Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Multi-stage Event-triggered Model Predictive Control for Automated Trajectory Drilling

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Abstract: In upstream Oil and Gas operations a well is drilled following a planned trajectory. The trajectory is designed to avoid hard formations and other wells while minimizing drilling time. The uncertainty of the environment, e.g. unknown rock hardness, effects negatively the efficiency of operation: drilling time increases due to frequent corrective control actions that must be taken to counteract disturbances and risk increases since process constraints may be violated. This paper proposes an event-triggered multi-stage model predictive control that aims at tackling both challenges. The event-triggering strategy tries to minimize the number of control actions sent to the actuators, while the multi-stage strategy improves constraints satisfaction despite uncertainties. The method is tested in simulation where unknown changes in rock hardness are considered. In comparison to a standard model predictive control approach, we show that using the combined event-triggered and multi-stage approach we improve constraints satisfaction and decrease the number of control actions.

Keywords: Model Predictive Control, Multi-stage, Event-triggered, Drilling Automation

1. INTRODUCTION

Drilling is an important stage in upstream Oil and Gas operations. Extraction efficiency is optimized by designing a well plan that avoids collision with other wells while maximizing the contact area with the reservoir. Often this results in particularly complex well plans. Once a well plan is designed, usually the trajectory control of the drilling bit is left to the experience of a directional driller who manually adjusts surface and down-hole control parameters that try to minimize the deviation of the drilling bit from the plan and the drilling time. Often manual control is not efficient and prone to errors. To improve drilling performance, we propose an automated strategy that uses Model Predictive Control (MPC) to compute the optimal input while satisfying process constraints (Rawlings and Mayne, 2009; Findeisen and Allgöwer, 2002). MPC has been successfully used for decades in many applications, firstly in chemical engineering and more recently in automotive and robotics.

In the present work, a Rotary Steerable Systems (RSS) is considered. In RSS, the bit trajectory is controlled by manipulating the forces on the ribs of the steering unit that push against the rock formation allowing continous steering (Fig. 2). Control actions are transmitted from surface to downhole through mud-pulse telemetry (MPT) (Fig. 3). The MPT system translates the signal (called downlink) from digital to analog by varying the mud flow which is continuously pumped into the drilling string. These flow variations are read and interpreted by a downhole receiver and then transmitted to the actuators. The MPT also transmits downhole measurements, e.g. bit inclination and azimuth, to the surface (uplink) but instead of flow variations, pressure waves traveling through the mud are used. The MPT has the advantage of not requiring any expensive wiring but downlinks have limited band width (only few bits of data per minute) and slow transmission times (up to 10 minutes). Since drilling operation is stopped while sending a downlink, the drilling time, and consequently the cost of operation, increases linearly with the number of downlinks. Furthermore, due to disturbances, e.g. unknown rock formation changes, frequent adjustments of the control actions are necessary to keep the bit on track. Hence, a control method that aims at decreasing the number of downlinks is necessary.

⋆ BM, MK and RF are affiliated to the International Max Planck Research School (IMPRS) for Advanced Methods in Process and Systems Engineering, Magdeburg.
This work was partly funded by the Federal Ministry of Economics and Technology (Germany).

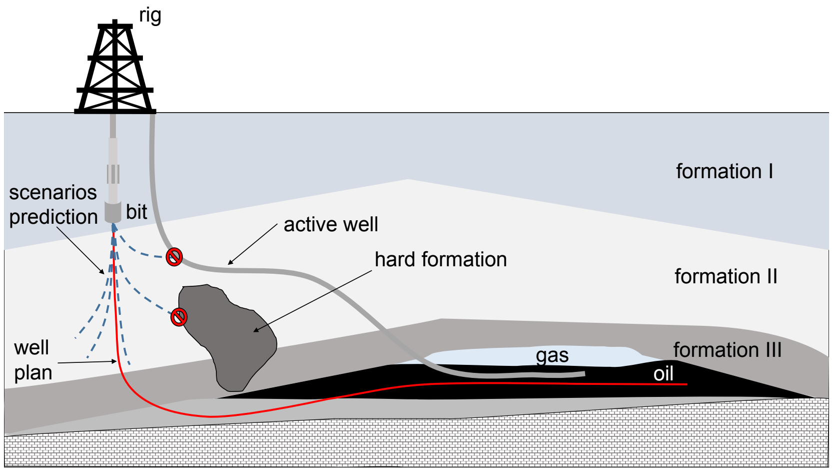


Fig. 1. Schematic representation of considered problem. The bit is required to follow a predefined well plan. Uncertainties, e.g. rock hardness, are taken into account in a scenario tree. Each prediction of the scenario reflects a certain combination of uncertainty realizations. This increases robustness to constraints violation (e.g. distance to active wells).

To minimize the number of downlinks, we propose an event-triggered model predictive controller (MPC). In event-triggered MPC a new control action is computed not at periodic time instances but only when a certain condition is met (Heemels et al., 2012; Lucia et al., 2016). Furthermore, to ensure safety, the controller must satisfy process constraints e.g. *Dog-Leg severity* (DLS) and position. The DLS indicates the local degree of curvature of the string. If the DLS exceeds the maximum allowed DLS, the string could break, causing a major economic loss. Furthermore, a large DLS causes friction that create problems while drilling and while running the case inside the hole prior to extraction. Since the interaction between bit and rock formation is uncertain a robust control approach is needed. In the present work, we assume that the uncertainties enter the problem formulation as uncertainties on the system parameters. To deal with this parametric uncertainty, we use multi-stage MPC (c.f. Lucia et al. (2013); Maiworm et al. (2015) and references therein). In multi-stage MPC a scenario tree representing a finite number of parameter realizations is spanned along the prediction horizon (Fig. 5). Each realization of a parameter is sampled from an uncertainty distribution representative of the underlying parametric uncertainty. Consequently, contrary to nominal MPC, multistage-MPC considers multiple system predictions instead of a single one when optimizing the inputs to the system. This increases the robustness against constraints violations.

Not many results using MPC for trajectory drilling are available. In Bayliss et al. (2015) a linear MPC was used for attitude control subject to delays. In Zhao et al. (2019) MPC was used to track the well plan trajectory. This contribution differs from the others mainly because it considers simultaneously uncertainties and downlink minimization.

The paper is structured as follows: in Section 2 some important mathematical definitions are briefly given. In Section 3 the general model formulation is introduced. Section 4 describes firstly the multi-stage approach, then the triggering condition and finally gives some conditions

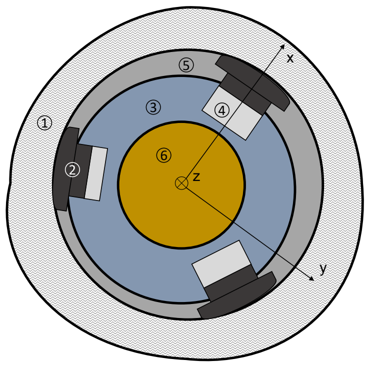


Fig. 2. Cross-section of a Rotary Steerable System. (1) Rock formation, (2) movable ribs, (3) string body, (4) hydraulics chambers, (5) bore hole, (6) mud chamber. The oil pressure in each chamber can be controlled independently. The asymmetry of the contact forces with the rock formation pushes the bit to the desired direction.

for recursive feasibility of the proposed MPC approach. In Section 5 the simulation results are shown, and finally Section 6 presents some conclusion and remarks.

For brevity for the remaining of the paper we shorten multi-stage event-triggered MPC to just multi-stage MPC.

2. NOTATION

$\mathbb{N}$ is the set of natural numbers. The notation $\mathbb{N}_{[a,b]}$ denotes the set of natural numbers between a and b, and $\mathbb{N}_{\geq a}$ the natural numbers greater or equal than a. $x_{l|k}$ denotes the prediction of x at time l computed at depth k. The Minkowski sum is defined as $A \oplus B := \{a + b | a \in A, b \in B\}$. $|a|_i$ denotes the i-norm of a while $\|x\|_A^2$ denotes the product $x^T Ax$ where x^T is the transpose of x.

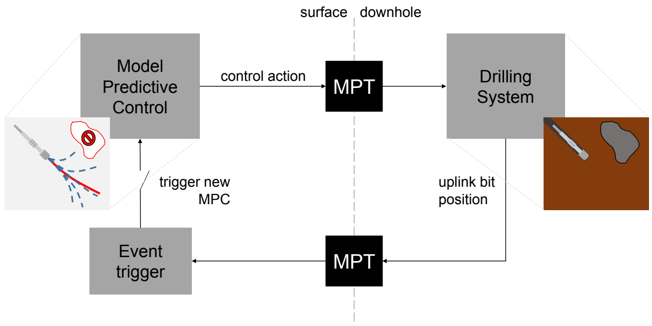


Fig. 3. Control structure with communication channels (MPT).

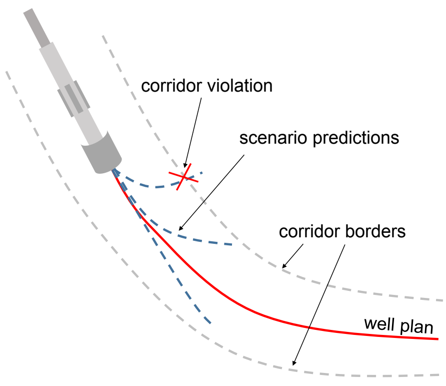


Fig. 4. Corridor based trigger: when at least one of the predicted positions lies outside of the corridor, a new MPC iteration is triggered.

3. MODELLING

The MPC uses a nonlinear discrete-depth model representing the steering response of the drilling system:

$$x_{k+1} = f(x_k, u_k, d_k), \tag{1}$$

where x is the vector of the system states. u is the input vector and d is the system parameters vector. These parameters are uncertain, since they change depending on the environment and the geometry of the drilling string (e.g. change in rock formation, bit type, weight on bit). Any model in the form of (1) can be used (e.g. the the models used in Zhao et al. (2019); Panchal et al. (2010)). Since the main goal of this paper is addressing the control strategy rather than the modelling approach, the detailed formulation of the model used for simulations is omitted.

4. MODEL PREDICTIVE CONTROL

4.1 Multi-stage MPC

Multi-stage MPC is one of method to deal with parametric uncertainties and belongs to the class of robust MPC approaches. Here, only the main idea of multi-stage MPC is given, for details refer to Lucia et al. (2013). The main idea is to represent uncertainty as a scenario tree. The tree starts from the current system state which is known (because measured or estimated, see Fig. 5). Since the future parameters values are unknown, the system branches into more possible directions in the state-space. Each branch

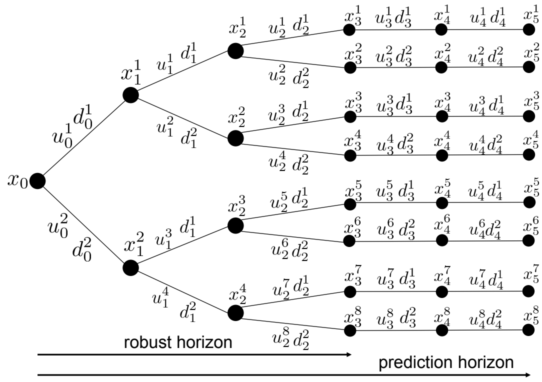


Fig. 5. Representation of the scenario tree. Each branch reflects a different uncertainty realization. After the robust horizon, the tree stops branching.

represents a different realization of the uncertain parameters. To decrease conservativeness, it is assumed that the value of the states will be available in the future, consequently for each scenario the control action can be adapted to counteract the uncertainty evolution. Hence, instead of a single state prediction, a larger number of predictions are considered, each of these having a different history of uncertainty realizations. This approach guarantees robust constraint satisfaction in case the uncertainty takes exactly the same values of those considered in the scenario tree. If uncertainties are not considered in the scenario, e.g. because they do not have discrete values, the method can give anyhow a good feedback sequence if the scenario tree is chosen properly.

To build the scenario tree, (1) is re-written as:

$$x^j_{k+1} = f(x^{p(j)}_k, u^j_k, d^{r(j)}_k), \tag{2}$$

where in x^j_k the lower index k represent the depth-stamp while the upper index j enumerates the different values that the variable can take at that depth-stamp (see Fig. 5). $p(j)$ represent the index of the parent state. For example in Fig. 5 the parent state of x^1_2 is x^1_1, this means that $p(j) = 1$. Same idea applies to $r(j)$.

Each state x^j_{k+1} depends on the parent state $x^{p(j)}_k$ on the control input u^j_k and on the uncertainty realization $d^{r(j)}_k \in [d^1_k, d^2_k, ..., d^s_k]$. Each time the scenario tree branches into other scenarios, more optimization variable are added to the problem. To maintain the problem solvable online, Lucia et al. (2013) introduced the concept of *robust horizon*. The branching is only allowed within the robust horizon. After that, no further branching is considered. This idea is based on the assumption that far away uncertainties will have little influence on the decisions the controller takes at the current time.

The scenario based MPC can be formulated as

$$\min_{u_k^j,\forall(j,k)\in I} \sum_{i=1}^{N_s} \omega_i, J_i \tag{3a}$$

subject to (3b)

$$x_{k+1}^j = f(x_k^{p(j)}, u_k^j, d_k^{r(j)}), \quad \forall(j,k) \in I, \tag{3c}$$

$$x_k^j \in \mathbb{X}, \quad u_k^j \in \mathbb{U} \quad \forall(j,k) \in I, \tag{3d}$$

$$u_k^j = u_k^l \quad \text{if} \quad x_k^{p(j)} = x_k^{p(l)} \quad \forall(j,k),(l,k) \in I, \tag{3e}$$

where J_i is the cost of each scenario, N_s is the number of scenarios, I is the set of indices (j,k) and ω_i are weights that can represent, for example, the probability of the $i-th$ scenario. In our case, we assume that all scenarios are equally likely, hence $\omega_i = 1/(N_s)$. Equations (3c),(3d) and (3e) are optimization constraints. The model equation must always be respected hence we impose (3c). Furthermore, additional constraints can be added with (3d), e.g. maximum distance to plan. Finally, (3e) represent the non-anticipativity constraints, these constraints make sure that inputs and states at each node are equal for each branch generated from that node. In other words, it avoids that control actions anticipate the future.

Objective function The cost of each scenario is defined as

$$J_i = \sum_{k=0}^{N_p-1} L_p(x_{k+1}^i) + \sum_{k=0}^{N_c-1} L_c(u_k^i) + V_{\mathrm{f}}(x_{N_p}), \tag{4}$$

$$\forall x_{k+1}^j, u_k^j \in S_i, \tag{5}$$

where S_i is the $i-th$ scenario, L_p, L_c and V_f are weight functions, N_p is the prediction horizon and N_c is the control horizon.

Equation (4) is written as

$$J_i = \sum_{k=0}^{N_p-1} \left(\|\alpha(x_k) - \hat{\alpha}_k\|_Q^2 + \|DLS_k\|^2\right) + \tag{6}$$

$$+ \sum_{k=0}^{N_c-1} \left(\|u_{k+1} - u_k\|_R^2\right) + \|\alpha(x_{N_p}) - \hat{\alpha}_{N_p}\|_E^2 + \|\epsilon\|_P^2.$$

where $\alpha(x_k)$ is a function that maps the current model states to position, inclination and azimuth of the bit and $\hat{\alpha}_k$ is the vector of reference values of the well plan at length k. DLS is the dog-leg severity and ϵ is the slack variable of the soft constraints (*c.f.* Section 4.1).

Constraints Three sets of nonlinear inequality constraints are considered: DLS, maximum input and maximum distance from plan. The DLS is defined as:

$$DLS = \frac{2}{\Delta s} \operatorname{asin}\left(\sqrt{\left(\sin(\frac{\Delta azi}{2})\right)^2 + \left(\sin(\frac{\Delta inc}{2})\right)^2 \beta}\right) \tag{7a}$$

$$\beta = \sin(inc_k)\sin(inc_{k+1}), \tag{7b}$$

where inc and azi are respectively inclination and azimuth at the bit, s is the drilled length, while $\Delta(\cdot) \triangleq (\cdot)_{k+1} - (\cdot)_k$. The normalized input needs to satisfy:

$$|u|_2 \leq 1. \tag{8}$$

Finally, maximum distance from plan is implemented as *soft constraint*:

$$c = \sqrt{(\pi(x) - \hat{\pi})^2} \leq r_{max} + \epsilon \tag{9a}$$

$$0 \leq \epsilon \leq \epsilon_{max} \tag{9b}$$

where $\pi(x)$ is the vector containing north, east and down coordinates of the bit (function of the model states) and similarly $\hat{\pi}$ is the vector of reference coordinates, defined as the closest point from the bit on the wellplan. r_{max} is the maximum distance and ϵ is a slack variable that can take any value between 0 and ϵ_{max}. Note from (6) and (9), that as long as the distance from plan is less than r_{max}, ϵ will be chosen as zero, and no extra cost will be added to the objective function. Nevertheless, the MPC could decide to violate r_{max} by choosing $0 < \epsilon \leq \epsilon_{max}$ and consequently paying an extra cost. The distance from plan is implemented as soft constraint to avoid running into infeasibility while solving the multi-stage MPC problem. Notice that the multi-stage approach produces multiple predictions, therefore the likelihood that one of these might violate the corridor constraints increases.

4.2 Triggering condition

The maximum distance to plan r_{max} defines a virtual corridor around the wellplan. If at least one of the scenarios predicts that the bit will lie outside the corridor within a certain prediction horizon, a new MPC iteration is triggered and new control parameters are downlinked (Fig. 4). If for all scenarios the bit is predicted to be inside the corridor, no new MPC iteration is necessary, and the old control parameters are used. Following the notation of Brunner et al. (2015) we define the triggering condition as

$$u_k = \kappa(x_{k_i}, k - k_i), \quad k \in \mathbb{N}_{[k_i, k_{i+1}-1]} \tag{10}$$

$$k_{i+1} = \inf\{k \in \mathbb{N}_{\geq k_i+1} | x_{l|k}^j \notin \mathcal{E}(\hat{x}_{k_i}, k - k_i), \tag{11}$$

$$\text{any} \quad l \in \mathbb{N}_{[k+1,N_c]}\} \tag{12}$$

where the set $\mathcal{E}$ is defined as

$$\mathcal{E}(\hat{x}_{k-k_i|k_i},) = \hat{x}_{k-k_i|k_i} \oplus \mathcal{R} \tag{13}$$

where

$$\mathcal{R} = \{x_k \in \mathbb{X}| \quad x_k - \hat{x}_k \perp \frac{d\hat{x}_k}{ds}, |x_k - \hat{x}_k|_2 \leq r_{max}\}. \tag{14}$$

Note that $d\hat{x}_k/ds$ represents the tangent to the wellplan. This means that a new MPC will be triggered when any of the predictions will be outside the set describing a corridor of radius r_{max} around the reference trajectory.

4.3 Recursive feasibility

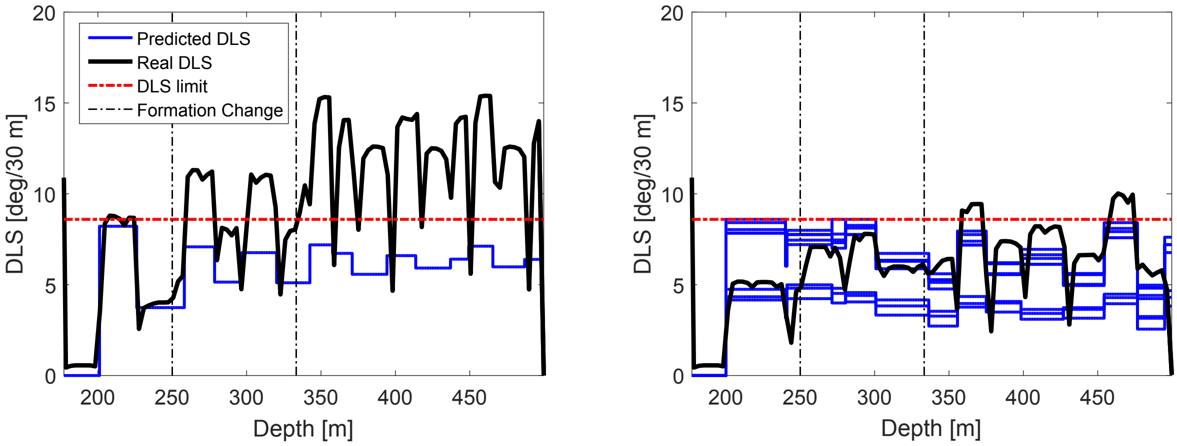
We prove recursive feasibility of MPC by using a common terminal control invariant region (Maiworm et al., 2015) adapted for the regulation problem. We assume that the following assumptions are true:

Assumption 1. The function $f(x,u,d)$, $L_p(x)$, $L_c(u)$ and $V_{\mathrm{f}}(x)$ are continuous and $f(x(s),u(s),d) = x(s), \quad \forall d \in \mathcal{D}$, $L_p(x(s)) = 0$, $L_c(u(s)) = 0$ and $V_{\mathrm{f}}(x(s)) = 0$ where $x(s)$ is a space-varying trajectory.

Assumption 2. The set $\mathbb{X}$ is closed, $\mathbb{U}$ is compact and $\hat{x}_k \in \mathbb{X} \quad \forall k \in [0,..,N_c]$.

Note that the model (2) can be interpreted as a set of models that differ from each other due to different parameter realizations:

$$x_{k+1}^j = f(x_k^{p(j)}, u_k^i, d_k^{r(j)}) \to x_{k+1} = f_j(x_k, u_k). \tag{15}$$



(a) DLS for the nominal MPC.

(b) DLS for multi-stage MPC. Note that every scenario predicts a different DLS.

Fig. 6. DLS comparison between nominal and multistage MPC. The DLS limit is imposed as hard constraint in (3d).

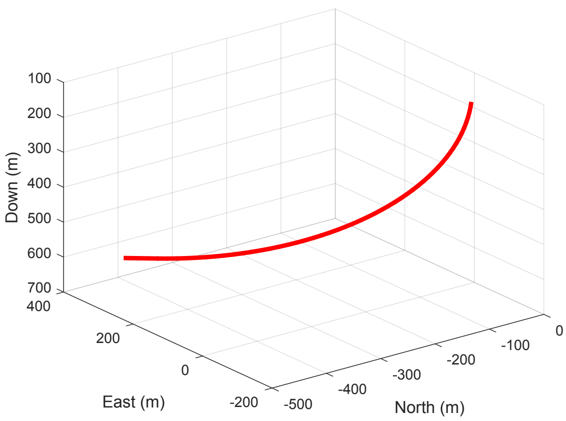


Fig. 7. Wellplan used for testing.

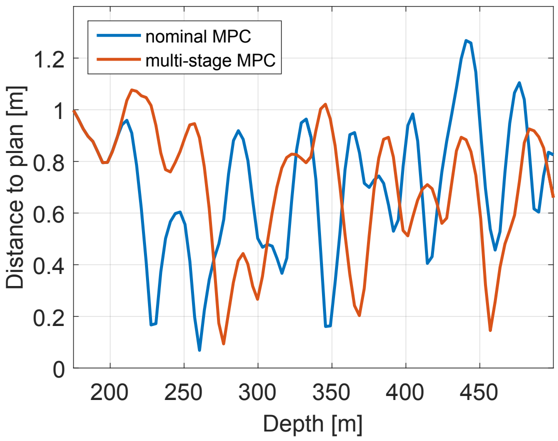


Fig. 8. Distance to plan comparison between nominal and multistage MPC.

Assumption 3. Suppose that each model

$$x_{k+1} = f_j(x_k, u_k) \tag{16}$$

has the same terminal control invariant region Ω_f for all $j = [1, ..., s]$ where s is the number of scenarios and Ω_f is defined as

$$\Omega_f = \{x \in \mathbb{X}, \exists u \in \mathbb{U} : f_j(x, u) \in \Omega_f \quad \forall j \in [1, ..., s]\} \tag{17}$$

Assumptions 1, 2 and 3 are enough to guarantee recursive feasibility of the multistage MPC if the a new MPC problem is solved at each sampling time. To extend the results to an event-based MPC we need a further assumption. Firstly, it is convenient to linearize the models around the current state

$$x_{k+1} = A_j x + B_j u. \tag{18}$$

Now, let Ω_{N_c} be the set of all initial conditions for which a solution of the optimal control problem exists. Furthermore assume $\Omega_0 := \Omega_f$ and let

$$\Omega_1 = \{x \in \mathbb{X}, \exists u \in \mathbb{U} : f_j(x, u) \in \Omega_0 \quad \forall j \in [1, ..., s]\} \tag{19}$$

Ω_1 is the set of all states which can be driven to Ω_f with an admissible control.

Assumption 4. We assume that

$$A_j \mathcal{R} \subseteq \Omega_{k+1} \quad \forall k \in \mathbb{N}_{\geq 0}, \forall j \in [1, ..., s]. \tag{20}$$

This ensures that if no triggering event occurs at a given point $k + i$ the system state x_{k+i} is still in set Ω_{k+1}. Since Ω_f is control invariant by Assumption 3, given (19) then Ω_1 is also control invariant. Thanks to Assumption 4 by recursion also Ω_{N_c} is control invariant, even though a new triggering event does not occur.

Proposition 1. If Assumptions 1-4 are true, then the event-triggered multi-stage MPC is recursively feasible. The proof follows from the discussion above.

5. RESULTS

The proposed approach is tested in simulation in a Simulink/Matlab environment and compared with a *nominal* event-triggered MPC, i.e. where no parametric uncertainty is considered. The software CasADi (Andersson et al., 2019) was used for the implementation. The simulation parameters are chosen as follows: the prediction horizon is $N_p = 30$, with depth-step of $1m$ resulting on a prediction length of 30m. Since we want to minimize the number of downlinks and given the limited bandwidth it is not possible to send the whole control sequence, the control horizon N_c is chosen as 1. The measurements are available at intervals ranging from 30 seconds to 5 minutes. The corridor radius is chosen as $r_{max} = 3$m. For the construction of the scenario we consider two possible realization of the parameter vector d: d_{min} and d_{max}. To keep computational time low we use a robust horizon of 1. The well plan used is shown in Fig. 7. To test the robustness of the controller, we simulate two changes in formation that cause step changes in the real system parameters. Until the second formation change, the real parameters are within the values considered by the multi-stage MPC i.e. $d_{min} < d_{real} < d_{max}$. In the last change, the real parameters are outside the values considered in the tree. This is to simulate the scenario when the real uncertainty is larger than the one considered in the scenarios. In Fig. 6 the predicted and actual DLS for nominal event-triggered MPC and multi-stage event-triggered MPC are shown. The vertical line represent the formation changes, while the red line is the maximum DLS. The parameter set used in nominal event-triggered MPC is set as $(d_{max} - d_{min})/2$. Due to the parametric uncertainty the nominal MPC does not respect the DLS constraints, and it reaches almost 100% violation after the second formation change. The proposed approach is more robust to parametric uncertainty. In the first and second formation, the DLS constraints are always respected. In the last formation change, despite the fact that the error on the parameters is outside the range considered by the scenario the constraints are only slightly violated. The distance to plan for the two MPC approaches is shown in Fig. 8. For the case considered both methods show similar performances. The multi-stage MPC was triggered, on average, every 23.2m while the nominal MPC every 21.7m. The triggering interval increased because the multi-stage MPC is better at avoiding violation the corridor constraints and consequently it will trigger less MPC calculations.

6. CONCLUSIONS

Automation of trajectory drilling presents a large number of challenges e.g. due to the uncertain environment and low band-width communication. This papers tackled the problem of downlink minimization and robust constraints satisfaction. We used an event-triggered multi-stage MPC to reduce the number of control actions and increase robustness to parametric uncertainty. Conditions for recursive feasibility were given. The method was tested in simulation and it showed to increase robustness to constraint violation also for the case where uncertainties were underestimated. In this work we assumed that all the states of the system were measurable and noise free. In future work, a state estimator will be added to consider the case when measurements are noisy or not available. Furthermore, to have more insight on the uncertainties effecting the model, real data from drilling will be used together with machine learning approaches.

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