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Event triggering control for dynamical systems with designable inter-event times

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Abstract: This paper presents a class of event-triggering rules for dynamical control systems with guaranteed positive minimum inter-event time (IET). We first propose an event-based function design with guaranteed control performance using a clock-like variable for general nonlinear systems, and later specialize them to general linear systems. Compared to the existing static and dynamic triggering mechanisms, the proposed triggering rules feature a robust global event-separation property, and can be easily implemented on practical digital platforms to meet various hardware limitations. Finally, several numerical simulations are given to illustrate the theoretical results.

Keywords: Event-triggered control, inter-event times, dynamic triggering mechanism, robustness analysis.

1. INTRODUCTION

The emerging application of wireless communication techniques in conventional feedback control systems creates a new type of control system that is often called networked control system (NCS) Zhang et al. (2015). NCS offers a variety of benefits including flexibility, maintainability and cost reduction of automation processes. Nevertheless, it also introduces considerable design challenges like extra energy consumption, and additional constraints in the closed-loop system, such as requiring a minimum communication bandwidth and a minimum control update frequency. In recent years, it has been shown that the event-triggered control paradigm is a promising solution compared to the commonly-used time-triggered schemes to update control input. Instead of continuous sampling and communication, an event-triggered control scheme can determine the time when the state information needs to be sampled and associated data need to be sent to update control law based on certain triggering rules; see Astrom and Bernhardsson (2002); Tabuada (2007). In event-triggered control, a key issue is the exclusion of possible Zeno triggering behavior. In this paper, we will present a practical event-triggering mechanism that guarantees a positive minimum inter-event time (IET), thus excluding the Zeno triggering for an event-triggered control system.

Related papers Event-triggered control has received considerable attention in recent years, and here we review some key developments in this field. The seminal paper Tabuada (2007) initialized a general event-based scheduling control for general nonlinear systems, and a typical analysis and design framework for event-triggered control systems was formally proposed. However, the minimum IET in the static triggering rule of Tabuada (2007) and many subsequent papers often requires measurement of entire system state variables, and the upper bound of the constructed triggering signal is fixed. That is, although the Zeno behavior can be excluded by guaranteeing the existence of a positive minimum IET, the adjustable range of inter-event times is limited and dependent of the system states. An event-triggered control scheme was proposed and analyzed for perturbed linear systems in Heemels et al. (2008), where the sampling is performed only when the tracking/stabilization error exceeds some bound. Another paper Astrom (2008) presented the architecture of a general structure for event-triggered control and discussed the relations to nonlinear systems. Various new notions related to the existence of the positive minimum IET, called event-separation property, was introduced in Borgers and Heemels (2014). The paper Borgers and Heemels (2014) also shows that some popular event triggering mechanisms do not ensure the event-separation property no matter how small the disturbance is. This is an important aspect in the design of triggering function for event-based control systems under perturbations. For more developments of event-based control and triggering function design, the reader is referred to the surveys Heemels et al. (2012); Zhang et al. (2016).

Unfortunately, for all of the above-mentioned work, it is worth mentioning that the upper bounds of those constructed triggering signals are hard to adjust. Specifically, although the Zeno behavior can be excluded by guaranteeing a strictly positive minimum IET, the resulting algorithms cannot flexibly adapt physical limitations of

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communications and devices if the upper bound of the minimum IET is constrained and IETs are difficult to be adjusted. In other words, those event triggering mechanisms, which do not hold the above properties, cannot be implemented very well on a digital platform.

Recently, to improve the results in Tabuada (2007), a dynamic triggering mechanism was formally proposed in Girard (2014). In this framework, an internal dynamic variable is introduced into the static triggering rule, which also helps to enlarge the delay between successive triggering time instants. However, similarly to what happens with static rules, the resulting minimun IET is still not easy to adapt hardware limitations. Also, the upper bound of triggering signal, which depends on real-time system states, is fixed. For other recent remarkable work on a dynamic triggering mechanism, we notice that in Tanwani et al. (2016) the authors considered the output feedback stabilization problem for general linear systems with event-triggered sampling and dynamic quantization and that in Brunner et al. (2018) a new type of event condition was proposed to be dependent on the states difference between the actual system and the nominal undisturbed system, which is triggered when the nominal states are equal to the states of the real system. For other interesting results on dynamic event-triggered control, we further refer the reader to the overviews Nowzari et al. (2019); Ding et al. (2017).

Contributions Based on the above discussion, we observe that under the static or dynamic triggering mechanisms, the variable range and the evolution rate of the constructed triggering signal cannot be freely designed. In addition, an investigation of the robustness issue for dynamic event triggering mechanism is lacking. These current shortcomings motivate the construction of an event triggering mechanism with the designable IETs and a robust global event-separation property. We remark that only in Berneburg and Nowzari (2019) was the designable IETs control discussed, in the context of multi-agent consensus control with single-integrator dynamics. In this paper, we aim to propose an event-triggered control scheme to realize an adjustable IETs with a strictly positive lower bound and guaranteed system convergence. The main contributions of this paper are summarized as below.

(1) We show new design and analysis approaches for determining an dynamic event triggering mechanism, which are applied to general nonlinear systems and also specialized to general linear systems.
(2) The freely designable IETs are realized. Compared to the static and dynamic event triggering mechanisms, we can adjust the variable range of an upper bound of the constructed triggering signal regardless of physical limitations of real-time system states.
(3) The proposed dynamic event-triggered strategy ensures the robust global event-separation property under state perturbations.

The rest of this paper is organized as follows. In Section 2, we review two event-triggered schemes in the literature and recall some preliminary background on event-triggered control, and present the problem formulation. The design and analysis framework of the IETs-designable event triggering mechanism for nonlinear systems is presented in Section 3. In Section 4, the framework presented in Section 3 is specialized to general linear control systems, while the robustness of the proposed algorithm is analyzed. In Section 5, two simulation examples are provided to illustrate the effectiveness of the theoretical results. Finally, conclusions and remarks concerning future work are given in Section 6.

Notations. Throughout this paper, $\mathbb{R}$ and $\mathbb{R}^n$ denote the set of real numbers and the n-dimensional Euclidean space, respectively. $\mathbb{R}_0^+$ is the set of non-negative real numbers. The notation $|\cdot|$ refers to the Euclidean norm for vectors and the induced 2-norm for matrices. The superscript $\top$ denotes vector or matrix transposition. A function $\alpha(r)$: $\mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said to be of class K_∞ if it is continuous, strictly increasing, $\alpha(0) = 0$, and $\alpha(r) \to +\infty$ as $r \to +\infty$.

2. EVENT-TRIGGERING MECHANISMS

We consider the control system of the form

$$\dot{x} = f(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m \tag{1}$$

with a state feedback law $u = k(x)$ that stabilizes the system, in which functions $f(\cdot,\cdot)$ and $k(\cdot)$ are Lipschitz continuous on compacts. Then the resulting closed-loop control system is

$$\dot{x} = f(x, k(x)). \tag{2}$$

Under the state feedback law $u = k(x)$, the state information of the plant needs to be available and accessed continuously so as to allow update of the input continuously. In event-triggered control, the state information is accessed when necessary and the control input is updated when certain events occur. This results in a discrete-time updated control law $u = -k(x(t_i)), t \in [t_i, t_{i+1})$, where t_i denotes the i-th triggering time instant. We note that if $t_{i+1} - t_i \to 0$ for some finite time t_i, the sampling process becomes impractical (a phenomenon termed Zeno triggering). Therefore, how to guarantee a positive minimum IET is one of the key issues in the design of event-triggered control.

As in Tabuada (2007), we define the measurement error as $e(t) = x(t_i) - x(t), t \in [t_i, t_{i+1})$. We assume that the closed-loop control system

$$\dot{x} = f(x, k(x + e)) \tag{3}$$

is input-to-state stable (ISS) with respect to $e(t)$. There thus exists an ISS-Lyapunov function V for class K_∞ functions $\bar{\alpha}, \underline{\alpha}, \alpha$, and γ satisfying the following inequalities

$$\begin{aligned} \underline{\alpha} \le V(x) \le \bar{\alpha}, \\ \dot{V}(x) \le -\alpha(|x|) + \gamma(|e|). \end{aligned} \tag{4}$$

2.1 Static Event Triggering Mechanism

The seminal paper Tabuada (2007) proposed an event triggering strategy for the system (3), with the following static triggering rule

$$t_{i+1} = inf\{t > t_i | \gamma(|e|) \geq \sigma\alpha(|x|)\}. \quad (5)$$

In event-triggered control systems, IETs should be designed to satisfy

$$t_{i+1} - t_i \geq \tau > 0, \forall i, \quad (6)$$

where τ is a positive constant or the minimum IET. It is shown in Tabuada (2007) that for all initial states $x(0) \in S$ where $S \subset \mathbb{R}^n$ is a compact set containing the origin, there exists $\tau > 0$ such that the sequence t_i determined by (5) satisfies (6) if $f(\cdot)$, $k(\cdot)$, $\alpha^{-1}(\cdot)$ and $\gamma(\cdot)$ are Lipschitz continuous on compacts and $0 < \sigma < 1$.

2.2 Dynamic Event Triggering Mechanism

The dynamic triggering mechanism, proposed in Girard (2014), is an extension of the static triggering scheme obtained through enlarging the variable range of the constructed triggering signal $\gamma(|e|)/\alpha(|x|)$. To explain this point, let us recall the triggering rule (5) and add a positive item in the right hand side of the inequality

$$\frac{\gamma(|e|)}{\alpha(|x|)} \geq \sigma + \frac{\eta}{\theta\alpha(|x|)},$$

where $\theta > 0$ is an adjustable parameter, and $\eta \geq 0$ is a virtual state to be designed. Note that the additive item $\frac{\eta}{\theta\alpha(|x|)}$ increases the upper bound of the comparison threshold of the triggering signal. The dynamic triggering rule can be thus given as below

$$t_{i+1} = inf\{t > t_i | \eta + \theta(\sigma\alpha(|x|) - \gamma(|e|)) \leq 0\}. \quad (7)$$

When the virtual state is designed as $\dot{\eta} = -\zeta(\eta) + \sigma\alpha(|x|) - \gamma(|e|)$, $\eta(0) = \eta_0 \in \mathbb{R}_0^+$, it has been proved in Girard (2014) that the inequality $\eta \geq 0$ is satisfied and that both the states $x(t)$ and the virtual variable η will converge to the origin asymptotically.

As can be observed from the above reviews, the adjustable range of IETs for static and dynamic triggering mechanisms is limited. To improve the implementability of a theoretical solution on a physical platform, this paper follows the design of a flexible dynamic event-triggered scheme that allows the variable range of IETs to be prescribed freely, and ensures a positive minimum IET independent of intrinsic system states.

3. DYNAMIC IETS-DESIGNABLE EVENT TRIGGERING CONTROL FOR NONLINEAR SYSTEMS

Through observing the two types of triggering mechanisms (5) and (7) reviewed in the last section, we find that the derivation of the minimum IET is dependent of the states x and measurement errors e, and the upper bounds of the constructed triggering signals can only be adjusted in a limited range for certain specific plants. In this section, we aim to propose a novel triggering mechanism with designable IETs for general nonlinear systems. In contrast to the static or dynamic triggering mechanism, the intuitive idea here is to create a triggering signal $Z(t)$, for which the variable range of the upper bound can be freely designed. We thus adopt the following event triggering rule

$$\begin{aligned} t_0 &= 0, \\ t_{i+1} &= inf\{t > t_i | Z(t) = 0\}, \end{aligned} \quad (8)$$

where $Z(t_i)$ will be reset to $\bar{Z}$ at a triggering instant and $\bar{Z} > 0$ is the first design parameter. Here, the variable $Z(t)$ takes a similar role to a countdown variable with an assigned upper bound $\bar{Z}$. The dynamics $\dot{Z}(t) = \omega(\varpi(x,e),\varepsilon) < 0$ are thus considered in the sequel of this paper for designing event triggering conditions, where $\varepsilon > 0$ is the second design parameter.

The first main result of this paper can be given as follows.

Theorem 1. Consider the nonlinear ISS closed-loop control system (3) with functions $f(\cdot,\cdot)$ and $k(\cdot)$ that are Lipschitz continuous on compacts. The event triggering mechanism is given as (8). The dynamics of the additional variable $Z(t)$ is chosen as $\dot{Z}(t) = \omega(\varpi(x,e),\varepsilon) < 0$, in which $\omega(\cdot,\cdot)$ and $\varpi(\cdot,\cdot)$ are functions of associated variables. Then, for all initial conditions $x(0)$, the closed-loop control system is guaranteed to converge to the origin asymptotically. Meanwhile, there exists designable IETs lower bounded by τ_1

$$\begin{aligned} \tau_1 &= \sqrt{\frac{1}{b\varepsilon}} \left\{ \mathbf{atan}[\sqrt{\frac{b}{\varepsilon}}(1+\bar{Z})] - \mathbf{atan}[\sqrt{\frac{b}{\varepsilon}}] \right\} > 0, \\ b &= L^2 \frac{|M|^2}{\lambda_{\min}(M)} \end{aligned} \quad (9)$$

with design parameters $\varepsilon, \bar{Z}$ to be detailed in the sequel, for the triggering sequence $(t_i)_{i \to +\infty}$.

Proof. We first analyze the stability. Choose the candidate Lyapunov function as $W = V + \frac{1}{2} Z e^\top M e$, where V is the ISS-Lyapunov function and M is a symmetric positive definite matrix. The countdown variable satisfies $Z \geq 0$ because of the triggering mechanism (8). Note that the derivative of W along the solution of (3) is $\dot{W} = \dot{V} + \frac{1}{2}\omega e^\top M e + Z e^\top M \dot{e}$. Because of $\dot{x} = -\dot{e}$ and inequality (4), it follows

$$\begin{aligned} \dot{W} &\leq -\alpha(|x|) + \gamma(|e|) + \frac{1}{2}\omega e^\top M e - Z e^\top M \dot{x} \\ &= -\alpha(|x|) + \gamma(|e|) + \frac{1}{2}\omega e^\top M e - Z e^\top M f(x, k(x+e)). \end{aligned}$$

Since $\omega < 0$, we have

$$\begin{aligned} \dot{W} \leq& -\alpha(|x|) + \gamma(|e|) + \frac{1}{2}\omega\lambda_{\min}(M)|e|^2 \\ &+ Z|M||e||f(x, k(x+e))|. \end{aligned}$$

The Lipschitz continuity on compact sets of $f(x,u)$ and $k(x)$ implies that $f(x, k(x+e))$ is also Lipschitz continuous, we can thus obtain $|f(x, k(x+e))| \leq L|x| + L|e|$ with the Lipschitz constant $L > 0$. These facts lead to

$$\begin{aligned} \dot{W} \leq& -\alpha(|x|) + \gamma(|e|) + \frac{1}{2}\omega\lambda_{\min}(M)|e|^2 \\ &+ Z|M||e|(L|x| + L|e|) \\ =& -\alpha(|x|) + \gamma(|e|) + \frac{1}{2}\omega\lambda_{\min}(M)|e|^2 \\ &+ ZL|M||e||x| + ZL|M||e|^2. \end{aligned}$$

In order to guarantee the asymptotic stability of the control system (3), we enforce the following inequality

$$\omega < \frac{2\alpha(|x|)}{\lambda_{\min}(M)} \cdot \frac{1}{|e|^2} - \frac{2\gamma(|e|)}{\lambda_{\min}(M)} \cdot \frac{1}{|e|^2} - \frac{2LZ|M|}{\lambda_{\min}(M)} \cdot \frac{|x|}{|e|} - \frac{2LZ|M|}{\lambda_{\min}(M)}.$$

If class K_∞ functions $\alpha(|x|) = \frac{1}{2}|x|^2$ and $\gamma(|e|) = L|M||x||e|$ are chosen, the variable ω further satisfies

$$\omega < \varpi = \frac{1}{\lambda_{\min}(M)} \cdot \frac{|x|^2}{|e|^2} - (1+Z)\frac{2L|M|}{\lambda_{\min}(M)} \cdot \frac{|x|}{|e|}.$$

By using the Young inequality $y_1 y_2 \le \frac{b}{2}y_1^2 + \frac{1}{2b}y_2^2$, with $b > 0$ and $y_1, y_2 \in \mathbb{R}$, we can obtain

$$-(1+Z)\frac{2L|M|}{\lambda_{\min}(M)} \cdot \frac{|x|}{|e|} \ge -b(1+Z)^2 - \frac{1}{b} \cdot \frac{L^2|M|^2}{\lambda^2_{\min}(M)} \cdot \frac{|x|^2}{|e|^2}$$

such that

$$\varpi \ge -b(1+Z)^2 + \left(\frac{1}{\lambda_{\min}(M)} - \frac{1}{b} \cdot \frac{L^2|M|^2}{\lambda^2_{\min}(M)}\right) \cdot \frac{|x|^2}{|e|^2}.$$

By letting the above variable b in the second item of the right hand side satisfy $b = L^2 \frac{|M|^2}{\lambda_{\min}(M)}$, we can write

$$\varpi - \varepsilon \ge -b(1+Z)^2 - \varepsilon$$

with the design parameter $\varepsilon > 0$. If we further design ω as

$$\omega = \begin{cases} min(0, \varpi) - \varepsilon, & e \ne 0, \\ -\varepsilon, & e = 0, \end{cases} \tag{10}$$

and consider two cases of e: (1) $e \ne 0$, if $\varpi < 0$, then $\omega = \varpi - \varepsilon$ and if $\varpi \ge 0, \omega = -\varepsilon \ge -b(1+Z)^2 - \varepsilon$, so is case (2) when $e = 0$. Note that because $\omega < \varpi$ can be always guaranteed by the design (10) of ω, we conclude that the Lyapunov function W decreases such that $x(t)$ converges to the origin asymptotically. Moreover, the dynamics of the countdown variable Z gives the estimate $\dot{Z} \ge -b(1+Z)^2 - \varepsilon$. Let ϕ be the solution of differential equation $\dot{\phi} = -b(1+\phi)^2 - \varepsilon$, then the IETs are lower bounded by the time τ_1 that it takes for ϕ to evolve from $\bar{Z}$ to 0. We therefore conclude the formula of τ_1 in (9). □

4. DYNAMIC IETS-DESIGNABLE EVENT TRIGGERING CONTROL OF GENERAL LINEAR SYSTEMS

In this section, we will specialize previous results to event-triggered control of general linear systems with designable IETs.

4.1 Basic Algorithm

Consider a general linear control system of the form

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{11}$$

where A and B are system matrices with proper dimensions, and we assume the system is controllable and observable. A feedback control law is designed as $u(t) = Kx(t)$ through pole assignment for stabilizing the system (11). The closed-loop control system is then obtained as below

$$\dot{x}(t) = (A + BK)x(t).$$

This thus implies that there exists a Lyapunov function $V = x^\top Px$ such that the symmetric positive definite matrix P satisfies

$$(A + BK)^\top P + P(A + BK) = -Q,$$

where Q is an arbitrary symmetric positive definite matrix. When the state-feedback control law $u(t) = Kx(t)$, which is updated in a continuous time manner, is executed on digital platforms and/or in a wireless communication environment, then it needs to be modified to use discrete-time updates. In this section, we formally propose an IETs-designable event triggering method to schedule the computation and communication resources and determine the triggering time that updates the feedback of the system states $x(t)$ into the closed-loop control system; i.e., the control is modified as $u(t) = Kx(t_i), t \in [t_i, t_{i+1})$, where t_i is the i-th triggering instant. Following the idea of the event-triggered control framework presented in Section 3, we define the measurement error $e(t) = x(t_i) - x(t), t \in [t_i, t_{i+1})$; the following closed-loop system is thus obtained

$$\dot{x}(t) = Ax(t) + BKx(t) + BKe(t). \tag{12}$$

Similar to the event triggering mechanism (8), we apply the same triggering rule for general linear systems, and define the additional event function dynamics as $\dot{Z}(t) = \omega(\varpi, \varepsilon) < 0$.

We can now give the second main contribution of paper.

Theorem 2. Consider the general linear control system (12) with the event triggering mechanism (8) for all initial conditions $x(0)$. Then, $x(t)$ asymptotically converge to the origin. Meanwhile, there exists designable IETs lower bounded by τ_2

$$\tau_2 = \sqrt{\frac{1}{b\varepsilon}}\left\{\textbf{atan}[\sqrt{\frac{b}{\varepsilon}}(1+\bar{Z})] - \textbf{atan}[\sqrt{\frac{b}{\varepsilon}}]\right\} > 0,$$
$$b = \frac{|PBK|^2}{\lambda_{\min}(P)\lambda_{\min}(Q)} \tag{13}$$

with design parameters $\varepsilon, \bar{Z}$ as the same in the statement of Theorem 1, for the triggering sequence $(t_i)_{i\to+\infty}$.

The proof, which follows similar ideas of the proof in Theorem 1, is omitted here due to space limit.

4.2 Robustness Analysis

We continue to consider the following perturbed linear system

$$\dot{x}(t) = Ax(t) + Bu(t) + Hd(t), \tag{14}$$

where H is a constant matrix with proper dimensions and $d(t)$ denote system disturbances.

***Proposition** 1.* Consider the general linear control system (14) with the event triggering mechanism (8) and any bounded disturbances $|d| \le \bar{d}$ for all initial conditions $x(0)$. Then there exists the same positive lower bound $\tau_3 = \tau_2$ for the designable IETs, implying that the minimum IET is robust to perturbations. Furthermore, suppose that the perturbations $d(t)$ are convergent to 0. Then the system states also asymptotically converge to the origin.

Proof. We first analyze the robustness properties of the algorithm. Recall the Lyapunov function candidate and its derivation

$$\dot{W} \leq -\frac{1}{2}\lambda_{\min}(Q)|x|^2 + |PBK||x||e| + \frac{1}{2}\omega\lambda_{\min}(P)|e|^2 \\ +Z|PA||x||e| + Z|PBK||x||e| + Z|PBK||e|^2 \\ +|PH||x|\bar{d} + Z|PH||e|\bar{d}.$$

Therefore, it is clear that the formula below is still satisfied

$$\omega < \varpi = \frac{\lambda_{\min}(Q)}{\lambda_{\min}(P)}\frac{|x|^2}{|e|^2} - 2(1+Z)\frac{|PBK|}{\lambda_{\min}(P)}\frac{|x|}{|e|}.$$

Following the same lines as in the proof of Theorem 2, it can be verified that the derivation process of the minimum IET is independent of the disturbances. We therefore conclude that the strictly positive minimum IET is still guaranteed in the presence of system perturbations $d(t)$. The convergent $d(t)$ implying convergent $x(t)$ is a consequence of the exponential stability of the linear system. □

5. NUMERICAL SIMULATIONS

In this section, we present simulations to show the effectiveness of the proposed theoretical results.

In order to compare the present results with the static and dynamic triggering mechanisms for linear systems in Tabuada (2007) and Girard (2014), we use the same linear plant model with the same controller and choose the same gains. Specifically, by choosing $A = [0 \quad 1; -2 \quad 3]$, $B = [0; 1]$, $K = [1 \quad -4]$, one can get $P = [1 \quad 1/4; 1/4 \quad 1]$ and $Q = [1/2 \quad 1/4; 1/4 \quad 3/2]$. Since $\frac{|PBK|^2}{\lambda_{\min}(P)\lambda_{\min}(Q)} = 54.61$, based on formula (13) we choose $b = 55$. Meanwhile, by adopting the same simulation setup in Berneburg and Nowzari (2019), $\bar{Z} = 1$ and $\varepsilon = 1$ are chosen as the design parameters. We set the initial states as $x_1 = 10, x_2 = 0$.

From Fig.1 and Fig.4, it can be found that the states x and measurement errors e converge to the origin asymptotically which validates the stability of the general linear control system with the IETs-designable event triggering mechanism. Based on the formula (13), we can calculate a lower bound of IETs as 9 ms, which is smaller than the simulation result of 36 ms, implying that the calculated lower bound of IETs may be conservative. The variable ω always keeps smaller than $-\varepsilon$ in Fig.3, which implies that the clock-like variable Z always decreases while the speed rate is changed throughout the whole countdown process. The evolution of the term "$\frac{1}{2}Ze^{\top}Pe$" is shown in Fig.4, which is not monotonous.

Next, we compute the eigenvalues of state matrix $A + BK$ in Postoyan et al. (2019) as $\lambda_1 = -0.5 + 0.866j$ and $\lambda_2 = -0.5 - 0.866j$, which are complex conjugates. Furthermore, it is noticed that $\pi/0.866 = 3.6277$ is very close to the period observed in Fig.5. All of these facts are consistent with the Theorem 3 in Postoyan et al. (2019), in which the planar linear system and static event condition $\|\hat{x}(t) - x(t)\| \geq \sigma\|x(t)\|$ in Tabuada (2007) were considered. Moreover, a set of different initial states $[10; 0], [-10; 0], [0; 10], [0; -10], [5; 5]$ is implemented in the same setting, and the results further validate the statement in Theorem 3. In addition, it is shown that the period of IETs is irrelevant to the initial states of the controlled system and the initial states might result in different phases. In the meantime, these findings also provide some hints for the connection between static and dynamic triggering mechanisms.

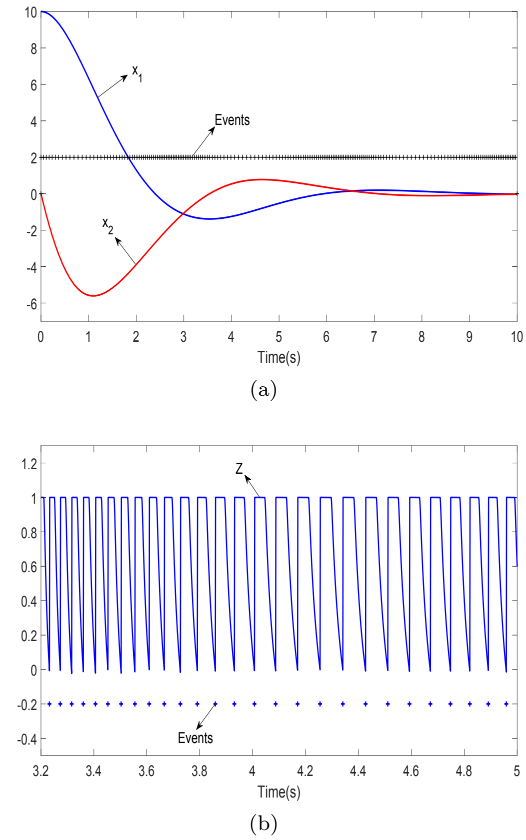

Fig. 1. Numerical simulation results of the dynamic IETs-designable event-triggered linear control system with $\bar{Z} = 1$ and $\varepsilon = 1$: (a) The trajectories and event triggering times, respectively; (b) The triggering events and the evolution of Z from 3.2 to 5s.

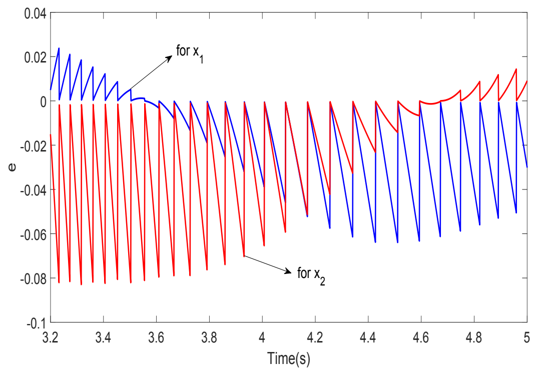

Fig. 2. The evolution of the measurement errors e_1, e_2 for the event-triggered linear control system.

6. CONCLUDING REMARKS

In this work, in order to improve certain crucial characteristics of IETs, we develop a new design and analysis frame-

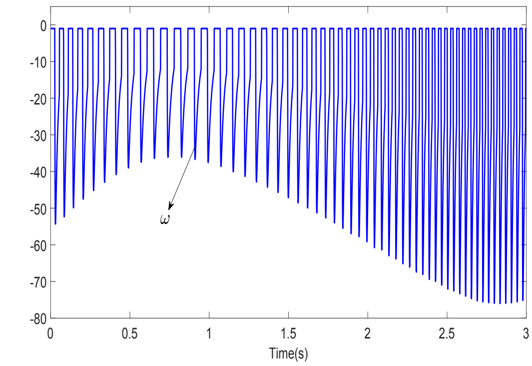

Fig. 3. The evolution of ω for the event-triggered linear control system.

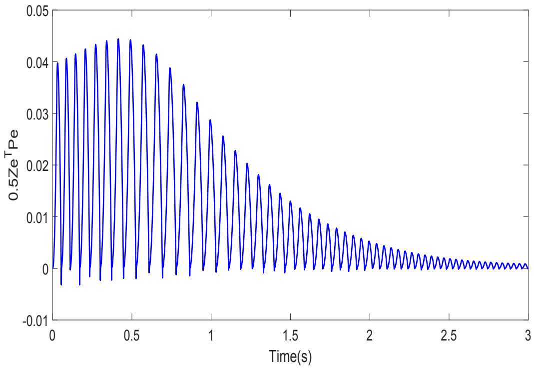

Fig. 4. The evolution of the item $\frac{1}{2}Ze^\top Pe$ in the Lyapunov function.

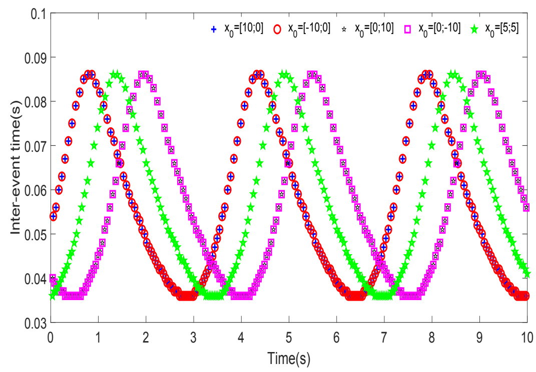

Fig. 5. The evolution of the IETs.

work for the event-triggered control system. An IETs-designable triggering mechanism has been established for nonlinear systems and general linear systems, respectively. Afterwards, the robustness issue of the proposed results is further considered. It is shown that the IETs-designable triggering mechanism guarantees Zeno-free triggering and the robust global event-separation property.

Currently, we are working on applying the proposed triggering mechanism to distributed control systems with general linear dynamics. It is also interesting to investigate other kinds of disturbances, such as time delay, DoS attack, or timing error, etc. In the future, we also plan to apply the IETs-designable event triggering mechanism to broader areas, such as industrial automation systems and cyber-physical systems.

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