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Consensus for Expressed and Private Opinions under Self-Persuasion ⋆

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Abstract: As recognized in psychological research, there is often a difference between an agent's expressed opinion and private opinion (or belief). This occurs for different reasons, such as political correctness or peer pressure. The opinion expressed by an agent is the result of pressure to follow the (average) opinions expressed by the group to which the agent belongs, or to follow group norms. The agent's private opinion is unknown to others, but evolved under the influence of other agents' expressed opinions. This paper proposes an opinion formation model based on the theory of bounded confidence, and studies the dynamic process of expressed and private opinions in time-varying networks. At the same time, the self-persuasion effect of agents in the dissonance between expressed and private opinions is considered. Here, group pressure establishes the motive force from private opinion to expressed opinion, while self-persuasion establishes the reverse connection. We find that group pressure can effectively reduce the gap of opinions between the group, but does not always promote consensus. Furthermore, the self-persuasion effect of agents can ensure the realization of group consensus.

Keywords: opinion dynamics, Hegselmann-Krause model, social network, self-persuasion, multi-agent systems.

1. INTRODUCTION

In recent years, the study of opinion formation and its dynamic evolution in a network has become a typical problem in social network analysis (Anderson et al. (2019); Noorazar et al. (2019); Etesami et al. (2015)). As a classic consensus problem in a multi-agent system, individuals can only communicate based on the limited information obtained from neighbors depicted by the network. Although the description of collective behavior involving psychosocial and individual emotions is a well-known challenge, a variety of agent-based opinion dynamics models have been studied.

According to the opinion value held by agents, opinion dynamic models can be divided into discrete and continuous models. For discrete models, we tend to express the pros and cons by using binary values. Famous examples include the voter model (Holley et al. (1975)), Sznajd model (Sznajd et al. (2000)), and majority-rule model (Galam (2002)). In more cases, opinions are described by continuous values to indicate attitudes within a range. In the case of continuous value, the early DeGroot model (DeGroot (1974)) and Friedkin model (Friedkin et al. (1990)) based on fixed network topologies are typical. In particular, time-varying network topology models based on the bounded confidence mechanism proposed by Hegselmann-Krause(HK) (Hegselmann et al. (2002)) and Deffuant-Weisbuch (Deffuant et al. (2000)) respectively have aroused considerable research interest. In the HK model, each agent communicates only with neighbors who share similar opinions within a given threshold to capture trends of sociological homogeneity.

In most existing opinion dynamics (including the above), the main assumption is that each agent has a single opinion on a given issue. However, in many cases, such as when a candidate is trying to attract the attention of voters, the expressed opinion of an agent may differ from its inner belief. As is recognized in the study of psychology, a discrepancy often exists between the expressed and private opinion (or belief) of an individual. This occurs for different reasons, such as political correctness or group pressure. For example, psychologist Asch's famous conformity experiment (Asch et al. (1951)), which inspired many researchers including Ye et al (Ye et al. (2019); Huang et al. (2014); Francisco et al. (2019); Shang (2019)). The agent's expressed opinion is the result of pressure to follow the average opinions expressed by the group to which the agent belongs, or to follow group norms. The private opinion of an agent is unknown to others, but evolved as a function under the influence of other agents' expressed opinions.

Although researchers have developed many models to describe the dissonance between expressed and private opinions caused by group pressure, current research lacks

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follow-up attention to this dissonance behavior. Models often assume that agents completely ignore the influence of this dissonant factor when updating private opinions. Therefore, these models based on group pressure cannot guarantee group consensus, which in many cases does not match consensus phenomena in real society. As a response mechanism to dissonance behavior, *self-persuasion* theory has a long history in social psychology, but its mathematical study in opinion dynamics is novel. Self-persuasion is an intermediate process in which an individual's external behavior causes an internal state change (Zimbardo et al. (1991)), often occurring when people reflect on a topic and change their attitudes (Petty et al. (2003)). In traditional media messages, self-persuasion has shown to be more effective than direct persuasion to change health-related cognition and behavior (Loman et al. (2018); Krischler et al. (2015); Damen et al. (2015)). In this form, self-persuasion effects also have been found in social media (Pingree (2007); Valkenburg et al. (2016); Greenberg et al. (2018)).

In this paper, we explore a new line of thought in opinion dynamics and consider the influence of group pressure and self-persuasion on opinion formation. Therefore, we propose a modified HK model, in which each agent in the network have both expressed and private opinions on a given topic. Under social influences, the agent's private opinion evolves from those opinions expressed by its neighbors, and agent's expressed opinion is affected by group pressure, showing the tendency to adopt public opinion. At the same time, the response mechanism to the dissonance between expressed and private opinions is considered by self-persuasion theory. Different from the fixed network topology adopted by Ye et al., we study the opinion evolution in a time-varying network topology based on the bounded confidence theory. We find that group pressure does not always promote group consensus, while self-persuasion theory is the perfect patch to ensure consensus.

The paper is organized as follows: in sections 2, some preliminaries are presented and we formulate our problem. In section 3, main results and proofs are presented. Section 4 includes numerical simulations to further verify our conclusion. Finally, in the section of concluding remarks, we give a summary of results of this paper and possible directions of future work.

2. PRELIMINARIES AND PROBLEM FORMATION

2.1 Graph Theory

Some basic concepts and notations of graph theory will be introduced. (1).Graph is a mathematical model of network, here we mainly discuss directed graphs. For a direct graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with nodes set $\mathcal{V} = \{1, 2, \cdots, n\}$ and edge set $\mathcal{E}$ which is defined that $(i, j) \in \mathcal{E}$ is an edge from node i to node j. (2).*Root*, in a direct graph $\mathcal{G}$, if there exist a path from i to j for any node $j \in \mathcal{V}$, we say i is a root of $\mathcal{G}$. Furthermore, if the length of the path from i to j is 1 for every node j, we say $\mathcal{G}$ is *strongly rooted* at node i, thus node i is a neighbor of every other node in $\mathcal{G}$. (3).*Composition*, consider $\mathcal{G}_1$ and $\mathcal{G}_2$ are directed graphs with the same node set, define formula $\mathcal{G}_1 \circ \mathcal{G}_2$ as the composition of $\mathcal{G}_1$ and $\mathcal{G}2$ with the following property: the directed edge (i, j) exists if and only if we can find another node k to make sure that edge (i, k) and (k, j) respectively belong to $\mathcal{G}_1$ and $\mathcal{G}_2$.

2.2 Review of Hegselmann-Krause (HK) model

Consider a group of n agents as $N = \{1, 2, \cdots, n\}$, and for each agent $i \in N$, his/her opinion at time t is represented by $y_i(t) \in [0, 1]$. Denote agents' bounded confidence set as $\varepsilon = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n\}$, each agent i only interacts with near neighbors whose opinions differ from his own not more than the certain confidence interval ε_i. The basic assumption of the discrete time HK model is that, all agents' opinions simultaneously update at each time. Hence model can be described by Eq. (1)

$$y_i(t+1) = |I(i, y(t))|^{-1} \sum_{j \in I} y_j(t) \tag{1}$$

where $I(i, y) = \{1 \le j \le n \mid |y_i - y_j| \le \varepsilon_i\}$ is agent i's neighbor set and $|I(i, y)|$ is the cardinality of $I(i, y)$. Eq. (1) indicates that each agent in the group updates it's opinion by considering all the neighboring agents' opinions with weight factor $|I(i, y)|^{-1}$, and always be a self-neighbor.

2.3 Group Pressure and Self-Persuasion

In the classical HK model, each agent only has a single opinion in the group. However, group pressure leads to conformity behavior, which has been investigated in social psychology, resulting in the dissonance between agent's expressed and private opinions. As shown in Fig. 1, expressed opinion is evolved from private opinion, and a dissonance often exists between them. Here, we try to consider, does an agent's expressed opinion have a negative impact on its private opinion? Self-persuasion theory provides us with a new perspective. Self-persuasion differs from other forms of persuasion because the means of influence are self-generated instead of externally provided (Brinol et al. (2012)). Research has shown that information that is generated by oneself is perceived as more accurate and trustworthy and therefore more persuasive than information generated by an external source (Hoch et al. (1989); Levin et al. (1988)). Expressing a position publicly can change subsequent behavior through the principle of commitment and consistency (Cialdini (2009)). Specifically, when an individual expresses a position publicly, he/she will change subsequent behaviors and attitudes to be in accordance with the expression.

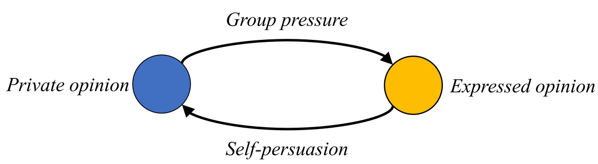


Fig. 1. Effect of group pressure and self-persuasion.

As summarized in Fig. 1, group pressure establishes the relevance from private opinion to expressed opinion, while self-persuasion establishes the backward connection.

2.4 Our Modified Hegselmann-Krause (M-HK) model

For a population of n agents, defining $V = \{1, 2, \cdots, n\}$, let $y_i(t)$ and $\hat{y}_i(t)$, $i \in V$, represent agent i's private and expressed opinions at time $t = 0, 1, \cdots, \infty$. The opinions are scaled to be $y_i(t), \hat{y}_i(t) \in [0, 1]$. Here, we regard y_i as agent's inner true opinion, and $\hat{y}_i$ as opinion open to the public under group pressure. Because of the information asymmetry, for each agent $i \in V$, only the expressed opinions of others $\hat{y}_j(t)$, $j \in V, j \neq i$ can be observed. In general, $y_i(t)$ and $\hat{y}_i(t)$ are not equal, and we assume that $y_i(0) = \hat{y}_i(0)$, $i \in V$ at the initial moment.

Now, we will present the dynamics process of our modified model which describes the evolution of an agent's expressed and private opinions under group pressure and self-persuasion.

- At each step, agent i first expresses an opinion $\hat{y}_i(t)$, then observes others' expressed opinions, $\hat{y}_j(t)$, $j \neq i$.
- Agent i updates his/her private opinion $y_i(t+1)$ based on the bounded confidence and self-persuasion.
- Agent i updates his/her expressed opinion $\hat{y}_i(t+1)$ under conformity pressure.

Therefore, our M-HK model is described by Eq. (2) and Eq. (3).

$$y_i(t+1) = \frac{1-\alpha_i}{1+|N_i(t)|}\left[y_i(t) + \sum_{j \in N_i(t)} \hat{y}_j(t)\right] + \alpha_i \hat{y}_i(t) \quad (2)$$

and determines his/her expressed opinion according to

$$\hat{y}_i(t+1) = (1-p_i)y_i(t+1) + p_i \hat{y}_{avg}(t) \quad (3)$$

where $N_i(t) = \{1 \leq j \leq n, j \neq i | \, |y_i(t) - \hat{y}_j(t)| \leq \varepsilon_i\}$ represents the set of $i's$ communicating neighbors at time t, the constant $\varepsilon_i \in (0, 1]$ is the confidence interval of agent i, and $|N_i(t)|$ is the cardinality counting of $N_i(t)$. The value $\hat{y}_{avg}(t) = \frac{1}{n}\sum_{j=1}^{n} \hat{y}_j(t)$ represents the group public opinion. Here we have constant $\alpha_i \in (0, 1]$ and $p_i \in (0, 1]$ to describe the level of self-persuasion and group pressure, respectively. Noteworthy, if $\alpha_i = 0$ and $p_i = 0$, the model will revert directly to the classic HK model.

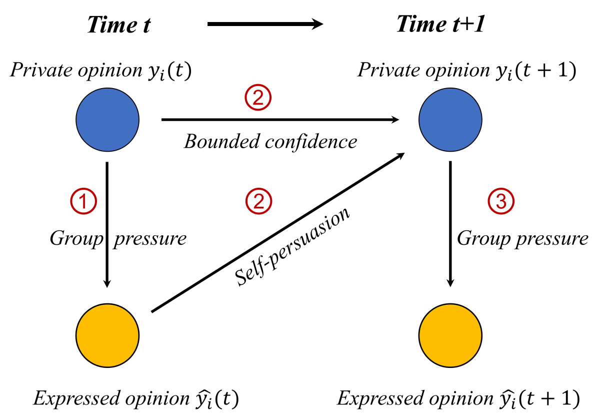


Fig. 2. Evolution of private and expressed opinions.

In the classic HK model, the convergence of opinions will be strongly affected by agents' confidence intervals, and relatively small confidence intervals can produce separate clusters. Here, in our M-HK model (2), both private and expressed opinions can reach a consensus regardless of variation in confidence interval. The formation of an agent's final private and expressed opinions will be influenced by group pressure and self-persuasion.

3. PROOF OF CONSENSUS ON M-HK MODEL

Theorem 1. In M-HK Model, suppose $\alpha_i \in (0, 1)$ and $p_i \in (0, 1)$, for all $i \in N$, then the consensus of private opinion $y(t)$ and expressed opinion $\hat{y}(t)$ both can be reached in finite time T_1^*, and we have $y(t) = \hat{y}(t)$, for $t > T_1^*$.

We will start with several related definitions and propositions.

Definition 1. Coefficient of Ergodicity (Sneta (2006)): For a stochastic matrix $M = \{m_{ij}\}$, define its coefficient of ergodicity as $\rho(M) = 1 - \min_{i,j} \sum \min\{m_{ik}, m_{jk}\}$.

Proposition 1. (see Sneta (2006)) For any stochastic matrices M_1 and M_2, it follows that $\rho(M_1 M_2) \leq \rho(M_1)\rho(M_2)$.

Definition 2. Scrambling matrix (Shen (2000)): For any non-negative n order matrix M, if there exists $k \in \{1, 2, \cdots, n\}$ such that for arbitrary i and j with $i, j \in \{1, 2, \cdots, n\}$, $i \neq j$, m_{ik} and m_{jk} are both positive, then we call M a scrambling matrix.

Furthermore, the scrambling matrix has the following property.

Lemma 1. Let $\mathcal{M} = \{M_i\}, i = 1, 2, \cdots$ be a compact set of scrambling stochastic. Then for each infinite sequence $M_{i_1}, M_{i_1}, \cdots$ there exist a row vector $\boldsymbol{c}$ such that

$$\lim_{j \to \infty} M_{i_j} M_{i_{j-1}} \cdots M_{i_1} = \mathbf{1}\boldsymbol{c} \quad (4)$$

Proof. First we indicate that for any stochastic matrix A and a row vector $\boldsymbol{c}$, we have $A(\mathbf{1}\boldsymbol{c}) = \mathbf{1}\boldsymbol{c}$ because of the fact that $A\mathbf{1} = \mathbf{1}$. For any $M \in \mathcal{M}$, we have $\min_{ij} \sum_{k=1}^{n} min\{m_{ik}, m_{jk}\} > 0$ because matrix M is scrambling. So $0 \leq \rho(M) < 1$ for any scrambling matrix M. With the property in Proposition 1, for any stochastic matrices M_1 and M_2, we have $\rho(M_1 M_2) \leq \rho(M_1)\rho(M_2)$, hence

$$\rho(M_{i_j} M_{i_{j-1}} \cdots M_{i_1}) \leq \rho(M_{i_j})\rho(M_{i_{j-1}}) \cdots \rho(M_{i_1})$$

Thus, as $j \to \infty$, we have $\rho(M_{i_j} M_{i_{j-1}} \cdots M_{i_1}) \to 0$ because of $0 \leq \rho(M) < 1$, which implies that the element-wise difference between any pair of rows in the product $M_{i_j} M_{i_{j-1}} \cdots M_{i_1}$ approaches 0. Then Eq. (4) holds.

To study the dynamic behavior of M-HK model, we can rebuild the communication relationships associated with our Model by using a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The system state can be described as $y = [y_1, y_2, \cdots, y_n]^\top$, $\hat{y} = [\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_n]^\top$. In order to describe the model formula in spatial form, we define W as the influence matrix, w_{ij} is the ij-element of the nonnegative influence matrix W, connoting the influence weight agent i assigns the opinion value of agent j. So the opinion formation process described by Eq. (2) and Eq.(3), is captured, for $t = 0, 1, \cdots, \infty$, by the discrete-time system (5).

$$\begin{bmatrix} y(t+1) \\ \hat{y}(t+1) \end{bmatrix} = \begin{bmatrix} W_{11}^t & W_{12}^t \\ W_{21}^t & W_{22}^t \end{bmatrix} \begin{bmatrix} y(t) \\ \hat{y}(t) \end{bmatrix} \quad (5)$$

Here, we build an new interactive network $\mathcal{G}[W^t]$ that contains $2n$ nodes $\mathcal{V}^* = \{1, 2, \cdots 2n\}$. Nodes $\mathcal{V}_p^* = \{1, 2, \cdots n\}$ represent all agents' private opinions $y(t)$, and $\mathcal{V}_e^* = \{n+1, \cdots 2n\}$ represent expressed opinions $\hat{y}(t)$. Set $\beta_i = (1-\alpha_i)/(1+|N_i(t)|)$, the submatrix $W_{11}^t = diag\{\beta_i\}$ and $W_{21}^t = diag\{(1-p_i)\,\beta_i\}$ are both diagonal matrices. W_{22}^t contains the link from $\hat{y}_{avg}(t)$ to $\hat{y}(t+1)$, having a specific structure, holding all elements to be positive. The off-diagonal elements of submatrix W_{12}^t represent the agents' communication neighbor sets which is changing over time, in particular, all the diagonal elements of W_{12}^t are positive as $\alpha_i > 0$

$$W^t = \left[\begin{array}{ccc:ccc} + & & 0 & + & & \\ & \ddots & & & \ddots & \\ 0 & & + & & & + \\ \hdashline + & & 0 & + & \cdots & + \\ & \ddots & & \vdots & + & \vdots \\ 0 & & + & + & \cdots & + \end{array}\right]$$

Lemma 2. Suppose that $\alpha_i(t) > 0$ and $p_i > 0$ for all $i \in N$, in graph $\mathcal{G}[W^t]$, for $\forall i \in V_p^*$, node i is a root of node j, $j \in V^*$.

Proof. (1). If $j \in V_e^*$, we can always find a path from i to j in the directed graph $\mathcal{G}[W^t]$, as $i \to i+n \to j$, for $w_{ii}, w_{i+n,i}, w_{j,i+n} > 0$; (2). If $j \in V_p^*$, we have W_{12}^t with positive diagonal elements, then we can find a path from $j+n$ to j, as $i \to i+n \to j+n \to j$, for $w_{ii}, w_{i+n,i}, w_{j+n,i+n}, w_{j,j+n} > 0$.

Thus, for $\forall i \in V_p^*$, node i is a root of all the other $2n-1$ nodes.

Theorem 2. In graph $\mathcal{G}[W^t]$, there exists a positive integer k, for any time-intervals $[t_{j_k}, t_{j_{k+1}})$, such that any node i from set V_p^* is a strong root of every other $2n-1$ nodes at time $t_{j_{k+1}}$ in the composition of graphs encountered along $[t_{j_k}, t_{j_{k+1}})$.

To prove Theorem 2, we need the following definition and results.

Lemma 3. (Proposition 3 in (Cao et al. (2008))) suppose $n > 1$ and let $\mathcal{G}_{p1}, \mathcal{G}_{p2}, \cdots, \mathcal{G}_{pk}$ be a finite sequence of rooted graphs in $\mathcal{G}$. If $\mathcal{G}_{p1}, \mathcal{G}_{p2}, \cdots, \mathcal{G}_{pk}$ are all rooted at v and $k \geq n-1$, then $\mathcal{G}_{pk} \circ \mathcal{G}_{pk-1} \circ \cdots \circ \mathcal{G}_{p1}$ is strongly rooted at v.

Proof. From lemma 3, we know that with $k \geq n-1$, the composition of graphs associated with $\mathcal{G}_t \circ \mathcal{G}_{t+1} \circ \cdots \circ \mathcal{G}_{t+k}$ is strongly rooted at any node i, for all $i \in V_p^*$, because i is a root of each $\mathcal{G}_t$, that means there exists a directed edge from node i to every other node in the composition of the corresponding graphs. Such that node i is a neighbor of every other $2n-1$ nodes at time $t_{j_{k+1}}$ in the composition of graphs encountered along $[t_{j_k}, t_{j_{k+1}})$.

Now we can present the consensus result of Theorem 1.

Proof of Theorem 1.

From Theorem 2, there exists a positive integer $k \geq n-1$, for any time-intervals $[t_{j_k}, t_{j_{k+1}})$, such that any node $i \in V_p^*$ is a neighbor of every other node in the composition of graphs encountered along $[t_{j_k}, t_{j_{k+1}})$. So the multiplication of matrix $\hat{W}^{\tau_j} = W^{t_{j_k}} W^{t_{j_k+1}} \cdots W^{t_{j_{k+1}-1}}$ has at least one positive column, that means $\hat{W}^{\tau_j}$ is a scrambling matrix.

Set $Y = [y_1, \cdots, y_n, \hat{y}_1, \cdots, \hat{y}_n]^\top$,

$$\begin{aligned} Y(t_{j_k}) &= W^{t_{j_k-1}} \cdots W^{t_0} Y(0) \\ &= (W^{t_{j_k-1}} \cdots W^{t_{j_{k-1}}}) \cdots (W^{t_{j_1-1}} \cdots W^{t_0}) Y(0) \\ &= \hat{W}^{\tau_{j-1}} \cdots \hat{W}^{\tau_0} Y(0) \end{aligned} \tag{6}$$

By the Lemma 1, There is always going to be a row vector $\boldsymbol{c}$ such that

$$\lim_{j \to \infty} \hat{W}^{\tau_j} \hat{W}^{\tau_{j-1}} \cdots \hat{W}^{\tau_0} = \mathbf{1c}$$

Set $Y_{ss} = \boldsymbol{c}Y(0)$, then we get $\lim_{j\to\infty} Y(t_{j_k}) = Y_{ss}\mathbf{1}$.

Since a consensus will be reached for opinion set $Y(t) = [y_1, \cdots, y_n, \hat{y}_1, \cdots, \hat{y}_n]^\top$ that contains both private opinions $y(t)$ and expressed opinions $\hat{y}(t)$. There exists a smallest T_0^* such that

$$|Y_{max}(T_0^*) - Y_{min}(T_0^*)| \leq \varepsilon$$

so from Eq. (2) and Eq. (3), the consensus of private opinion $y(t)$ and expressed opinion $\hat{y}(t)$ are both reached after $T_1^* = T_0^* + 1$, and we have $y(t) = \hat{y}(t)$, for $t > T_1^*$.

Here, we can establish a new relationship between the scrambling matrices and rooted graphs, by consider the situation where each agent of V_p^* is strong rooted across some finite-length intervals. Hence, in M-HK model, opinion consensus can be reached in finite time.

4. SIMULATIONS

In this section, we make further investigation about the role of group pressure p and self-persuasion α through model simulation, consider a simple network of n=50 agents whose initial private and expressed opinions are uniform distributed in the space $y(0) = \hat{y}(0) \in [0, 1]$, and we set $\varepsilon_i = \varepsilon$, $\alpha_i = \alpha$, $p_i = p$, for all $i \in V$, without loss of generality.

4.1 With Self-Persuasion ($\alpha > 0$)

As proved in section 3, M-HK model can reach consensus in finite time if $\alpha_i \in (0, 1)$ and $p_i \in (0, 1)$, for all $i \in N$. Fig. 3 shows the convergence time of our M-HK model with different group pressure p. It is counterintuitive that the convergence time and group pressure are not negatively correlated, which means group pressure does not always accelerate the convergence rate of M-HK model. In addition, we find that the convergence time curve shows common characteristics at different confidence intervals $\varepsilon = 0.05, 0.10, 0.15$, that is, the curve decreases with the increase of group pressure p, and then breaks at the critical point and rises rapidly. The reason for the inflection point is that, the converge to consistency of M-HK model is the result of the joint effect of group pressure p and self-persuasion α. In other words, group pressure cannot alone guarantee the realization of group consensus without considering self-persuasion, which will be discussed in the next subsection.

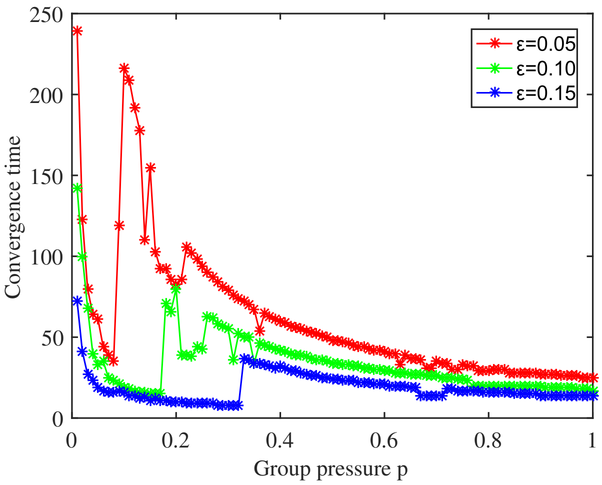
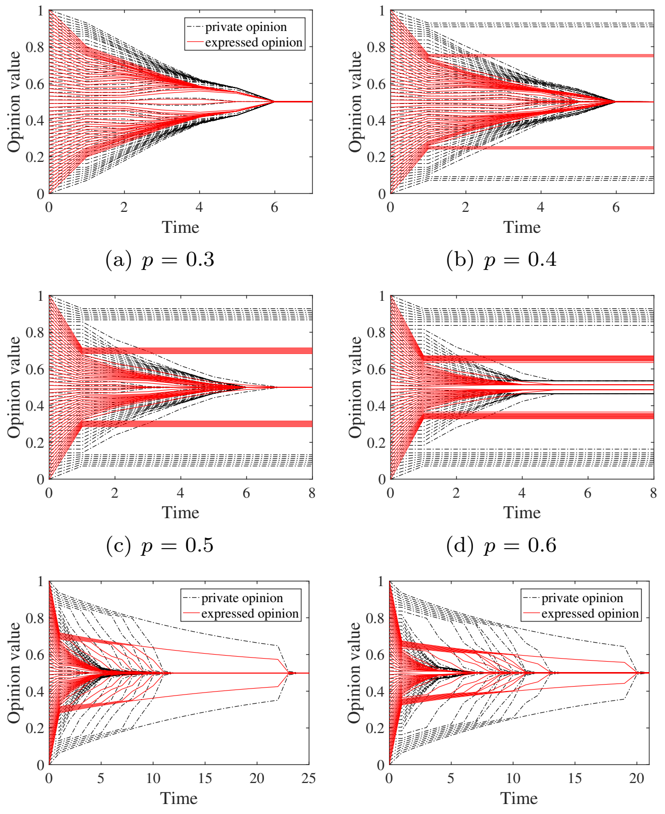


Fig. 3. Convergence time of M-HK model with different group pressure p. The result refers to n = 50, α=0.1, and ε=0.05, 0.1, 0.15.



(a) $p = 0.3$ (b) $p = 0.4$

(c) $p = 0.5$ (d) $p = 0.6$

(e) $p = 0.5$, α=0.1 (f) $p = 0.6$, α=0.1

Fig. 4. Time evolution of $n = 50$ agents with different group pressure level p.

4.2 Without Self-Persuasion ($\alpha = 0$)

The time evolution of the n = 50 group with different pressure level p is shown in Fig. 4. we set ε=0.15, and the initial profiles $y(0)$ and $\hat{y}(0)$ are uniformly distributed in the space $[0, 1]$. As pressure level p increases, the final opinions step from consensus (conformity) to fragmentation (plurality). From Fig. 4 (a), an appropriate pressure level can guarantee the consensus of the group. However, when the pressure level gradually grows larger, the marginal agents will split from the group, and the number of marginal agents thus increases with the pressure level, see Fig. 4 (b)-(d). Interestingly, the polarization of the two central clusters can be observed in Fig. 4 (d) with p=0.6. On the other side, Fig. 4 (e)-(f) shows the contribution of self-persuasion to the realization of group consensus. Finally, we find the gap of expressed opinions is smaller than private opinions in the group from Fig. 4 (b)-(d), implicating that, in real life, the actual divergence of group opinions could be much bigger than the apparent divergence.

Then, with group population n and initial profile $y(0)$ fixed, we further investigate the role of confidence interval ε and conformity pressure p of ensuring group consistency. Under a certain confidence interval ε, as shown in Fig. 5, we can always find a maximum pressure level p_c to ensure the convergence of the model, and p_c increases monotonically with ε. It indicates that when an individual has a large confidence interval, the more pressure he/she can accept in the group without being isolated.

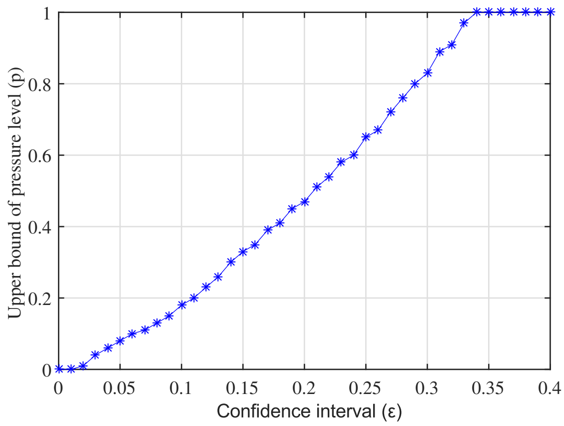


Fig. 5. The maximum p_c with different ε for consensus.

5. CONCLUSION

In this paper, we use the conformity phenomenon caused by group pressure in social psychology, to build a dynamic model including private and expressed opinions on the basis of Hegselmann-Krause opinion dynamics. The dissonance between agent's private and expressed opinions is determined by group pressure in social networks. Then, the self-persuasion theory is introduced as a complement to the inverse link between private and expressed opinions. Thus, we propose an new modified HK model to investigate the evolution characteristics of both private and expressed opinions under the common effect of group pressure and self-persuasion in time-varying networks. We find that group pressure can not always guarantee group consensus, while self-persuasion theory, as a dissonance response mechanism, provides a perfect patch for the formation of group consensus. In the future, more methods reflecting complex human behavior can be proposed based on this model.

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