# Adaptive Control of a Vehicular Platoon with Unknown Parameters and Input Variations

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**Abstract:** A switching adaptive control algorithm for automating connected vehicles in a rigid platoon pattern is proposed here. A second-order nonlinear model for the follower vehicles running on the highways is adopted and it is assumed that the parameters of the vehicles's model, including the mass, aerodynamic drag and tire drag, are fully unknown and their values cannot be used in arriving at the control laws. Furthermore, some uncertainties and external perturbations are added to the model to consider the effects of always present modeling errors, un-modeled dynamics and external time varying perturbations on the vehicles. Besides, control input variations are inserted into the nonlinear model of the platoon to represent actuator fluctuations. Subsequently, a robust adaptive control scheme is established so that the asymptotic stability of each vehicle in the platoon is guaranteed, and this is demonstrated using the Lyapunov stability criterion. A novel spacing error variable is also introduced to achieve the global string stability for the whole platoon. Following a comprehensive mathematical analysis, a computer simulation example is presented to illustrate the effectiveness as well as the performance of the proposed control system.

*Keywords:* Autonomous vehicle platooning, Adaptive controller, Control input fluctuation, String stability.

## 1. INTRODUCTION

With the penetration of electronics and software in vehicular/highway systems, automatic vehicle technology utilizing onboard sensing, computing, and communication devices is developing rapidly in recent years. Smart vehicle platooning is an intelligent transportation approach that aims to organize a finite number of automatic vehicles into a rigid string on a highway with short distance and harmonized velocity to track the path of a leader agent. Platooning with small inter-vehicle spacing can bring several important benefits to the smart cities. Reduction in carbon emissions, improvement of the traffic flow congestion, reduction of the aerodynamic drag for lessening fuel consumption, safety increase of the driver and passenger by omitting the effects of human factors, journey time reduction and avoiding fatigue from driving especially during long travels are just some examples of the benefits of smart platooning systems Li et al (2017). Accordingly, during the last decade, a variety of control algorithms, such as nonlinear consensus-based control Li et al (2018), adaptive control Harfouch et al (2018), sliding mode control Li et al (2019), model predictive control Tuchner and Haddad (2017) and neuro-fuzzy control Lin and Nguyen (2019), have been utilized to make sure that the connected vehicle platoons have a correct performance for forming a fully automatic intelligent transportation system.

However, most of the above-mentioned works have trusted on a linear dynamical model for the vehicles appeared in the platoons without considering the effects of modeling errors and un-modeled dynamics. Wu et al (2019) have derived a distributed variable structure control algorithm for platoon control of nonlinear heterogeneous vehicles with a class of generic topologies. Gao et al (2018) have introduced an adaptive sliding mode control technology for stabilization of error dynamics of the nonlinear vehicular platooning systems considering uncertainties. Although, these works have adopted a nonlinear model for vehicles in the platoons, they have ignored a complete guarantee for the string stability for the whole platoon. In fact, the concept of the string stability for an interconnected dynamical system indicates the uniform boundedness of the state of all the connected agents. In our case, for connected vehicle platooning problems, the string stability implies that the tracking (spacing) errors should not amplify downstream from vehicle to vehicle for safety purposes. As a result, in vehicular platooning applications, each vehicle in the interconnected system should be controlled to maintain the desired space and velocity between the vehicles and to guarantee the overall stability of the whole system (i.e.

the string stability). Furthermore, it is known that the string stability of the platooning system cannot be ensured even when the desired space and velocity are kept among the platoon vehicles and a stable platoon can be string unstable Swaroop and Hedrick (1996).

As a matter of fact, when a controller is implemented in practical situations, limitations of the actuators cause some unknown nonlinearities in the control input. Such nonlinearities can be modeled using a control variation part. Ma et al (2019) have designed robust distributed sliding mode controllers for smart vehicular transportation systems with uncertainties. However, that research has neglected the compensation of the control input variations. Platoon control of vehicular systems with dead-zone nonlinearity in the control input without string stability analysis has been investigated in Guo et al (2017). In Guo et al (2018a) and Guo et al (2018b), two adaptive controllers have been developed for connected vehicles in a platoon with input saturation. The problem of longitudinal control of vehicle platoons with control gain uncertainties has been solved via an adaptive intelligent backstepping output recurrent cerebellar model articulation controller in Peng (2010). Nonetheless, there are always some steady state errors in the spacing errors of the previous approaches Guo et al (2017)-Peng (2010) and the spacing errors are guaranteed to precisely converge to zero.

Generally speaking, in practice, it is hard to exactly determine the values of the parameters of the vehicular systems as some of them may be uncertain and even time varying. Reference Kwon and Chwa (2014) proposed adaptive sliding mode controllers for platooning of vehicular systems with unknown parameters. Also, Zhu and Zhu (2019) provided an adaptive backstepping control scheme for the platoon control of connected vehicles with uncertain parameters. An adaptive neuro-fuzzy controller for automatic vehicular platoon systems was derived by Lin and Nguyen (2019) for some connected vehicles with unknown parameters. Two adaptive Lyapunov-based control algorithms were developed by Chehardoli and Ghasemi (2019) and Zhu and Zhu (2018) to achieve the asymptotic stability for the spacing errors of the vehicle platooning transportations under the existence of some unknown parameters. However, there are a number of shortcomings with most of the aforementioned works as they either did not consider the effects of control input uncertainties, they are not robust against uncertain terms and external disturbances, they have used the derivative of the accelerations in control inputs which its computation might bring some noises to the computations, they have failed to provide a rigorous proof for the string stability of the overall platoon or their approaches are complex to be implemented in practice.

Inspired by the above discussions, the main purpose of this article is to propose an adaptive efficient robust control scheme for connected vehicles in a platoon. First, we propose a new spacing error variable to construct a string stable structure. The proposed spacing error variable not only uses the position errors of the vehicles, but also it utilizes the velocity errors of the vehicles to result in a more effective approach. The effect of modeling uncertainties, unknown parameters and external perturbations as well as control gain variations are fully taken into account. Also, there is no need for any information about the bounds of the uncertain terms and external fluctuations. Contrary to some previous works, the proposed control strategy in this research does not use the acceleration derivative in the control input which makes it more convenient for implementation. After analyzing the robust stability as well as the string stability of the platooning model, computer simulations verify the robustness and efficacy of the introduced vehicle platooning automatic control strategy.

The reminder of this study is structured as follows. In Section 2, the vehicle model is described briefly and the platooning problem is formulated along with the necessary assumptions. Section 3 gives the details of the proposed robust adaptive control technique. Section 4 provides an illustrative computer example for the control platform. Finally, concluding remarks and future perspectives are presented in Section 5.

## 2. PROBLEM FORMULATION

A platoon of $N$ follower vehicles and a leader are organized in a line to drive on a straight road in a string (see Fig. 1). The longitudinal motion dynamics of the follower vehicles with uncertain parts and control input fluctuations is considered as a second-order nonlinear differential equation as follows:

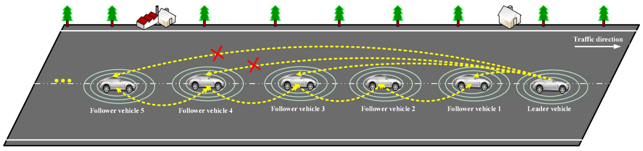


Fig. 1. Connected vehicles in string running on a road Gao et al (2018)

$$\ddot{x}_i = \frac{(u_i + \Delta u_i) - c_i \dot{x}_i^2 - F_i + \Delta f_i(X_i, t)}{M_i} \tag{1}$$

where $x_i$ presents the position of the $i$th vehicle in the platoon with respect to an inertial frame, $u_i$ stands for the control torque to be designed later, $\Delta u_i$ is the control input variations, $c_i x_i^2$ shows the aerodynamic drag force of the vehicle, $F_i$ presents the mechanical drag of the $i$th vehicle, $M_i$ is the mass of the vehicle, $\Delta f_i(X_i, t)$ stands for a time varying uncertainty and external disturbance term and $X_i = [x_i, \dot{x}_i, \ddot{x}_i]^T$.

Assumption 1. Without loss of generality, we consider the term $\Delta f_i(X_i, t) + \Delta u_i$ as a lumped uncertainty and we suppose that it is bounded as follows.

$$\|\Delta f_i(X_i, t) + \Delta u_i\| \leq \alpha_i \|X_i\| + \beta_i \tag{2}$$

where $\alpha_i$ and $\beta_i$ are two unknown positive constants.

Assumption 2. We assume that the constant parameters $c_i$, $F_i$ and $M_i$ of the vehicle dynamics (1) are unknown.

Assumption 3. It is assumed that the communication topology of the platoon enables vehicle $i$ to have access to the lead vehicle position ($x_0$), velocity ($v_0$) and acceleration ($a_0$). Also, the information of the position, velocity and acceleration of the $(i-1)$th vehicle is assumed to be available for the $i$th vehicle via proper communications.

According to the communication policy mentioned above, and assuming that the length of the $i$th vehicle in the platoon is $L_i$ with $P_i$ as the desired vehicle spacing between the vehicles $i$ and $(i-1)$, we propose a new spacing error formation for the platooning organization as follows:

$$z_i = e_i^x + \lambda_i e_{i,0}^x + \gamma_i e_i^v \tag{3}$$

where $e_i^x = x_i - x_{i-1} + L_i + P_i$ is the spacing error between $i$th and $(i-1)$th vehicle, $e_{i,0}^x = x_i - x_0 + \sum_{j=1}^{i}(L_j + P_j)$ presents the spacing error between $i$th and leader vehicle, $e_i^v = v_i - v_0$ stands for the velocity error between $i$th and the leader vehicle, the subscripts $i$ and 0 stand for the ith vehicle and the leader, respectively, and $\lambda_i$ and $\gamma_i$ are two positive constants representing the coupling strengths of the position and velocity errors, respectively.

## 3. MAIN RESULTS

### *3.1 Adaptive Controller Design*

This section presents a Lyapunov-based adaptive control strategy to guarantee the stability of the platoon system without having any prior knowledge about the parameters of the vehicles as well as the bounds of the lumped uncertainty. It is well-known that the main idea of the direct adaptive control theory is to introduce some proper adaptation rules for the unknown parameters of the system so that the overall stability of the system can be ensured without explicit usage of the values of the unknown parameters in the control signals. Since there are three unknown parameters in the vehicle dynamics (i.e. $c_i$, $F_i$ and $M_i$) as well as two unknown parameters in the bounds of the lumped uncertainty term (i.e. $\alpha_i$ and $\beta_i$), we should define five corresponding adaptation laws for compensating their effects. Before proceeding to the derivation of the adaptation rules, the dynamics of the proposed spacing errors (3) are obtained as follows.

Taking time derivative of (3), one obtains

$$\dot{z}_i = \dot{x}_i - \dot{x}_{i-1} + \lambda_i \dot{x}_i - \lambda_0 v_0 + \gamma_i \dot{v}_i - \gamma_0 \dot{v}_0 \tag{4}$$

It is clear that the above equation can be rewritten as follows.

$$\dot{z}_i = (\lambda_i + 1)\dot{x}_i - \dot{x}_{i-1} + \gamma_i \ddot{x}_i - \lambda_0 v_0 - \gamma_0 a_0 \tag{5}$$

It is noted that, in general, every constant parameter in the system can be written as a coefficient of a nonlinear function $A_i(X_i)$ (e.g. $A_i(X_i)$ for the parameter $c_i$ is $\dot{x}_i^2$ and for the parameter $\beta_i$ is 1). Noting this, and in order to select some intelligent adaptation rules for the control signal, we propose the $A_i(X_i)$ functions of the unknown parameters multiplied by the spacing error variable $z_i$ as the adaptation rules. The utilization of the spacing error in the adaptation rules will guarantee that the adaptation parameters will converge to some fixed values as the spacing errors reach zero. Based on the above discussion, the proper adaptation rules are:

$$\begin{aligned}
&\dot{\hat{c}}_i = -r_i z_i v_i^2, \hat{c}_i(0) = c_{i0} \\
&\dot{\hat{F}}_i = -n_i z_i, \hat{F}_i(0) = F_{i0} \\
&\dot{\hat{\alpha}}_i = s_i \|z_i\|^2, \hat{\alpha}_i(0) = \alpha_{i0} \\
&\dot{\hat{\beta}}_i = w_i \|z_i\|, \hat{\beta}_i(0) = \beta_{i0} \\
&\dot{\hat{M}}_i = q_i z_i[(\lambda_i + 1)v_i - v_{i-1} - \lambda_i v_0 - \gamma_i a_0], \hat{M}_i(0) = M_{i0}
\end{aligned} \tag{6}$$

where $r_i$, $n_i$, $s_i$, $w_i$ and $q_i$ are positive constants acting as learning factors for the corresponding adaptation rules and $c_{i0}$, $F_{i0}$, $\alpha_{i0}$, $\beta_{i0}$ and $M_{i0}$ are initial states for the adaptation parameters.

It is now possible to propose a suitable control law to assure the asymptotic stability for the spacing error dynamics (3). Here, a switching adaptive control law is proposed as follows.

$$u_i = -\{\frac{1}{\gamma_i}\dot{\hat{M}}_i \hat{M}_i + \dot{\hat{c}}_i \hat{c}_i + \dot{\hat{F}}_i \hat{F}_i + \dot{\hat{\alpha}}_i \hat{\alpha}_i + \dot{\hat{\beta}}_i \hat{\beta}_i + k_i sign(z_i)\} \tag{7}$$

where $k_i$ is a positive constant switching gain and if $z_i = 0$, then $sign(z_i) = 0$.

It is noted that the term $sign(z_i)$ in the control input will guarantee the fast convergence of the spacing errors to zero. On the other hand, to evade the possible shocks on the control signals, one can replace it by a smooth function like $tanh(z_i)$.

**Theorem 1**. Consider the platoon spacing error dynamics (3) with the Assumptions 1-3. If this system is controlled via the switching control signal (7) along the adaptation laws in (6), then the spacing errors will converge to zero asymptotically.

Proof. Choose a Lyapunov function candidate for each vehicle in the platoon in the form of

$$V_i = \frac{1}{2}(M_i z_i^2 + \frac{\gamma_i \tilde{c}_i^2}{r_i} + \frac{\gamma_i \tilde{F}_i^2}{n_i} + \frac{\gamma_i \tilde{\alpha}_i^2}{s_i} + \frac{\gamma_i \tilde{\beta}_i^2}{w_i} + \frac{\tilde{M}_i^2}{q_i}) \tag{8}$$

where $\tilde{c}_i = \hat{c}_i - c_i$, $\tilde{F}_i = \hat{F}_i - F_i$, $\tilde{\alpha}_i = \hat{\alpha}_i - \alpha_i$, $\tilde{\beta}_i = \hat{\beta}_i - \beta_i$ and $\tilde{M}_i = \hat{M}_i - M_i$ are estimation errors for the unknown parameters $c_i$, $F_i$, $\alpha_i$, $\beta_i$ and $M_i$, respectively.

Taking time derivative of the Lyapunov function with respect to time, one obtains

$$\dot{V}_i = M_i z_i \dot{z}_i + \frac{\gamma_i \tilde{c}_i \dot{\hat{c}}_i}{r_i} + \frac{\gamma_i \tilde{F}_i \dot{\hat{F}}_i}{n_i} + \frac{\gamma_i \tilde{\alpha}_i \dot{\hat{\alpha}}_i}{s_i} + \frac{\gamma_i \tilde{\beta}_i \dot{\hat{\beta}}_i}{w_i} + \frac{\tilde{M}_i \dot{\hat{M}}_i}{q_i} \tag{9}$$

Inserting the spacing error dynamics in (5) into the above equation, this yields

$$\begin{aligned}
\dot{V}_i &= M_i z_i((\lambda_i + 1)\dot{x}_i - \dot{x}_{i-1} + \gamma_i \ddot{x}_i - \lambda_i \dot{v}_0 - \gamma_i a_0) \\
&+ \frac{\gamma_i \tilde{c}_i \dot{\hat{c}}_i}{r_i} + \frac{\gamma_i \tilde{F}_i \dot{\hat{F}}_i}{n_i} + \frac{\gamma_i \tilde{\alpha}_i \dot{\hat{\alpha}}_i}{s_i} + \frac{\gamma_i \tilde{\beta}_i \dot{\hat{\beta}}_i}{w_i} + \frac{\tilde{M}_i \dot{\hat{M}}_i}{q_i}
\end{aligned} \tag{10}$$

Introducing the vehicle dynamics (1) to (10), one has

$$\begin{aligned}
\dot{V}_i &= M_i z_i((\lambda_i + 1)\dot{x}_i - \dot{x}_{i-1} + \frac{\gamma_i}{M_i}u_i - \frac{\gamma_i c_i}{M_i}v_i^2 - \frac{\gamma_i}{M_i}F_i \\
&+ \frac{\gamma_i}{M_i}(\Delta f_i(X_i, t) + \Delta u_i) - \lambda_0 v_0 - \gamma_0 a_0 \\
&+ \frac{\gamma_i \tilde{c}_i \dot{\hat{c}}_i}{r_i} + \frac{\gamma_i \tilde{F}_i \dot{\hat{F}}_i}{n_i} + \frac{\gamma_i \tilde{\alpha}_i \dot{\hat{\alpha}}_i}{s_i} + \frac{\gamma_i \tilde{\beta}_i \dot{\hat{\beta}}_i}{w_i} + \frac{\tilde{M}_i \dot{\hat{M}}_i}{q_i}
\end{aligned} \tag{11}$$

Simplifying the above equation, we will have

$$\begin{aligned}
\dot{V}_i &= z_i\{M_i((\lambda_i + 1)v_i - v_{i-1} - \lambda_0 v_0 - \gamma_0 a_0) \\
&+ \gamma_i u_i - \gamma_i c_i v_i^2 - \gamma_i F_i + \gamma_i(\Delta f_i(X_i, t) + \Delta u_i)\} \\
&+ \frac{\gamma_i \tilde{c}_i \dot{\hat{c}}_i}{r_i} + \frac{\gamma_i \tilde{F}_i \dot{\hat{F}}_i}{n_i} + \frac{\gamma_i \tilde{\alpha}_i \dot{\hat{\alpha}}_i}{s_i} + \frac{\gamma_i \tilde{\beta}_i \dot{\hat{\beta}}_i}{w_i} + \frac{\tilde{M}_i \dot{\hat{M}}_i}{q_i}
\end{aligned} \tag{12}$$

It is obvious that

$$\dot{V}_i \le z_i\{M_i((\lambda_i+1)v_i - v_{i-1} - \lambda_0 v_0 - \gamma_0 a_0) \\ +\gamma_i u_i - \gamma_i c_i v_i^2 - \gamma_i F_i\} + \gamma_i\|z_i\|\|\Delta f_i(X_i,t) + \Delta u_i\| \\ +\frac{\gamma_i\tilde{c}_i\dot{\hat{c}}_i}{r_i} + \frac{\gamma_i\tilde{F}_i\dot{\hat{F}}_i}{n_i} + \frac{\gamma_i\tilde{\alpha}_i\dot{\hat{\alpha}}_i}{s_i} + \frac{\gamma_i\tilde{\beta}_i\dot{\hat{\beta}}_i}{w_i} + \frac{\tilde{M}_i\dot{\hat{M}}_i}{q_i} \tag{13}$$

Based on Assumption 1, one gets

$$\dot{V}_i \le z_i\{M_i((\lambda_i+1)v_i - v_{i-1} - \lambda_0 v_0 - \gamma_0 a_0) \\ +\gamma_i u_i - \gamma_i c_i v_i^2 - \gamma_i F_i\} + \gamma_i\|z_i\|(\alpha_i\|X_i\| + \beta_i) \\ +\frac{\gamma_i\tilde{c}_i\dot{\hat{c}}_i}{r_i} + \frac{\gamma_i\tilde{F}_i\dot{\hat{F}}_i}{n_i} + \frac{\gamma_i\tilde{\alpha}_i\dot{\hat{\alpha}}_i}{s_i} + \frac{\gamma_i\tilde{\beta}_i\dot{\hat{\beta}}_i}{w_i} + \frac{\tilde{M}_i\dot{\hat{M}}_i}{q_i} \tag{14}$$

Inserting the adaptation rules in (3) into the above inequality, one obtains

$$\dot{V}_i \le z_i\{M_i((\lambda_i+1)v_i - v_{i-1} - \lambda_0 v_0 - \gamma_0 a_0) \\ +\gamma_i u_i - \gamma_i c_i v_i^2 - \gamma_i F_i\} + \gamma_i\|z_i\|(\alpha_i\|X_i\| + \beta_i) \\ -\gamma_i(\hat{c}_i - c_i)z_i v_i^2 - \gamma_i(\hat{F}_i - F_i)z_i + \gamma_i(\hat{\alpha}_i - \alpha_i)\|z_i\|^2 \\ +\gamma_i(\hat{\beta}_i - \beta_i)\|z_i\| + (\hat{M}_i - M_i)z_i\times \\ ((\lambda_i+1)v_i - v_{i-1} - \lambda_i v_0 - \gamma_i a_0) \tag{15}$$

After some mathematical simplifications, we have

$$\dot{V}_i \le z_i\gamma_i u_i - \gamma_i\hat{c}_i z_i v_i^2 - \gamma_i\hat{F}_i z_i + \gamma_i\hat{\alpha}_i\|z_i\|^2 + \gamma_i\hat{\beta}_i\|z_i\| \\ +\hat{M}_i z_i((\lambda_i+1)v_i - v_{i-1} - \lambda_i v_0 - \gamma_i a_0) \tag{16}$$

Introducing the switching control input in (7) into the above equation, one has

$$\dot{V}_i \le -z_i\gamma_i\{\frac{1}{\gamma_i}\dot{\hat{M}}_i\hat{M}_i + \dot{\hat{c}}_i\hat{c}_i + \dot{\hat{F}}_i\hat{F}_i + \dot{\hat{\alpha}}_i\hat{\alpha}_i + \dot{\hat{\beta}}_i\hat{\beta}_i \\ +k_i sign(z_i)\} - \gamma_i\hat{c}_i z_i v_i^2 - \gamma_i\hat{F}_i z_i + \gamma_i\hat{\alpha}_i\|z_i\|^2 + \gamma_i\hat{\beta}_i\|z_i\| \\ +\hat{M}_i z_i((\lambda_i+1)v_i - v_{i-1} - \lambda_i v_0 - \gamma_i a_0) \tag{17}$$

Noting to the fact $z_i sign(z_i) = |z_i|$ and based on the adaptation rules in (6), the above inequlity becomes

$$\dot{V}_i \le -k_i|z_i| \le 0 \tag{18}$$

Therefore, using a final Lyapunov function as $\sum_{i=1}^{N} V_i$ the conclusion $\sum_{i=1}^{N}\dot{V}_i \le \sum_{i=1}^{N} -k_i|z_i| \le 0$ is made. Thus, according to the Lyapunov stability theory, the error states of the platoon will attain zero asymptotically. ■

### 3.2 String Stability Analysis

String stability ensures each vehicle in the platoon maintains a desired safety distance from its leading vehicle and avoids collision. The (strong) string stability definition is given below Kwon and Chwa (2014).

Definition 1. *(strong) String stability*: Origin $e_i = 0$, with $i \in N$ with the dynamics in (1), is string stable in the strong sense if error propagation transfer function $H_i(s) = \frac{E_{i+1}(s)}{E_i(s)}$ satisfies $\|H_i(s)\| \le 1$ for all $i \in N$, where $E_i(s)$ stands for the Laplace transform of $e_i$. Then, it can be shown that the tracking error of the platoon system is uniformly bounded. It is noted that although the considered vehicles' dynamics in this work are nonlinear, the proposed spacing errors are linear. So, one can use the string stability concept for the model.

**Theorem** 2. The connected vehicle platooning system with spacing error dynamics (3) is string stable under the robust adaptive controller (6) and (7), if the parameter $\lambda_i$ is chosen greater than zero.

Proof. First it is noted that according to the definition of the spacing error in (3) one can reach the following relations.

$$e_{i,0}^x = e_i^x + \sum_{j=1}^{i-1} e_j^x = e_{i-1,0}^x + e_i^x \tag{19}$$

$$\dot{e_i^x} = \dot{x}_i - \dot{x}_{i-1} = v_i - v_{i-1} \tag{20}$$

Subsequently, according to the stability of the origin for the spacing error dynamics provided by the proposed adaptive controller and proved in Theorem 1, we have

$$z_i = e_i^x + \lambda_i e_{i,0}^x + \gamma_i e_i^v = 0 \Rightarrow \lambda_i e_{i,0}^x = -e_i^x - \gamma_i e_i^v \tag{21}$$

$$z_{i-1} = e_{i-1}^x + \lambda_{i-1}e_{i-1,0}^x + \gamma_{i-1}e_{i-1}^v = 0 \tag{22}$$

Inserting (21) to (19), one can conclude

$$\lambda_{i-1}e_{i,0}^x = -(1+\lambda_i)e_i^x - \gamma_i e_i^v \tag{23}$$

Introducing the above equation to (22), one gets

$$e_{i-1}^x - (1+\lambda_i)e_i^x - \gamma_i e_i^v + \gamma_{i-1}e_{i-1}^v = 0 \tag{24}$$

Using $e_i^v = v_i - v_0$ and $e_{i-1}^v = v_{i-1} - v_0$, the above equation turns into

$$e_{i-1}^x - (1+\lambda_i)e_i^x - (v_i - v_{i-1}) = 0 \tag{25}$$

Taking time derivative of $e_i^x = x_i - x_{i-1} + L_i + p_i$, one reaches $\dot{e}_i^x = \dot{x}_i - \dot{x}_{i-1} = v_i - v_{i-1}$. Adopting this result in (25), one can get

$$e_{i-1}^x - (1+\lambda_i)e_i^x - \dot{e}_i^x = 0 \tag{26}$$

Taking the Laplace transform of (26), we have

$$E_{i-1}^x(s) = (1+\lambda_i)E_i^x(s) + sE_i^x(s) \tag{27}$$

It follows from (27)

$$H_i(s) = \frac{1}{1+\lambda_i+s} \tag{28}$$

One can easily check that for $s = j\omega, \lambda_i > 0$ the maximum magnitude of this transfer function will be always less than 1. Thus, the string stability of the platoon is guaranteed.■

## 4. SIMULATION RESULTS

This section provides some illustrative numerical simulations to validate the effective performance of the developed adaptive switching controller for platoon formation of autonomous connected vehicles. We consider a platoon of five follower vehicles indexed by $1, 2, 3, 4$ and 5 to track the path of a leader indexed by 0. Without loss of generality and for simplicity, we choose an identical vehicle length of 8 for all the followers with a desired spacing distance of 2. The parameters of the vehicles' dynamics are set as $c_i = 0.008$, $f_i = 0.001$ and $M_i = 1100$. The time varying model uncertainties and external disturbances are chosen as $\Delta f_i(X_i,t) = 0.5sin(x_i) + 0.3cos(t)$. Furthermore, the control input variations are selected as $\Delta u_1 = 0.3cos(u_1)$, $\Delta u_2 = 0.35sin(5u_2)$, $\Delta u_3 = -0.35tanh(5u_3)$, $\Delta u_4 = 0.4cos(0.1u_4)$ and $\Delta u_1 = -0.3tanh(2u_5)$. The parameters of the proposed control scheme are set as

$k_i = 0.1$, $\lambda_i = 0.9$, $\gamma_i = 0.5$, $r_i = 1$ and $n_i = s_i = w_i = q_i = 10$. The initial conditions of the adaptation parameters are all set to zero. To implement a more practical situation, non-zero initial states are given for the platoon spacing error variables. As a result, the initial conditions for the vehicles' positions and velocities are selected as follows: $(x_0, v_0) = (70, 18)$, $(x_1, v_1) = (60, 17.5)$, $(x_2, v_2) = (50, 17)$, $(x_3, v_3) = (40, 16.5)$, $(x_4, v_4) = (30, 16)$ and $(x_5, v_5) = (20, 15.5)$ . And, the time evolution of the leader vehicle's acceleration is adopted as follows.

$$a_0 = \begin{cases} 0 & \text{for } t < 5 \\ -0.3(t-5) & \text{for } 5 \le t < 8 \\ -0.8 & \text{for } 8 \le t < 11 \\ 0.3(t-11) - 0.8 & \text{for } 11 \le t < 17 \\ 0.8 & \text{for } 17 \le t < 20 \\ 0.3(20-t) + 0.8 & \text{for } 20 \le t < 23 \\ 0 & \text{for } 23 \le t \le 30 \end{cases} \quad (29)$$

Fig. 2 illustrates the time evolutions of the positions of the vehicles controlled via the proposed robust adaptive switching controller. Obviously, the developed controller is successful in providing safe paths for the follower vehicles without colliding with each other while tracking the leader. Also, a desired inter-vehicle spacing between the vehicles are ensured. The time histories of the velocities of the vehicles in the platoon are shown in Fig. 3. It can be seen that the desired velocity of the leader vehicle is tracked by the follower vehicles within a reasonable time range. The time histories of the vehicles' accelerations appear in Fig 4. It is clear that the connected follower vehicles track the time varying desired acceleration (29) via successful implementation of the introduced switching adaptive controller. Fig. 5 shows the time evolutions of the spacing errors. One sees that the spacing errors converge to zero which implies that the string stability of the platoon is indeed guaranteed in spite of the lumped uncertainties as well as control input fluctuations. The time evolutions of the adopted adaptation parameters are depicted in Fig. 6. One can observe that the adaptation parameters are bounded and converge to some fixed values. It means that the internal stability of the derived control system is not destroyed using the adaptation scheme.

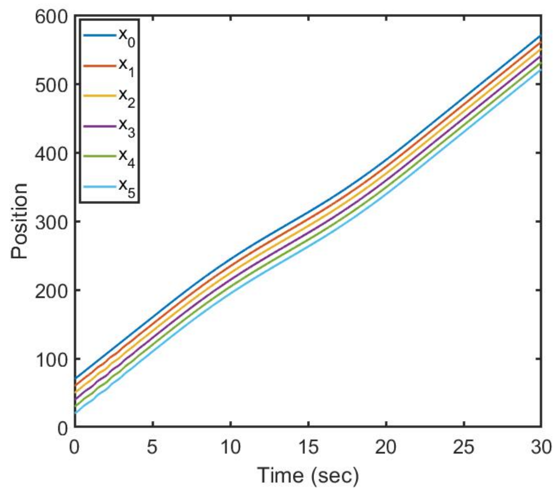


Fig. 2. Time evolutions of the vehicles' positions

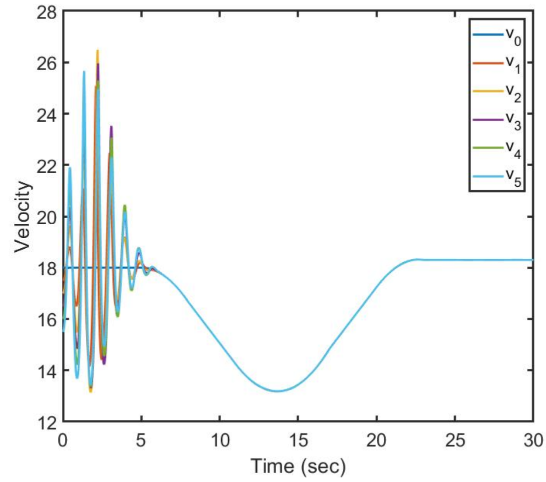


Fig. 3. Time evolutions of the vehicles' velocities

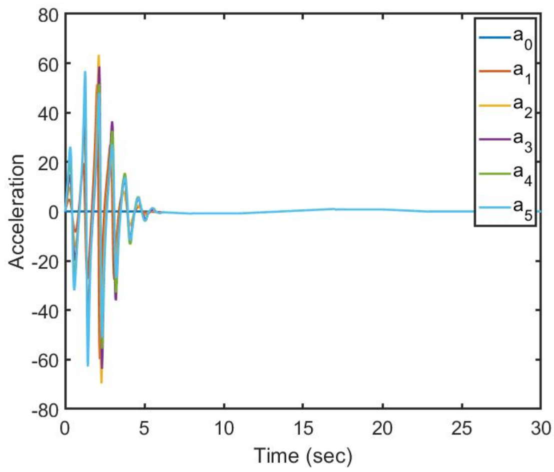


Fig. 4. Time evolutions of the vehicles' accelerations

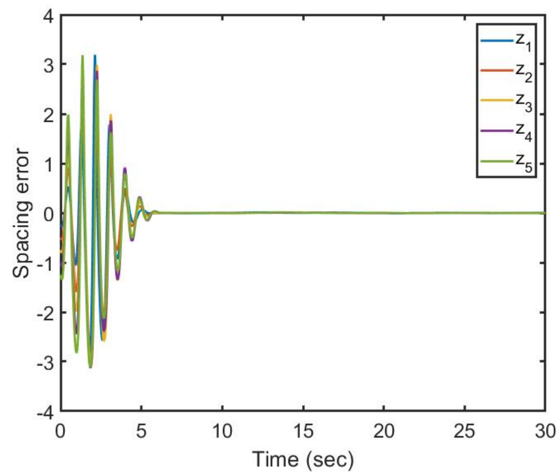


Fig. 5. Time evolutions of the spacing errors

## 5. CONCLUDING REMARKS

This study addressed the design of a robust and adaptive control algorithm for autonomous intelligent connected vehicles to follow a leader in a highway transportation sys-

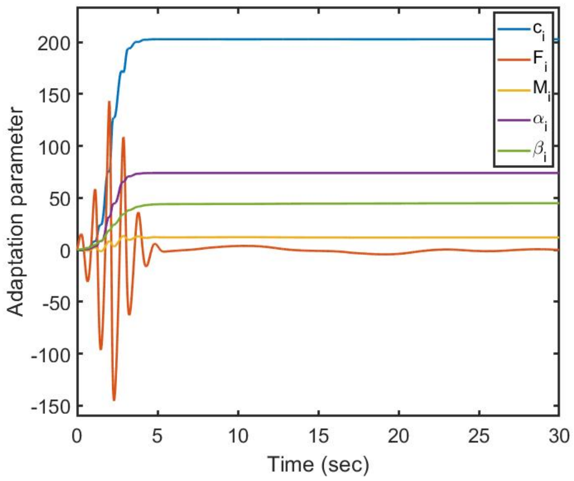


Fig. 6. Time evolutions of the adaptation parameters

tem. Without knowing the exact values of the parameters of the vehicles, the main idea was to develop a switching control methodology such that the stability of the platoon was guaranteed. Moreover, the effects of modeling errors and control input uncertainties were taken into account without requiring any prior information about the bounds of the uncertain terms. This is more realistic compared to reported ideal situations considered in the previous studies. Accordingly, a novel spacing error was introduced to not only realize an efficient approach for individual vehicle's performance in the platoon, but also to guarantee strong string stability of the overall system. After rigorous stability analysis, some illustrative numerical simulations were given to show that the proposed adaptive controller is an effective approach for platooning of the connected vehicles with no needs for the derivatives of the accelerations in which it avoids the possible noises captured while taking time derivatives of the variables. Development of a fault tolerant robust controller for the same problem as well as adding saturation nonlinearities for the accelerations of the vehicles are of interest of the authors to be remained as future works.

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