y_{\Delta\tau}$, $L_{\Delta\tau}$ and $N_{\Delta\tau}$ respectively denote the differential torque effects on lateral direction, roll rate and yaw rate, and they are all to be estimated. Theorem 1 can be applied after the left-hand side of (24) is set as $\delta[x]$.

We have two flight data shown as black lines in Figs. 4 and 5. Using the two sets of flight data, we first estimate the input matrix, i.e. $[Y_{\Delta\tau}\ L_{\Delta\tau}\ N_{\Delta\tau}]^T$, via the least square method. They are estimated as follows:

$$Y_{\Delta\tau} = -0.04825,\ L_{\Delta\tau} = 0.00440,\ N_{\Delta\tau} = 0.00189. \tag{25}$$

The optimal η_q's are given as 61.192 for the first data and 68.033 for the second data. The simulation results with $B(\hat{\theta})$ in (25) using the input measured in flight data are shown as red lines in Figs. 4 and 5. The Bode plots using the above values are shown as red lines in Fig. 6.

Next, it is artificially supposed that the following gain constraints should be satisfied.

$$\begin{cases} |G_{11}(j\omega, \hat{\theta})| < 0.05, & 0 \le \omega \le 0.1 \\ |G_{21}(j\omega, \hat{\theta})| < 2 \times 10^{-3}, & 0 \le \omega \le 0.1 \\ |G_{31}(j\omega, \hat{\theta})| < 0.5 \times 10^{-3}, & 0 \le \omega \le 0.1 \end{cases} \tag{26}$$

We then apply Theorem 1, and the following are obtained.

$$Y_{\Delta\tau} = -0.04824,\ L_{\Delta\tau} = 0.00435,\ N_{\Delta\tau} = 0.00112. \tag{27}$$

The optimal η_q's are given as 61.192 for the first data and 68.034 for the second data, which means that the values of the cost functions are almost the same as those via the least square method. The simulation results with $B(\hat{\theta})$ in (27) using the input measured in flight data are shown as blue lines in Figs. 4 and 5. The Bode plots using the above values are shown as blue lines in Fig. 6. It is verified that the artificially imposed constraints are satisfied.

It is also confirmed that the peak gains in the latter case are reduced compared to the former case, and this property consequently reduces amplitudes in simulations shown in Figs. 4 and 5.

4. CONCLUSIONS

We consider parameter estimation problem for input matrix in Linear Time-Invariant (LTI) systems. In contrast to the methods in literature, it is supposed that some gain constraints for some frequency ranges, which should be

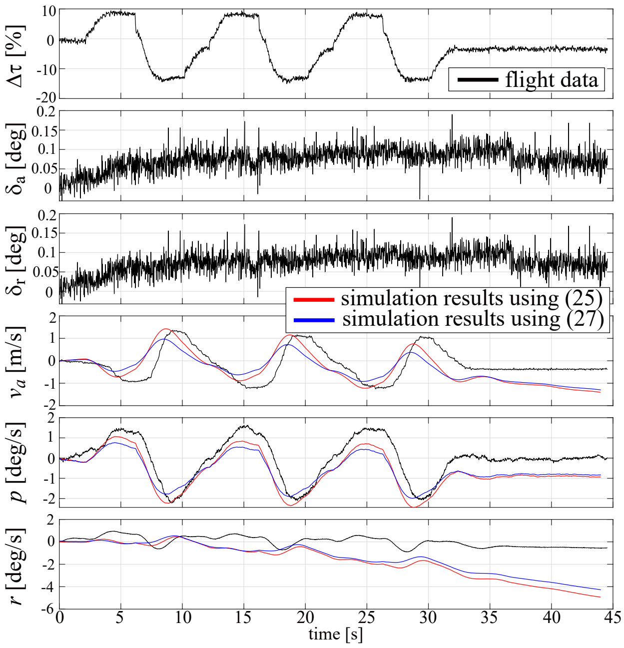

Fig. 4. Flight data #1 for asymmetric torque input (black lines), and simulation results with estimated $B(\hat{\theta})$ in (25) (red lines) and estimated $B(\hat{\theta})$ in (27) (blue lines)

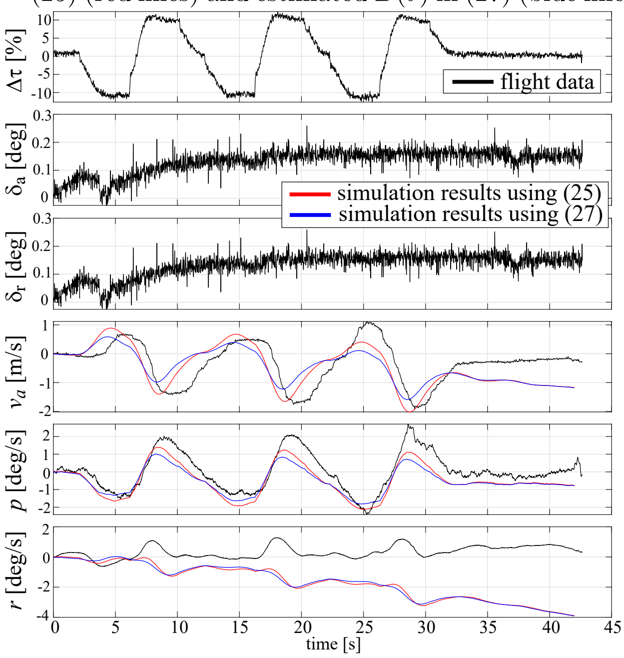

Fig. 5. Flight data #2 for asymmetric torque input (black lines), and simulation results with estimated $B(\hat{\theta})$ in (25) (red lines) and estimated $B(\hat{\theta})$ in (27) (blue lines)

satisfied, are given beforehand, and an estimation method based on the least square method with consideration of the supposed gain constraints is proposed by using Generalized KYP (GKYP) lemma. Our method has the limitation for the applicability; that is, the state transition matrix is supposed to be completely known and only the input matrix has parameters to be estimated. However, we show a practical application example, i.e. asymmetric torque effect estimation problem, and also show the effectiveness of the proposed method using the actual flight data.

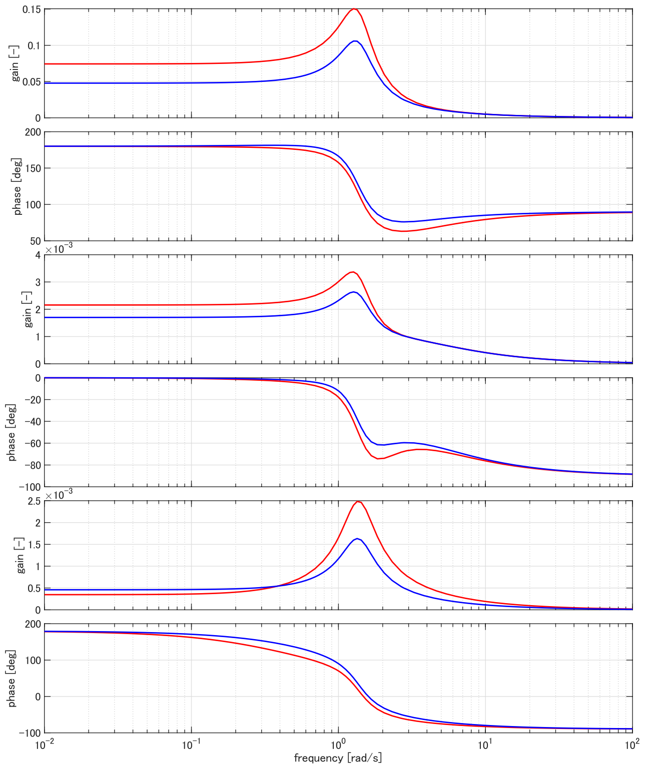

Fig. 6. Bode plots of $G(z, \hat{\theta})$ using $B(\hat{\theta})$ in (25) (red lines) and $B(\hat{\theta})$ in (27) (blue lines)

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