Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

A variable stochastic admittance control framework with energy tank

Francesco Cordoni [*], **Luca Di Persio** [*], **Riccardo Muradore** [*]

[*] *University of Verona - Department of Computer Science*
Strada le Grazie, 15 Verona, 37134-ITALY
e-mail: {francescogiuseppe.cordoni, luca.dipersio, riccardo.muradore}@univr.it

Abstract: In this paper we address the problem of implementing a stochastic variable admittance control. Both the variable part of the admittance control and the noise affecting the system may concur to the instability of the system. We propose an energy tank approach, based on the theory of stochastic port–Hamiltonian systems and weak passivity, where the energy dissipated by the stochastic system, if any, is stored into the tank to implement the desired actions. As we consider a non–vanishing noise, we need to consider weaker notion of passivity and convergence. We will show how the notion of weak passivity can be properly defined so that equipping a stochastic system with a suitable energy tank, variable admittance control can be efficiently implemented. We prove that the overall system is weakly passive and it converges toward an invariant measure. Simulation results show the effectiveness of the derived theoretical framework.

Keywords: Variable admittance control, Stochastic port–Hamiltonian systems, Passivity, Ultimately stochastic passive

1. INTRODUCTION

Last decades have reported an increasing attention to Human-Robot Interaction (HRI). It is in fact clear how robots can improve as well as facilitate several human tasks. While human can easily adapt to the surrounding environment, the same cannot be said for robots. Therefore an efficient way of controlling the interaction and the response of the robots to interaction forces needs to be implemented.

One of the most common control scheme in HRI is the impedance/admittance control, Hogan (1985); Calanca et al. (2015, 2017), where for instance the dynamical behavior of the robot is adapted by modifying the parameters of stiffness, damping and inertia. Several works have been published in last years, proposing efficient control schemes, Buchli et al. (2011); Tsumugiwa et al. (2002). One of the major issue in developing such strategies, it to consider the effect that the interaction with an unknown environment (like humans) has on the stability of the overall system.

Within this framework, the *Passivity theory* approach has been proven to be particularly suited to address stability problem of robotic systems interacting with unknown environments, see, e.g., Hatanaka et al. (2015). In fact it is not possible to describe mathematically the environment in a general formulation, so that the interaction of the robot with an unknown environment is quite a delicate point to address. Typically the passivity assumption holds for the environment and the so-called passivity-based controller (PBC) can be derived to ensure the stability of the overall system considered as time-invariant dynamical model.

However, in HRI it is often required to change at runtime the control law to render different stiffness: such time-dependent controller can lead to non–passive system, with consequent instability. Recently, a new approach has been introduced, see, e.g., Ferraguti et al. (2015, 2013); Franchi et al. (2012); Franken et al. (2011), to derive less conservatives control strategies for the time-varying scenarios. The main idea consists in equipping the system with energy tanks where the energy dissipated by the system is stored. This stored energy can be used to implement in a passive way actions that would have led to a destruction of the passivity of the system, e.g. Ferraguti et al. (2015, 2013).

The mentioned results have been established considering deterministic systems. Since any system is affected by measurement noise at the sensing system and the uncertainty on the mathematical model can be often modelled as a stochastic input, it turns out that the previous results should be extended to explicitly take into account the randomness. The noise can be also used in the HRI scenario for modelling the behaviour of the unknown environment. For stochastic systems with energy tank, the noise adds a further effect on the energy stored into the tank, so that the control scheme has to be carefully implemented.

The passivity theory for stochastic systems is quite delicate. In particular an assumption to guarantee (asymptotic) stability of a stochastic system is that the noise must vanish at the equilibrium, see Florchinger (1999). The generalization of such result for additive noise is still missing.

The aim of this work is to present an energy tank approach to efficiently implement a variable admittance control in presence of noise. We will address the case of an additive noise, which we stress to be more complicated than, for

[⋆] This work was partly supported by the project MIUR "Department of Excellence" 2018-2022.

instance, a linear multiplicative noise. This is due to the fact that the noise will not vanish at the equilibrium, so novel suitable (*weaker*) notions of convergence and passivity must be taken into account. To the best of our knowledge the only attempt in literature to model a stochastic system with additive noise and energy tank is Cordoni et al. (2019a), where a stochastic energy tank approach has been used to establish *ultimately stochastic passivity* to ensure convergence of a master–salve system towards the limiting invariant distribution under a delayed PD-like control scheme. In order to endow the stochastic system with a proper energy tank, we will exploit the theory of port–Hamiltonian systems, see, e.g. van der Schaft et al. (2014), recently extended to the stochastic case in Cordoni et al. (2019b).

The paper is structured as follows: in Section 2 we introduce the general setting of stochastic differential equations with additive noise equipped with energy tank. Section 3 is devoted to passivity and stability, in a suitable sense, of a robotic system with variable admittance control. In Section 4 we present some numerical results validating previous theoretical results.

2. ENERGY TANK FOR STOCHASTIC PORT–HAMILTONIAN WITH ADDITIVE NOISE

From a purely stochastic perspective the additive noise case is simpler than the multiplicative case, whereas from a stability point of view the constant diffusion complicates the analysis. In fact, the noise does not vanish at the desired equilibrium, causing the process not to converge in any standard probabilistic sense. This affects also passivity since passivity is usually used to stabilize and control nonlinear (stochastic) systems, see Florchinger (1999).

To overcome the problem, in Fang and Gao (2016) a weaker notion of passivity has been proposed. Broadly speaking, the passivity of the system is not defined on the whole space but only outside a ball centred at a specified state. This definition has several desirable implication regarding limiting distribution of the system, implying what is called asymptotic weak stability. One of the main property of the considered weak notion of passivity is its close link with the invariant measure of system. It turns out that this weak notion of passivity is tailor-made to deal with stochastic equation with additive noise.

We will focus on the stochastic PHS perturbed by a general additive Brownian noise

$$\begin{cases} dX(t) &= \left(\left[J(X(t)) - R(X(t)) \right] \partial_x H(X(t)) \right) dt + \\ & \quad + g(X(t)) u dt + \nu dW(t) \\ y(t) &= g^T(X(t)) \partial_x H(X(t)) \,, \end{cases} \tag{1}$$

where $X \in \mathbb{R}^n$, $J = -J^T$ and $R > 0$ are two given matrices of suitable dimensions; $H : \mathbb{R}^n \to \mathbb{R}$ is the *Hamiltonian function*, representing the total energy of the system; $u \in \mathcal{U}$ represents the signal for the control input, while $y \in \mathcal{Y} := \mathcal{U}^*$ is the output. We have denoted by $\mathcal{U}^*$ the dual space of $\mathcal{U}$ whereas ∂_x is the gradient operator with respect to x. The constant $n \times d$ matrix ν multiplies the d-dimensional Brownian motion $W(t)$ adapted to a reference filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P}\right)$ satisfying usual assumptions, namely right–continuity and saturation by $\mathbb{P}$–null sets.

We address in the present work the case of an additive noise; to develop a robust setting we have chosen to consider the notion of weak passivity introduced in Fang and Gao (2016). This is due to the fact that, this notion of passivity, is particularly suited to consider asymptotic behaviour of the system and its limiting stable distribution, with particular attention to ergodicity and to the associated invariant measure. We recall that an *invariant measure* ρ for the stochastic system (1) is the measure for which it holds

$$\int_{\mathbb{R}^n} \mathbb{P}\left(X^x(t) \in A \right) \rho(dx) = \rho(A) \,, \, \forall \, A \in \mathcal{B}(\mathbb{R}^n) \,,$$

for every $t \in [0, \infty)$, see, Khasminskii (2011). Above, we have denoted by X^x the solution with initial value x; we remark that in the present section, as standard in stochastic analysis, X denotes the stochastic process whereas x is its initial value. Standard stability notion for nonlinear stochastic differential equation requires that the noise vanishes at the desired equilibrium, see Florchinger (1999). To drop the vanishing noise condition several weak notions of stability have been considered in literature. In Satoh and Saeki (2015) a notion of bounded stability in probability have been used. Another approach is to directly study the convergence in distribution, looking directly at the *Fokker–Planck equation* describing the evolution of the probability measure, see Khasminskii (2011); Liberzon and Brockett (2000); Zhu (2006). As mentioned we have chosen in the present work to consider the weak passivity notion given in Fang and Gao (2016), which appears to be strictly related to the recurrence of the underlying system and in turn to the limiting invariant measure. As done also in Cordoni et al. (2019a), we decided to chose a different name rather than *weak passivity* just to highlight the connection between the notion of *weak stability* and the already known notion of *ultimately bounded stability*. We will show how a stochastic system can be endowed with an energy tank such that the overall (ultimately) passivity is preserved and so the ultimately stochastic stability guaranteed.

We next introduce the definition of *weak passivity*, that we will call *ultimately stochastic passivity*, for a stochastic system, see, e.g. (Fang and Gao, 2016, Def. 4.3).

Definition 1. [Ultimately stochastic passivity] The non–linear controlled stochastic system of the form

$$\begin{cases} dX(t) = \mu(X(t), u(t)) dt + \nu(X(t)) dW(t) \,, \\ y(t) = h(X(t)) \,, \end{cases} \tag{2}$$

is said to be *ultimately stochastic passive* with respect to a storage function $V(t, x)$ if for any (t, x) such that, fixed x_C such that $\|x - x_C\| \geq C$, for a given constant $C > 0$ called passivity radius, it holds

$$\mathcal{L}V(t, x) \leq h^T(x) u \,,$$

where $\mathcal{L}$ is the infinitesimal generator

$$\mathcal{L}V(t, x) = \partial_t V(t, x) + \partial_x V(t, x) \mu + \frac{1}{2} Tr \left[\partial^2_{xx} V(t, x) \nu \nu^T \right] \,.$$

If further it exists $\delta_C > 0$ such that for $\|x - x_C\| \geq C$ it holds

$$\mathcal{L}V(t, x) \leq h^T(x) u - \delta_C \|x - x_C\|^2 \,,$$

then system (2) is said to be *strictly ultimately stochastic passive*.

For the sake of simplicity, we will consider H to be a quadratic function of the state, i.e. $H(X(t)) = \frac{1}{2}X^T(t)\Lambda X(t)$, with Λ a positive definite symmetric constant matrix of suitable dimensions. Then, the following holds

Proposition 2. The stochastic PHS (1) with quadratic Hamiltonian function is *strictly ultimately stochastic passive.*

Proof. The infinitesimal generator of the Itô process SPHS (1) $\mathcal{L}$ is

$$\begin{aligned}\mathcal{L}H(x) &= \partial_x^T H(x)\left(\left[J(x) - R(x)\right]\partial_x H(x) + g(x)u\right) + \\ &\quad + \frac{1}{2}Tr\left[\partial_x^2 H(x)\nu\nu^T\right] \\ &= -\partial_x^T H(x)R(x)\partial_x H(x) + \partial_x^T H(x)g(x)u + \\ &\quad + \frac{1}{2}Tr\left[\partial_x^2 H(x)\nu\nu^T\right].\end{aligned}$$

From the quadratic form of the Hamiltonian function the first and the third terms can be rewritten as

$$\begin{cases}\partial_x^T H(x)R(x)\partial_x H(x) & = x^T\Lambda R(x)\Lambda x\,, \\ \frac{1}{2}Tr\left[\partial_x^2 H(x)\nu\nu^T\right] & = Tr\left[\frac{\nu\nu^2}{2}\Lambda\right].\end{cases}$$

Hence there exist positive constants $C > 0$ and $\delta_C > 0$ such that, for $\|x\| \geq C$ it holds

$$x^T\Lambda R(x)\Lambda x - Tr\left[\frac{\nu\nu^2}{2}\Lambda\right] \geq \delta_C\|x\|^2\,,$$

and, for $\|x\| \geq C$, we have

$$\mathcal{L}H(x) \leq \partial_x^T H(x)g(x)u = y^T(t)u(t) - \delta_C\|x\|^2\,,$$

which is the definition of *strictly ultimately stochastic passivity* (1). □

Remark 3. The notion of ultimately stochastic passivity can be thought as follows: for X converging to $X^* = 0$ we have that the system is not passive, as the noise keeps injecting energy into the system preventing the system from stabilizing at $X^* = 0$. Nonetheless, the system cannot exhibits non stationary behaviours since, as soon as the process exits a suitable ball of radius C, the system becomes passive and the energy injected by the noise into the system is strictly less then the one dissipated, so that the system recovers stability. It follows that, at large time, the system will be forced to stay in a fixed domain and keeps moving around $X^* = 0$.

Proposition 2 already highlights one of the main advantages of the new adopted notion of *ultimately stochastic passivity.* It is in fact known that, due to the energy injected by the noise into the system, a stochastic PHS may not be passive under standard assumptions used in the deterministic setting, see, Cordoni et al. (2019a); Satoh and Fujimoto (2012). Using the proposed weaker notion of passivity on the contrary, the SPHS (1) is *ultimately stochastic passive* with no further assumptions on the diffusion term.

Due to the fact that the SPHS (1) is ultimately stochastic passive, then we can endow the system with a (virtual) energy tank. Energy tank, see, Ferraguti et al. (2015, 2013); Franchi et al. (2012); Franken et al. (2011) for the deterministic case or Cordoni et al. (2019a) for the stochastic case, represents an efficient way to modulate "non–passive" controls to preserve the passivity constraint. If the level of the energy within the tank allows to implement the command, there is no modulation, otherwise the commands is decreased in order to "use" only the available energy.

The power dissipated by the stochastic system (1), if positive, is

$$\begin{aligned}&\left(\partial_x^T H(x)R(x)\partial_x H(x) - \frac{1}{2}Tr\left[\partial_x^2 H(x)\nu\nu^T\right]\right)^+ \\ &=: \left(D(x) + D_S(x)\right)^+ =: \mathcal{D}^+(x) > 0\,,\end{aligned} \tag{3}$$

where $(f)^+ := \max\{f, 0\}$ is the positive part of the function f.

The SPHS endowed with the *energy tank* is given by

$$\begin{cases}dX(t) & = \left(\left[J(X(t)) - R(X(t))\right]\partial_x H(X(t))\right)dt + \\ & \qquad + g(X(t))udt + \nu dW(t)\,, \\ dx_E(t) & = \frac{\sigma}{x_E(t)}\mathcal{D}^+(x)\,dt + \\ & \qquad + \left(\frac{1}{x_E(t)}\left(\sigma P^{in}(t) - P^{out}(t)\right) + u_E\right)dt\,, \\ y_1(t) & = \left(y(t), y_E(t)\right)^T\,,\end{cases} \tag{4}$$

where x_E is the state associated with the stored energy within the tank. The energy level at time t is given by $E(x_E(t)) = \frac{1}{2}x_E^2(t)$, whereas P^{in} and P^{out} denote the incoming and outcoming power flows that the tank can exchange with other tanks. The power port (u_E, y_E) is used to exchange power with the extarnal world and it holds

$$y_E = \frac{dE(x_E(t))}{dt} = x_E(T)\,.$$

We can thus obtain

$$\dot{E} = x_E(t)\dot{x}_E(t) = \sigma\mathcal{D}^+(x) + \left(\sigma P_\alpha^{in}(t) - P_\alpha^{out}(t)\right) + u_E y_E\,.$$

As explained in Ferraguti et al. (2015), the parameter $\sigma \in \{0, 1\}$ is used to enable or disable the dissipated energy storage in case the energy stored in the tank reaches the upper bound set for safety reason. Also, the tank has to be initialized so that $E(x_E(0)) > T_{min}$ and energy extraction is prohibited if $E(x_E(t)) < T_{min}$. The energy stored into the tank is used to implement the passive–preserving u_E as close as possible to the desired input u.

3. VARIABLE STOCHASTIC ADMITTANCE CONTROL WITH ADDITIVE NOISE

Admittance control and impedance control, Villani and De Schutter (2008), are well–known and highly used control schemes for implementing desired behaviour in human-robot interaction scenarios. Given an interaction model, that is a dynamic relation between the applied force and the pose error, the corresponding pose of the robot is tracked so that the tracking error is minimized. A typical admittance scheme is reported in Figure 1.

We consider the non–linear stochastic equation

$$\Lambda(x)\ddot{x} + \mu(x, \dot{x})\dot{x} + F_g(t)x = F_{ext} + F_\tau + \nu\dot{W}(t)\,, \tag{5}$$

where x are the Cartesian coordinates of the end–effector, F_{ext} is the vector of external forces, F_τ is the wrench due to the controlled torques and F_g is the wrench due to gravity. Also $\Lambda = \Lambda^T > 0$ and μ are the $n-$dimensional matrices representing respectively inertia and centrifugal and Coriolis forces. At last $\dot{W}$ if the formal time derivative of a $d-$dimensional Brownian motion and ν is a $n \times d$ constant matrix.

Let $x_d(t)$ be a desired configuration; the goal of the admittance control is to make the robot behaves as a linear mass-spring-damping system

$$\Lambda_d \ddot{\tilde{x}} + D_d \dot{\tilde{x}} + K_d \tilde{x} = F_{ext} + \nu \dot{W}(t)\,, \tag{6}$$

with $\tilde{x}(t) := x(t) - x_d(t)$ the pose error. The constant matrices Λ_d, D_d and K_d are the $n-$dimensional symmetric and positive definite desired inertia, damping and stiffness matrices, respectively.

3.1 Problem formulation

The main goal of the present work is to generalize equation (6) to consider variable admittance control allowing higher flexibility to the interactive robot. More formally, denoting by $\Lambda_d(t)$, $D_d(t)$ and $K_d(t)$ some $n-$dimensional time–varying symmetric and positive definite inertia, damping and stiffness matrices, respectively, the desired interaction model takes the form

$$\Lambda_d(t) \ddot{\tilde{x}} + D_d(t) \dot{\tilde{x}} + K_d(t) \tilde{x} = F_{ext} + \nu \dot{W}(t)\,. \tag{7}$$

As mentioned in the introduction, the main drawback in introducing a variable interaction model is that time-dependence affects the overall passivity of the system. It is worth stressing that in the present setting both time–varying coefficients and noise concur to destroy the passivity of the system, so that in developing the energy tank we need to take into account both effects.
According to Ferraguti et al. (2015), we assume that above matrices separate into constant and variable positive–definite terms as

$$\begin{cases} \Lambda_d(t) = \Lambda_c + \Lambda_v(t)\,, \\ K_d(t) = K_c + K_v(t)\,, \end{cases}$$

so that equation (7) can be rewritten as

$$\begin{cases} \begin{pmatrix} dq(t) \\ dp(t) \end{pmatrix} &= \left(\begin{pmatrix} 0 & I \\ -I & -D_d(t) \end{pmatrix} \begin{pmatrix} \partial_q H_c \\ \partial_p H_c \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} F_{ext} \right) dt \\ & \quad - \begin{pmatrix} 0 \\ I \end{pmatrix} \left(K_v(t) q + \Lambda_v(t) \ddot{q} \right) dt + \begin{pmatrix} 0 \\ \nu \end{pmatrix} dW(t)\,, \\ y &= \Lambda_c^{-1} p\,, \end{cases} \tag{8}$$

with total energy given by

$$H_c(q,p) := \frac{1}{2} q^T K_c q + \frac{1}{2} p^T \Lambda_c^{-1} p\,,$$

and p, q denoting momentum and position.
Due to the time-dependence of the inertia and of the stiffness matrices, together with the stochastic noise, energy is injected into the stochastic port-Hamiltonian dynamics; these additions may therefore destroy the passivity of the overall system, which holds true for the time–independent and deterministic case.

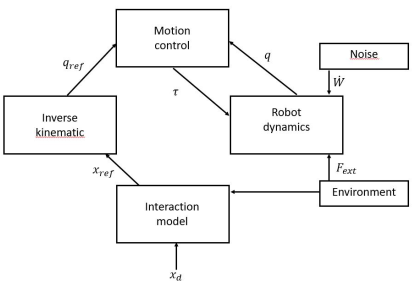

Fig. 1. Typical admittance scheme with motion controller and external noise

We therefore augment the system with an energy tank alike equation (4), considering thus the SPHS of the form

$$\begin{cases} \begin{pmatrix} dq(t) \\ dp(t) \end{pmatrix} &= \left(\begin{pmatrix} 0 & I \\ -I & -D_d(t) \end{pmatrix} \begin{pmatrix} \partial_q H_c \\ \partial_p H_c \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} F_{ext} \right) dt + \\ & \quad + \begin{pmatrix} 0 \\ I \end{pmatrix} w dt + \begin{pmatrix} 0 \\ \nu \end{pmatrix} dW(t)\,, \\ dx_E(t) &= \left(\frac{\sigma}{x_E(t)} \left(p^T \Lambda_c^{-1} D_d(t) \Lambda_c^{-1} p - \frac{1}{2} Tr\left[\Lambda_c^{-1} \nu \nu^T \right] \right)^+ \right) dt \\ & \quad - \frac{w^T}{x_E(t)} \Lambda_c^{-1} p dt\,, \\ y &= \Lambda_c^{-1} p\,, \end{cases} \tag{9}$$

with the energy function of the tank equal to

$$E(x_E(t)) = \frac{1}{2} x_E^2(t)\,.$$

Since we must guarantee at any time a minimum level of energy stored in the tank, we set

$$\omega := \begin{cases} -\left(K_v(t) q + \Lambda_v(t) \ddot{q} \right)\,, & E(x_E(t)) > T_{min}\,, \\ 0, & \text{otherwise}\,. \end{cases}$$

We remark that in equation (9), differently from the deterministic case, we cannot expect the system to dissipate energy, i.e. we cannot always store energy into the tank. This fact explains the specific form for the equation of the energy tank in equation (9). If p is close enough to 0, then the term

$$p^T \Lambda_c^{-1} D_d(t) \Lambda_c^{-1} p - \frac{1}{2} Tr\left[\Lambda_c^{-1} \nu \nu^T \right]\,,$$

becomes negative, meaning that the system is no longer dissipating energy and we cannot store energy. Therefore, as done with the parameter σ, we stop energy storing into the tank.
The energy stored in the tank is exploited for implementing the command input due to the variability of the stiffness and inertia parameters. If there is some energy stored in the tank, the desired interaction model can be implemented, otherwise, the variable parts of the admittance parameters are not implemented. In this sense priority is always given to preserve the passivity of the system, so that in case no variable control can be implemented exploiting available energy in the tank, then a constant behaviour is implemented. This computation is done by endowing the SPHS with the mathematical model of the energy tank through the power preserving interconnection

$$\begin{cases} u &= \frac{w}{x_E} y_E = \frac{w}{x_E} x_E = \omega\,, \\ u_E &= -\frac{w^T}{x_E} y\,. \end{cases}$$

We refer the interested reader to Franchi et al. (2012); Franken et al. (2011) for a more detailed treatment of the topic, and to Cordoni et al. (2019b) for the formal proof that the system connection is a SPHS as well.

Proposition 4. The SPHS with energy tank (9) is *strictly ultimately stochastic passive* with respect to the energetic port (p, F_{ext}).

Proof. The total energy of the system is

$$\begin{aligned} H_T(q,p,x_E) &= H_c(q,p) + E(x_E) = \\ &= \frac{1}{2} q^T K_c q + \frac{1}{2} p^T \Lambda_c^{-1} p + \frac{1}{2} x_E^2(t)\,. \end{aligned}$$

We therefore have

$$\mathcal{L}H_T = -p^T\Lambda_c^{-1}K_v(t)\Lambda_c(t)p + \frac{1}{2}Tr\left[\Lambda_c^{-1}\nu\nu^T\right] + p\Lambda_c^{-1}F_{ext} + \\ + \left(\sigma p^T\Lambda_c^{-1}K_v(t)\Lambda_c^{-1}p - \frac{1}{2}Tr\left[\Lambda_c^{-1}\nu\nu^T\right]\right)^+ . \tag{10}$$

If $\|p\| \geq C$, for a suitable positive constant $C > 0$, we have that it exists $\epsilon > 0$ such that

$$\sigma p^T\Lambda_c^{-1}K_v(t)\Lambda_c^{-1}p > \frac{1}{2}Tr\left[\Lambda_c^{-1}\nu\nu^T\right] > \epsilon > 0,$$

so that we obtain from equation (10)

$$\mathcal{L}H_T = -p^T\Lambda_c^{-1}K_v(t)\Lambda_c(t)p + \frac{1}{2}Tr\left[\Lambda_c^{-1}\nu\nu^T\right] + p^T\Lambda_c^{-1}F_{ext} + \\ + \sigma p^T\Lambda_c^{-1}K_v(t)\Lambda_c^{-1}p - \frac{1}{2}Tr\left[\Lambda_c^{-1}\nu\nu^T\right] = \\ = -p^T\Lambda_c^{-1}\left(K_v(t) - \sigma K_v(t)\right)\Lambda_c(t)p + p^T\Lambda_c^{-1}F_{ext} . \tag{11}$$

Since $K_v(t) > 0$ and $(K_v(t) - \sigma K_v(t)) \geq 0$ we get that, for $\|p\| \geq C$, the following inequality holds

$$\mathcal{L}H_T \leq p^T\Lambda_c^{-1}F_{ext} - \delta_C\|p\|^2 ,$$

which prove the system (9) is *strictly ultimately stochastic passive* at the port (p, F_{ext}). □

In the deterministic context, passivity of a given system ensures the stability in free motion, i.e. when $F_{ext} = 0$. As mentioned we cannot expect in the present situation stability around a given state as the noise keeps moving the system away from the stable point. Nonetheless, above introduced weak notion of passivity ensures that the system converges, as $t \to \infty$ to a system whose probability law is given by the invariant measure of the original system.

Proposition 5. There exists a unique invariant measure (ρ^q, ρ^p, ρ^E) for the SPHS with energy tank (9) in free-motion. In particular it holds that, for any Borel set A

$$\lim_{t\to\infty} \mathbb{P}\left((q(t), p(t), x_E(t)) \in A|\,\mathcal{F}_0\right) = (\rho^q, \rho^p, \rho^E)(A) . \tag{12}$$

It also holds that $(q(t), p(t), x_E(t))$ converges weakly to (ρ^q, ρ^p, ρ^E), i.e. for any $\varphi \in C_b$, it holds

$$\lim_{t\to\infty} \mathbb{E}_{(\bar{q},\bar{p},\bar{x}_E)}\left[\varphi\left(q(t), p(t), x_E(t)\right)\right] = \\ = \int \varphi((\bar{q},\bar{p},\bar{x}_E))(\rho^q,\rho^p,\rho^E)\,(d\bar{q}, d\bar{p}, d\bar{x}_E) . \tag{13}$$

Proof. Proposition 4 implies that in case of free–motion, $F_{ext} = 0$, there exists $\epsilon > 0$ such that $\mathcal{L}H_T < -\epsilon$, for any $\|p\| \geq C$. Thus, using (Fang and Gao, 2016, Lemma 1 - Lemma 2), for any initial state $(\bar{q}, \bar{p}, \bar{x}_E)$ such that $\|(\bar{q}, \bar{p}, \bar{x}_E)\| \geq C$ it holds

$$\mathbb{E}\left[\tau|\,(\bar{q}, \bar{p}, \bar{x}_E)\right] \leq \frac{H((\bar{q}, \bar{p}, \bar{x}_E))}{\epsilon} ,$$

being τ the first exit time to $\|(\bar{q}, \bar{p}, \bar{E})\| \geq C$.
Note further that the SPHS with energy tank (9) satisfies the dissipativity assumption (Cosso et al., 2016, Assumption (H1) (ii)), so that by (Cosso et al., 2016, Proposition 2.1) a unique invariant measure (ρ^q, ρ^p, ρ^E) exists and $(q(t), p(t), x_E(t))$ converges weakly to (ρ^q, ρ^p, ρ^E). □

From a control theoretic point of view, the convergence to the invariant measure and the ergodicity of the process characterize a stable behaviour of the stochastic system. Therefore, as the invariant measure is shaped around a certain state, the system will evolve, as time goes to infinity, around such a state. Thus, the requirement that passivity holds outside a given ball implies that, even if the path sometimes exits a desired region, the passivity will force the system back into the region immediately.

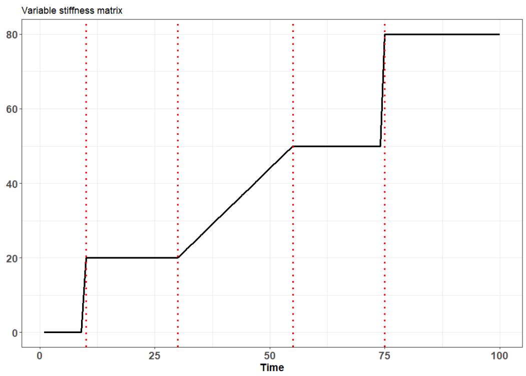

Fig. 2. Variable Stiffness matrix along $x-$axis

4. SIMULATION RESULTS

The present section reports simulations for the SPHS with energy tank (9) for a planar system, i.e. with dimension of the state equal to two ($n = 2$, $q = (q_1, q_2)$).
Let the inertia and damping matrices be given by

$$\Lambda_d = diag(0.5, 0.9)\,[kg] , \\ D_d = diag(50, 10)\,[Ns/m] .$$

The time–varying stiffness matrix is given in Figure 2 as a function of time, whereas the constant part is equal to $diag(10, 10)\,[N/m]$. Vertical dashed lines report changes in stiffness over time: this allows to highlight when and how momentum, position and energy tank vary according to stiffness changes. The constant diffusion matrix takes the value $\nu = diag(0.5, 0.8)$. Figure 3 reports the evolution the position q for the stochastic system (9): q_1 in the bottom right panel and q_2 in the top left panel. Top right panel in Figure 3 reports the empirical distribution of (q_1, q_2). It is evident how position does not converge to 0 reaching a stable steady state, so that asymptotic zero tracking error is not achieved. Nonetheless, long–time behaviour remains

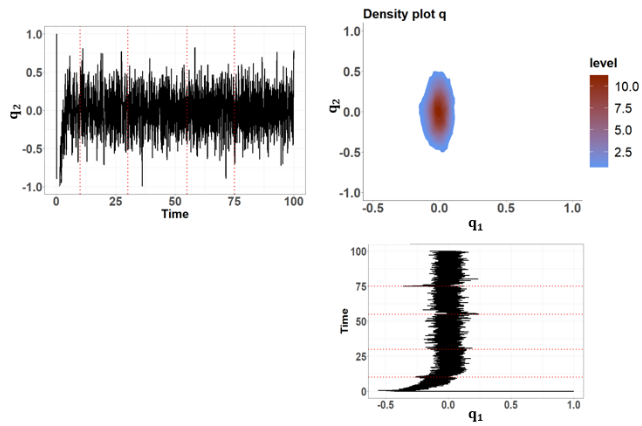

Fig. 3. Tracking errors (q_1, q_2) and empirical density distribution

bounded in a region which depends on the noise intensity and the asymptotic system is characterized by the associated invariant measure. Figure 4 shows momentum error $p = (p_1, p_2)$ (top panel) and energy stored into the tank E (bottom panel); again dashed red lines refer to changes in stiffness.

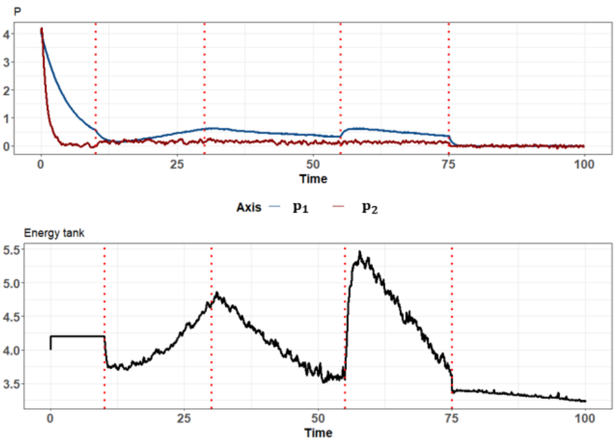

Fig. 4. Momentum (p_1, p_2) (top panel) and energy tank evolution E (bottom panel)

5. CONCLUSION

The present paper introduce a robust framework in which PHS and energy tank theory can be properly generalize to consider stochastic perturbations. In particular introducing suitable weak notion of passivity and convergence, we have shown how a stochastic system can be equipped with an energy tank in order to guarantee passivity and stability in a suitable sense, under mild assumptions. We have thus shown how the present setting can be used to implement variable admittance control, so that a time varying control can be efficiently implemented guaranteeing the convergence of the controlled system towards a desired reference configuration. Further research will focus on exploiting stochastic port–Hamiltonian framework to derive formal results regarding control of mechanical system, such as how *ultimately passivity* can be adapted to the classical deterministic energy shaping of PHS.

REFERENCES

Buchli, J., Theodorou, E., Stulp, F., and Schaal, S. (2011). Variable impedance control a reinforcement learning approach. *Robotics: Science and Systems VI*, 153–160.

Calanca, A., Muradore, R., and Fiorini, P. (2015). A review of algorithms for compliant control of stiff and fixed-compliance robots. *IEEE/ASME Transactions on Mechatronics*, 21(2), 613–624.

Calanca, A., Muradore, R., and Fiorini, P. (2017). Impedance control of series elastic actuators: Passivity and acceleration-based control. *Mechatronics*, 47, 37–48.

Cordoni, F., Di Persio, L., and Muradore, R. (2019a). Bilateral teleoperation of stochastic port-hamiltonian systems using energy tank. *(submitted)*.

Cordoni, F., Di Persio, L., and Muradore, R. (2019b). Stochastic port–hamiltonian systems. *arXiv preprint arXiv:1910.01901*.

Cosso, A., Fuhrman, M., and Pham, H. (2016). Long time asymptotics for fully nonlinear bellman equations: a backward sde approach. *Stochastic Processes and their Applications*, 126(7), 1932–1973.

Fang, Z. and Gao, C. (2016). Stochastic weak passivity based stabilization of stochastic systems with nonvanishing noise. *arXiv preprint arXiv:1602.07406*.

Ferraguti, F., Preda, N., Manurung, A., Bonfe, M., Lambercy, O., Gassert, R., Muradore, R., Fiorini, P., and Secchi, C. (2015). An energy tank-based interactive control architecture for autonomous and teleoperated robotic surgery. *IEEE Transactions on Robotics*, 31(5), 1073–1088.

Ferraguti, F., Secchi, C., and Fantuzzi, C. (2013). A tank-based approach to impedance control with variable stiffness. In *2013 IEEE International Conference on Robotics and Automation*, 4948–4953. IEEE.

Florchinger, P. (1999). A passive system approach to feedback stabilization of nonlinear control stochastic systems. *SIAM Journal on Control and Optimization*, 37(6), 1848–1864.

Franchi, A., Secchi, C., Son, H.I., Bulthoff, H.H., and Giordano, P.R. (2012). Bilateral teleoperation of groups of mobile robots with time-varying topology. *IEEE Transactions on Robotics*, 28(5), 1019–1033.

Franken, M., Stramigioli, S., Misra, S., Secchi, C., and Macchelli, A. (2011). Bilateral telemanipulation with time delays: A two-layer approach combining passivity and transparency. *IEEE transactions on robotics*, 27(4), 741–756.

Hatanaka, T., Chopra, N., and Spong, M.W. (2015). Passivity-based control of robots: Historical perspective and contemporary issues. In *2015 54th IEEE Conference on Decision and Control (CDC)*, 2450–2452. IEEE.

Hogan, N. (1985). Impedance control: An approach to manipulation: Part i-ii.

Khasminskii, R. (2011). *Stochastic stability of differential equations*, volume 66. Springer Science & Business Media.

Liberzon, D. and Brockett, R.W. (2000). Nonlinear feedback systems perturbed by noise: Steady-state probability distributions and optimal control. *IEEE Transactions on Automatic Control*, 45(6), 1116–1130.

Satoh, S. and Fujimoto, K. (2012). Passivity based control of stochastic port-hamiltonian systems. *IEEE Transactions on Automatic Control*, 58(5), 1139–1153.

Satoh, S. and Saeki, M. (2015). Bounded stability of nonlinear stochastic systems. *SICE Journal of Control, Measurement, and System Integration*, 8(2), 181–187.

Tsumugiwa, T., Yokogawa, R., and Hara, K. (2002). Variable impedance control based on estimation of human arm stiffness for human-robot cooperative calligraphic task. In *Proceedings 2002 IEEE International Conference on Robotics and Automation*, volume 1, 644–650.

van der Schaft, A., Jeltsema, D., et al. (2014). Port-hamiltonian systems theory: An introductory overview. *Foundations and Trends® in Systems and Control*, 1(2-3), 173–378.

Villani, L. and De Schutter, J. (2008). Force control. *Springer handbook of robotics*, 161–185.

Zhu, W. (2006). Nonlinear stochastic dynamics and control in hamiltonian formulation. *Applied Mechanics Reviews*, 59(4), 230–248.