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Pareto-Improving Pricing Schemes for Route Assignment of Heterogeneous Users ⋆

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Abstract: Traffic congestion constitutes a major problem in commercial areas having negative effects on travel times, fuel consumption and other operational costs. Additionally, the continuously increasing use of GPS technologies has made drivers to make routing decisions in an effort to minimize their own individual travel time which is known to lead to an inefficient road usage. In this paper, we propose a novel pricing scheme to alleviate traffic congestion by controlling the freight routing decisions through a coordination mechanism. The proposed mechanism asks the truck drivers to declare their Origin-Destination (OD) pair and their individual Value Of Time (VOT) and guarantees that every participant truck driver will be better-off compared to the User Equilibrium (UE) by providing them individual incentives to truthfully declare their VOT while leading to a budget balanced on average mechanism. The optimum route assignment and the resulting pricing scheme can be calculated by solving a nonconvex optimization problem. To reduce the dimensionality of the problem, we propose a second pricing scheme and we prove that satisfies the aforementioned characteristics. Finally, the evaluation of our approach using the Sioux Falls network shows that the proposed pricing schemes can make the network approach the System Optimum (SO) solution.

Keywords: Road pricing, Optimization, Mechanism design, Traffic assignment, Truck

1. INTRODUCTION

Transportation services contributed $1,066.9 billion to the United States (U.S.) gross domestic product (GDP) in 2016, (Bureau of Transportation Statistics, 2018). Additionally, trucking contributed to the largest amount of the freight modes adding a value of $288.2 billion in the U.S. GDP, (Bureau of Transportation Statistics, 2019). These statistics demonstrate the necessity for the creation of an efficient freight load balancing system.

The increased usage of routing apps has led drivers to make routing decisions in a manner that minimizes their own individual travel time. This behavior of network users not only leads to an inefficient road usage which is known in the literature as User Equilibrium (UE), (Wardrop, 1952), but it additionally has several negative externalities such as the increase in traffic in cities bordering highway from users taking local routes to avoid congestion, (Thai et al., 2016). On the other hand, a System Optimum (SO) solution, i.e. a situation where the network users cooperate such that the total travel time of the network is minimized, cannot be applied in practice since some drivers may be worse-off compared to the UE solution.

To address this problem, one of the most frequently proposed solutions is congestion pricing, (Beckmann et al., 1956). However, several concerns regarding equity and fairness considerations of congestion pricing have been addressed, (Giuliano, 1994). Under the assumption that the Origin-Destination (OD) demand of the drivers is deterministic, Guo and Yang (2010) studied possible solutions by proposing a combined congestion pricing scheme with uniform revenue refunding for heterogeneous users where each user has his/her own Value of Time (VOT) and guaranteed that this scheme is Pareto-improving, i.e. every user will be better-off compared to the UE. Recently, Gu et al. (2018) studied the public acceptance of congestion pricing, while Tscharaktschiew and Evangelinos (2019); Simoni et al. (2019) studied congestion pricing techniques in the presence of autonomous vehicles. However, Koster et al. (2018) showed that congestion pricing under preference heterogeneity can lead to politically unacceptable solutions since low income travelers may have to pay a higher tax.

In this work, we design Pareto-improving pricing schemes to alleviate traffic congestion through a coordinated freight

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routing mechanism where we take into account the existence of truck drivers with different VOT. The importance of taking into account heterogeneity in VOT when studying pricing schemes had been mentioned by van den Berg and Verhoef (2011). Unlike several previous works that made assumptions about the distribution that the user heterogeneity may follow, e.g. (Wang et al., 2018; Zhu et al., 2014), in our work, we propose that the coordinator asks the truck drivers to pick their VOT from a set of N possible options and subsequently designs a pricing scheme guaranteeing that each individual user will have an incentive to truthfully declare his/her VOT while concurrently leading to a budget balanced on average mechanism. The optimum pricing scheme can be calculated by solving a nonconvex optimization problem. Subsequently, we propose a second pricing scheme to approximate the optimum solution and we mathematically prove that satisfies the aforementioned properties. Last, the application of our method in the Sioux Falls network demonstrates that both pricing schemes outperform the UE solution and can approach the SO solution.

The rest of the paper is organized as follows. In Section 2, we describe the network model used and we formulate the optimization problems through which we can calculate the UE and and the SO solutions. The optimum pricing scheme and its approximate solution are presented in Section 3 and the simulation results of our approach are provided in Section 4. Section 5 presents the conclusions.

2. MATHEMATICAL MODEL

2.1 Problem Formulation

We consider a non-atomic game theoretic model where the OD demand for the truck drivers is assumed to be stochastic. A similar model was also used in Kordonis et al. (2019). In this work, we extend this model in the case of heterogeneous users, i.e. in the presence of truck drivers with different VOT. We assume that the coordinator asks the truck drivers to declare their VOT by providing them with a finite number of choices/classes. Let $w = 1, \ldots, N$ represent the classes of users with different VOT.

The transportation network is described by a graph $G = (V, L)$ where V is the set of nodes and L is the set of links in the network. Let X_{lT} be the number of trucks in the road segment l of the network and let X_{lp} be the corresponding number of passenger vehicles.

Let us denote by $d_w^j \in [0, D]$ the number of truck drivers belonging to the class w with OD pair j. In our model, we assume that the variables d_w^j are random and their probability distribution is known to all the truck drivers. We further assume that X_{lp} are also random variables but their distribution is independent of the decisions of the trucks drivers, similar to Papadopoulos et al. (2019b). Letting $\alpha_{w,r}^j$ represent the proportion of truck drivers belonging to the class w with OD pair j who follow route $r \in R_j$, we can define the number of trucks traversing the road segment l as:

$$X_{lT}(\alpha) = \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r \in R_j : l \in r} d_w^j \alpha_{w,r}^j \tag{1}$$

Table 1. Notation Used

Variable	Meaning
$G = (V, L)$	The graph representing the transportation network
N	The number of classes with different VOT
v	The number of OD pairs
d_w^j	Demand of truck drivers belonging to class w and traveling in OD pair j
R_j	The set of possible routes in OD pair j
Xlp	Number of passenger vehicles traversing the road segment l
X_{lT}	Number of trucks traversing road segment l
$\alpha_{w,r}^{c,j}$	The fraction of truck drivers belonging to class w with OD pair j who follow route $r \in R_j$ during the demand realization c
$F_{j,r}^w$	Expected travel time of a truck driver with VOT belonging to the class w, travelling in the OD pair j and following route r
$E[T_{tr}]$	Expected total travel time of the truck drivers in the network
$E[T_{tr}^{mon}]$	Expected total monetary cost of the truck drivers in the network
$J_{c,j,r}^{UE}$	Travel time of a truck driver with an OD pair j who follows route r during the demand realization c at the UE
s_w	Value of Time (VOT) of the class of truck drivers w
c	The realization of the Origin-Destination (OD) demand of the truck drivers
p_c	Probability of the realization c of the OD demand of the truck drivers
$E[T_s]$	Expected total travel time of the network
$E[T_p]$	Expected total travel time of the passenger vehicles in the network
$J_{w,r}^{M,c,j}$	The travel time of a truck driver belonging to the class w with an OD pair j who follows route r during the demand realization c under the suggestions of the mechanism M
$\tau_{w,r}^{c,j}$	Payment (made or received) of a truck driver belonging to the class w with an OD pair j who follows route r during the demand realization c
$A_{c,j}^{UE}$	Average travel time of a truck driver with an OD pair j during the demand realization c at the UE

2.2 User Equilibrium

In the absence of cooperation, it is known that the drivers of a transportation network are trying to minimize their own individual travel time leading to a situation known as the User Equilibrium (UE) or the first Wardrop principle, (Wardrop, 1952).

Let $F_{w,r}^j(\alpha)$ be the expected travel time of a truck driver with VOT belonging to the class w, travelling in the OD pair j and following route r. Then, $F_{w,r}^j(\alpha)$ is given by:

$$F_{w,r}^j(\alpha) = E\left[\sum_{l \in r} C_{lT}(X_{lp}, X_{lT}(\alpha))\right] \tag{2}$$

where $X_{lT}(\alpha)$ is given by (1) and α is a set of variables that is defined as follows:

$$\alpha = \{\alpha_{w,r}^j : w = 1, \ldots, N, j = 1, \ldots, v, r \in R_j\}$$

Additionally, $C_{lT}(X_{lp}, X_{lT}(\alpha))$ is assumed to be a known nonlinear function representing the travel time of a truck driver traversing the road segment l when there exist X_{lp} passenger vehicles and $X_{lT}(\alpha)$ trucks on it.

Note that in a UE solution, it holds that:

$$F^j_{w,r}(\alpha) \leq F^j_{w,r'}(\alpha)$$

for any $r' \neq r$, where $r, r' \in R_j$. In other words, in a UE solution, no truck driver has an incentive to unilaterally change his/her routing decision since his/her expected travel time will be higher.

It has been shown that there are possibly many non-equivalent UE solutions, see e.g. Kordonis et al. (2019). In order to compute a specific UE, let us first introduce some quantities. We define the expected total travel time of the truck drivers in the network to be:

$$E[T_{tr}(\alpha)] = E\left[\sum_{l\in r} X_{lT}(\alpha) C_{lT}(X_{lp}, X_{lT}(\alpha))\right] \quad (3)$$

while their expected total monetary cost is:

$$E[T^{mon}_{tr}(\alpha)] = \sum_c \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r\in R_j} p_c d^c_{j,w} \alpha^{UE}_{j,w,r} s_w J^{UE}_{c,j,r} \quad (4)$$

where c is the realization of the Origin-Destination demand of the truck drivers under the assumption that its distribution has finite support, p_c is the probability of this realization, $d^c_{j,w}$ is the demand of truck drivers with VOT belonging to class w for the OD pair j under the realization c, s_w is the VOT of the class of truck drivers w and $J^{UE}_{c,j,r}$ is the travel time of a truck driver with an OD pair j who follows route r during the demand realization c at the UE.

Using the aforementioned definitions, we can calculate the UE of the network by solving the following constrained optimization problem:

$$\begin{aligned} \underset{\alpha,\delta}{\text{minimize}} \quad & \lambda E[T_{tr}(\alpha)] + (1-\lambda) E[T^{mon}_{tr}(\alpha)] \\ \text{subject to} \quad & 0 \leq \alpha^j_{w,r} \perp F^w_{j,r}(\alpha) - \delta^j_w \geq 0, \ \forall j, w, r \\ & \sum_{r\in R_j} \alpha^j_{w,r} = 1, \ \forall j, w \end{aligned} \quad (5)$$

where δ^j_w is a set of free variables that are used in order to solve the equilibrium optimization problem (5) with complementarity constraints, Facchinei and Pang (2007) and λ is a weighting factor such that $\lambda \in [0, 1]$. The notation $\perp$ denotes the complementarity constraint and means that either $\alpha^j_{w,r} = 0$ or $F^w_{j,r}(\alpha) - \delta^j_w = 0$. Note that the UE solution may not be unique. Therefore, in the equilbrium optimization problem (5), we are looking for the UE solution which minimizes a weighted combination of the expected total travel time of the truck drivers and their expected total monetary cost given by (3) and (4), respectively.

2.3 System Optimum

The situation where the drivers cooperate in a manner which contributes to the minimization of the total travel time of the network is known as the System Optimum (SO) or the second Wardrop principle, (Wardrop, 1952). Given the network characteristics and definitions described in sections 2.1 and 2.2, we can calculate the SO solution of the network by solving the following optimization problem:

$$\begin{aligned} \underset{\alpha(\cdot)}{\text{minimize}} \quad & \lambda E[T_s(\alpha)] + (1-\lambda) E[T^{mon}_{tr}(\alpha)] \\ \text{subject to} \quad & \sum_{r\in R_j} \alpha^{c,j}_{w,r} = 1, \ \forall c, j, w \\ & \alpha^{c,j}_{w,r} \geq 0, \ \forall c, j, w, r \end{aligned} \quad (6)$$

where $E[T_s(\alpha)]$ is the expected total travel time of the network and is given by:

$$E[T_s(\alpha)] = E[T_p(\alpha)] + E[T_{tr}(\alpha)] \quad (7)$$

where $E[T_p(\alpha)]$ is the expected total travel time of the passenger vehicles in the network and $E[T^{mon}_{tr}(\alpha)]$ is given by:

$$E[T^{mon}_{tr}(\alpha)] = \sum_c \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r\in R_j} p_c d^c_{j,w} \alpha^{c,j}_{w,r} s_w J_{c,j,r} \quad (8)$$

Note that in the optimization problem (6), the set of the decision variables α depends on the demand realization c compared to the equilibrium optimization problem (5) where each truck driver acts in an independent manner and makes his/her routing decision by only knowing the probability distribution of the OD demand for the rest of truck drivers.

It is well known in the literature, (Koutsoupias and Papadimitriou, 1999), that there is an inefficiency between the UE and the SO solutions. On the other hand, enforcing the SO solution is not possible since some drivers may get harmed while some other others may get benefit compared to the UE solution. Therefore, in the next section, we present a Pareto-improving pricing scheme, i.e. a pricing scheme that makes every truck driver better-off compared to the UE solution that concurrently allows the transportation network to approach the SO solution.

3. VALUE OF TIME BASED PRICING

One of the main reasons for the inefficiency in the road network is the lack of cooperation of the network users. Therefore, in this section we propose the use of a coordination mechanism whose purpose is to collect information from the truck drivers related to their Origin-Destination (OD) pair and their Value of Time (VOT) and then assign them a route such that a social welfare cost is minimized. In our work, our objective is the minimization of a weighted combination of the expected total travel time of the network (passenger vehicles + trucks) and the expected total monetary cost of the truck drivers. To address the problem of the SO solution where some drivers may get harmed while some others may get benefit compared to the UE solution, the coordinator uses some additional pricing schemes that need to satisfy some specific characteristics.

First, we are interested about Pareto-improving pricing schemes, i.e pricing schemes that guarantee that every truck driver will be better-off compared to the UE solution. Additionally, in contrast with the vast majority of literature, e.g. (Liu and Nie, 2017; Tian et al., 2013), which makes assumptions about the distribution of user heterogeneity, we design a mechanism where its coordinator asks the truck drivers to pick their desired VOT through a set of N possible options. Therefore, it is important to guarantee that the mechanism will not be exploitable and that every truck driver will truthfully declare his/her VOT. Note that this property is not straightforward to guarantee since it is possible that many truck drivers will be willing to declare that their VOT is higher than their true one such that they can be assigned to the fastest possible route. Last, we need to guarantee that the overall mechanism will be budget balanced on average.

Before formulating the optimization problem through which we can calculate the optimum way of assigning the truck drivers into the network α^* as well as the optimum monetary scheme τ^*, let us mathematically describe the properties that this pricing scheme needs to satisfy:

(1) The pricing scheme is Pareto-improving, i.e every truck driver will be better-off compared to the UE solution if:

$$\sum_c \sum_{r\in R_j} p_c \alpha_{w,r}^{c,j}(J_{w,r}^{M,c,j} + \frac{1}{s_w}\tau_{w,r}^{c,j}) \leq \sum_c p_c A_{c,j}^{UE},\ \forall j, w \tag{9}$$

where $J_{w,r}^{M,c,j}$ is the travel time of a truck driver belonging to the class w with an OD pair j who follows route r during the demand realization c under the suggestions of the mechanism M, $\tau_{w,r}^{c,j}$ is the payment made or received by each truck driver who is travelling in the OD pair j, following route r during the demand realization c and $A_{c,j}^{UE}$ is the average travel time of a truck driver with an OD pair j during the demand realization c at the UE. As mentioned earlier, s_w expresses the VOT of the class of truck drivers w. Letting s_w have units $\frac{\$}{hr}$ and $\tau_{w,r}^{c,j}$ be measured in \$, then equation (9) guarantess that every truck driver will be better-off compared to the UE in time units.

(2) Each truck driver will be willing to truthfully declare his/her VOT if:

$$\begin{aligned}\sum_c \sum_{r\in R_j} p_c \alpha_{i,r}^{c,j}(J_{i,r}^{M,c,j} + \frac{1}{s_i}\tau_{i,r}^{c,j}) \leq \\ \leq \sum_c \sum_{r\in R_j} p_c \alpha_{k,r}^{c,j}(J_{k,r}^{M,c,j} + \frac{1}{s_i}\tau_{k,r}^{c,j}),\ \forall j, i, k\end{aligned} \tag{10}$$

According to the aforementioned definitions, equation (10) states that each truck driver with a VOT belonging to a fixed class i has a lower expected cost (time + payment) in time units if he/she truthfully declares the class i rather than declaring any other fixed class $k \neq i$.

(3) The mechanism is budget balanced on average if:

$$\sum_c \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r\in R_j} p_c d_{c,j}^{w} \alpha_{w,r}^{c,j} \tau_{w,r}^{c,j} = 0 \tag{11}$$

i.e. if the expected total payments made and received by the coordinator of the mechanism are equal to zero.

3.1 Optimum Pricing Scheme

Using equations (9)-(11), we can calculate the optimum way of assigning the truck drivers into the network α^* as well as the optimum pricing scheme τ^* by solving the following nonconvex optimization problem:

$$\begin{aligned}\underset{\alpha(\cdot),\tau(\cdot)}{\text{minimize}} \quad & \lambda E[T_s(\alpha)] + (1-\lambda)E[T_{tr}^{mon}(\alpha)] \\ \text{subject to} \quad & G_{j,w} \leq \sum_c p_c A_{c,j}^{UE},\ \forall j, w \\ & G_{j,ii} \leq G_{j,ik},\ \forall j, i, k \\ & \sum_c \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r\in R_j} p_c d_{c,j}^{w} \alpha_{w,r}^{c,j} \tau_{w,r}^{c,j} = 0 \\ & \sum_{r\in R_j} \alpha_{w,r}^{c,j} = 1,\ \forall c, j, w \\ & \alpha_{w,r}^{c,j} \geq 0,\ \forall c, j, w, r\end{aligned} \tag{12}$$

where $G_{j,w}, G_{j,ii}$ and $G_{j,ik}$ are given by the following equations:

$$G_{j,w} = \sum_c \sum_{r\in R_j} p_c \alpha_{w,r}^{c,j}(J_{w,r}^{M,c,j} + \frac{1}{s_w}\tau_{w,r}^{c,j}) \tag{13}$$

$$G_{j,ii} = \sum_c \sum_{r\in R_j} p_c \alpha_{i,r}^{c,j}(J_{i,r}^{M,c,j} + \frac{1}{s_i}\tau_{i,r}^{c,j}) \tag{14}$$

$$G_{j,ik} = \sum_c \sum_{r\in R_j} p_c \alpha_{k,r}^{c,j}(J_{k,r}^{M,c,j} + \frac{1}{s_i}\tau_{k,r}^{c,j}) \tag{15}$$

The first constraint of (12) together with equation (13) guarantee that the optimum pricing scheme τ^* is Pareto-improving, i.e. every truck driver will be better-off compared to the UE while the second constraint of (12) together with (14) and (15) guarantee that every truck driver will have an incentive to truthfully declare his/her VOT or otherwise his/her expected average cost (time + payment) is going to be higher. The third constraint of (12) guarantees that the mechanism is going to be budget balanced on average while the last two constraints are feasibility constraints of the optimization problem (12). Additionally, setting $\tau = 0$, it is straightforward to show that the UE solution always satisfies the constraints of (12) guaranteeing that a solution to the nonconvex optimization problem (12) always exists.

To reduce the dimensionality and accelerate the computational time needed to solve the optimization problem (12), in the next subsection, we approximate the solution of (12) by defining a pricing scheme $\hat{\tau}$ for which we derive sufficient conditions that need to be satisfied such that a solution with the desired characteristics described by the equations (9)-(11) exists.

3.2 Approximate Solution

To approximate the optimum solution obtained by solving the optimization problem (12), we need to find a pricing scheme $\hat{\tau}$ that is Pareto-improving, guarantees that each truck driver will be willing to truthfully declare his/her VOT and additionally leads to an overall budget balanced mechanism.

Let us define the following pricing scheme:

$$\begin{aligned}\hat{\tau}_{w,r}^{c,j} = & s_w(A_{c,j}^{UE} - \bar{J}_{w,r}^{M,c,j}) \\ & + \frac{s_w}{\sum_{l=1}^{N} s_l} \frac{E[T_{tr}^{mon,M}] - E\left[T_{tr}^{mon,UE}\right]}{\sum_{j=1}^{v} d_{c,j}^{w}}\end{aligned} \tag{16}$$

In the theorem that follows, we derive necessary and sufficient conditions that need to be satisfied such that the pricing scheme given by (16) satisfies the desired characteristics described by (9)-(11).

Before stating the theorem, let us first introduce the following inequalities:

$$E[T_{tr}^{mon,M}] \leq E[T_{tr}^{mon,UE}] \tag{17}$$

$$\begin{aligned}&\left(1-\frac{s_k}{s_i}\right)\sum_c p_c A_{c,j}^{UE}\\&+\frac{1}{\sum_{w=1}^N s_w}\sum_c p_c \frac{E[T_{tr}^{mon,M}]-E[T_{tr}^{mon,UE}]}{\sum_{j=1}^v d_{c,j}^i} \leq\\&\leq\left(1-\frac{s_k}{s_i}\right)\sum_c\sum_{r\in R_j} p_c\bar{\alpha}_{k,r}^{c,j}\bar{J}_{k,r}^{M,c,j}\\&+\frac{s_k}{s_i}\frac{1}{\sum_{w=1}^N s_w}\sum_c p_c\frac{E[T_{tr}^{mon,M}]-E[T_{tr}^{mon,UE}]}{\sum_{j=1}^v d_{c,j}^k}, \forall j,i,k\end{aligned} \tag{18}$$

where $E[T_{tr}^{mon,M}]$ is the expected total monetary cost of the truck drivers under the mechanism suggestions and $E[T_{tr}^{mon,UE}]$ is the corresponding cost at the UE.

Theorem 1. The pricing scheme (16) is Pareto-improving, guarantees that each truck driver will have an incentive to truthfully declare his/her VOT and leads to a budget balanced on average mechanism if and only if (17) and (18) hold.

Proof. The proof is similar to the proof of Theorem 1 of Papadopoulos et al. (2019c) and thus it is omitted.

Theorem 1 states that (17) and (18) are necessary and sufficient conditions to guarantee that the pricing scheme (16) satisfies the desired characteristics as described by (9)-(11). Now, letting $Z_{j,i,k}$ and $Y_{j,i,k}$ be the left and right parts of the inequality (18), respectively, we can approximate the optimum solution of (12) by solving the following optimization problem:

$$\begin{aligned}\underset{\alpha(\cdot)}{\text{minimize}}\quad & \lambda E[T_s(\alpha)]+(1-\lambda)E[T_{tr}^{mon}(\alpha)]\\ \text{subject to}\quad & E[T_{tr}^{mon}(\alpha)] \leq E[T_{tr}^{mon,UE}]\\ & Z_{j,i,k} \leq Y_{j,i,k},\ \forall j,i,k\\ & \sum_{r\in R_j}\alpha_{w,r}^{c,j}=1,\ \forall c,j,w\\ & \alpha_{w,r}^{c,j}\geq 0,\ \forall c,j,w,r\end{aligned} \tag{19}$$

It is straightforward to show that a solution to the optimization problem (19) always exists since the UE solution satisfies all of its constraints.

Note that the optimization problem (19) has to be solved only with respect to the decision variables α, i.e we only need to find a way to route the truck drivers into the network such that the constraints of (19) are satisfied. Letting $\hat{\alpha}^*$ be the optimum solution obtained by solving the optimization problem (19), then using the result of Theorem 1, we can guarantee that the pair $(\hat{\alpha}^*, \hat{\tau})$, where $\hat{\tau}$ is given by (16), satisfies the conditions described by (9)-(11). In other words, the pair $(\hat{\alpha}^*, \hat{\tau})$ makes everyone better-off compared to the UE solution, guarantees that each truck driver will have an incentive to truthfully declare his/her VOT and additionally leads to a budget balanced on average mechanism.

4. EXPERIMENTS

To validate the theoretical results and demonstrate the performance of the optimum pricing scheme and its approximate solution compared to the UE and the SO solutions, we carried out simulation experiments in the benchmark Sioux Falls transportation network, (LeBlanc et al., 1975). The Sioux Falls network consists of 24 nodes and 76 links.

In our experiments, we assumed that the cost of each link in the network corresponds to travel time and is given by a Bureau of Public Roads (BPR) function, (Sheffi, 1985), of the following form:

$$C_{lT}(X_{lp}, X_{lT}) = y_a + y_b\left(\frac{X_{lp}+3X_{lT}}{y_c}\right)^4 \tag{20}$$

where y_a, y_b and y_c are constants and their values were chosen to be similar to the ones adopted in Kordonis et al. (2019) [1] . The number of passenger vehicles at each road segment of the network was considered to be constant and equal to the values used in Kordonis et al. (2019) [2] . As far as it concerns the truck drivers, we assumed that they have 6 available Origin-Destination (OD) pairs, namely $(1,7),(1,11),(10,11),(10,20),(15,5)$ and $(24,10)$ and that they follow the 10 least congested routes, similar to Papadopoulos et al. (2019a). We further assumed that the coordinator of the mechanism asks the truck drivers to pick their VOT between two different options, i.e $s_1 = 200\$/hr$ and $s_2 = 50\$/hr$. The OD demand of the truck drivers was assumed to take one of the following two equiprobable values:

$$d_1 = \begin{bmatrix}3 & 4.5 & 6 & 3 & 14 & 3.6\\ 1 & 2.8 & 5.4 & 7 & 9 & 2\end{bmatrix}, d_2 = \begin{bmatrix}5 & 1.8 & 3.9 & 15 & 6.4 & 2.4\\ 6 & 5.5 & 1.8 & 6.5 & 11 & 6\end{bmatrix}$$

where each column of d_1 and d_2 corresponds to the demand of truck drivers for each OD pair and each row denotes a different class of users. Last, the weighting factor λ in the objective functions of the optimization problems (5), (6), (12) and (19) was chosen to be $\lambda = 0.9$. The simulation results are presented in Table 2.

Table 2. Simulation Results

Metric	UE	SO	OPS	APS
$E[T_{tr}]$	53574.4	49082.5	49049.8	49866.6
$E[T_{tr}^{mon}]$	117941.4	104445.5	104647.5	105999.3
$E[T_s]$	167160.7	157924.8	157934.9	159526.0

where OPS stands for the Optimum Pricing Scheme while APS stands for its approximate solution or else, Approximate Pricing Scheme.

As can be clearly seen from the simulation results in Table 2, OPS achieves 8.45% reduction in the expected total travel time of the truck drivers $E[T_{tr}]$ compared to the UE solution and it is even slightly better from the SO solution. On the other hand, APS also achieves a significant reduction in $E[T_{tr}]$ providing a solution which approaches the SO. Regarding the expected total

[1] These values can be found in this link.
[2] These values can be found in this link.

monetary cost of the truck drivers $E[T_{tr}^{mon}]$, it can be observed that both OPS and APS outperform the UE and approach the SO solution. Last, it is clear that both pricing schemes significantly reduce the expected total travel time of the network, providing a solution that is close to the SO.

5. CONCLUSION

In this work, we proposed two Pareto-improving pricing schemes that can make a transportation network approach the SO solution by controlling the routing decisions of the truck drivers through a coordination mechanism. In contrast with the vast majority of literature studying pricing schemes under user heterogeneity, we propose that the coordinator asks the truck drivers to declare their VOT and then we design Pareto-improving pricing schemes that mathematically guarantee that every user will have an incentive to truthfully declare his/her VOT while concurrently leading to a budget balanced on average mechanism. Additionally, the simulation results using the Sioux Falls network showed that the designed pricing schemes can significantly improve both the expected total travel time of the truck drivers and their associated monetary cost, as well as the expected total travel time of the network.

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