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Method of identification of kinematic and elastostatic parameters of multilink manipulators without external measuring devices

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Abstract: The paper deals with a method of identification of kinematic and elastostatic parameters of multilink industrial manipulators. This method does not require complex and expensive equipment for high-precision external measurements of position and orientation of the working tool in the Cartesian coordinate system. The method gives simple and cheap means to parameters identification and their implementation allows significantly increase the dynamic accuracy of the movement of working tools of serial manipulators along spatial trajectories during the performance of various technological operations of real production. The simulation is considered.

Keywords: robots manipulators, identification and control methods, kinematic parameters, elastostatic

1. INTRODUCTION

Currently, industrial manipulators (IM) are used not only to perform simple transport operations, but also as a basis for the creation of robotic machining centers. In this case the accuracy of the positioning of the working tool (WT) in the Cartesian (base) coordinate system has great importance.

The real accuracy of the IM WT positioning in base coordinate system is determined mainly by the accuracy of a kinematic model of the IM, since its controller determines the position of the WT by means only this model and the IM joints angles. At the same time the real model of the IM can differ from the model used by the controller, due to inaccuracies in the manufacture and assembly of the mechanics of the IM. Therefore, one way to increase the accuracy of the IM WT movement is to define more precisely the kinematic parameters of the IM model.

Today, there are many methods for calibrating the IM kinematic parameters. These methods are based on using of external high-precision measuring devices that measure the Cartesian coordinates and orientation of the IM flange, see, for example, Alici el al. (2005), Dumas et al. (2011) and so on.

Also, the optical measuring systems that measure the relative displacements of the IM flange can be used for its calibration, see Kolyubin et al. (2015) and Luo et al. (2018). In some cases, special calibration devices (calibration plates) or calibrated IM of the same model can be used for some IM models, see Soe-Knudsen et al. (2017).

Therewith heavy WT, such as milling spindles, which installed on the IM and used to perform machining operations, can cause sufficiently large external forces and torques applied to the IM flange. In this case elastostatic effects, which are caused by the low stiffness of the IM links and their actuators, begin to appear. Presence of these effects leads to additional WT deviations. These deviations cannot be determined on basis of measuring the angles of rotation of the IM joints. Therefore, it is necessary to take into account both the IM kinematic model and the model of its elastostatic to ensure the accuracy of the IM WT movement. Methods for determining the parameters of the IM elastostatic model are presented in Klimchik el al. (2014, 2017). These methods are based on using of external measuring devices to determine small deviations of the IM flange when known force and torque values are applied to it.

The requirement to use expensive external measuring devices and a rather complicated procedure for calculating these parameters are the main disadvantages of existing methods for calibrating the parameters of the IM models. The method for identifying the IM kinematic parameters was proposed in Gubankov et al. (2018). This method does not require the use of any specialized equipment. Only the data from the angle sensors of the actuators of all degrees of mobility are used to tune the parameters of the IM kinematic model. Moreover, these angles are measured when the IM WT approaches to the same spatial point with different orientations of the WT. However, this method does not take into account the elastostatic effects of the IM. Therefore, in this work the specified method will be modified to allow simultaneous identification of the kinematic and elastostatic parameters of the IM.

2. PROBLEM STATEMENT

In the paper we will consider series IM. Kinematic model of such IM with consideration of elastostatic effects described by the following expression, Klimchik el al. (2014):

$$\boldsymbol{T_f} = D(\boldsymbol{\Phi},\boldsymbol{Q},\boldsymbol{\zeta}), \quad (1)$$

$$D(\boldsymbol{\Phi},\boldsymbol{Q},\zeta) = \prod_{i=1}^{6} T_i(\varphi_i, q_i, \zeta_i), \quad (2)$$

$$T_i = \begin{bmatrix} \cos(q_i+\theta_i+\zeta_i) & -\sin(q_i+\theta_i+\zeta_i)\cos(\alpha_i) & \sin(q_i+\theta_i+\zeta_i)\sin(\alpha_i) & a_i\cos(q_i+\theta_i+\zeta_i) \\ \sin(q_i+\theta_i+\zeta_i) & \cos(q_i+\theta_i+\zeta_i)\cos(\alpha_i) & -\cos(q_i+\theta_i+\zeta_i)\sin(\alpha_i) & a_i\sin(q_i+\theta_i+\zeta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\boldsymbol{T_f} = \begin{bmatrix} \boldsymbol{R_f} & \boldsymbol{X_f} \\ 0 & 1 \end{bmatrix} \in R^{4\times4}$ is a homogeneous transformation describing the position and orientation of the manipulator end link (flange) in the base coordinate system; $R_f \in R^{3\times3}$ is a matrix of orientation of flange in base frame; $X_f \in R^{3\times1}$ is a position vector of flange in base frame; $\boldsymbol{\Phi} = [\varphi_1,...,\varphi_6]^T$, $\varphi_i = [a_i, d_i, \alpha_i, \theta_i]$, $i = \overline{(1,6)}$ is a matrix of transformation parameters of Denavit-Hartenberg convention; i is a joint number; $\boldsymbol{Q} = (q_1,\dots,q_6)^T$ is a vector of IM generalized coordinates (rotation angels of actuators); $\boldsymbol{\zeta} \in (\zeta_1,...,\zeta_6)^T$ is vector of generalized coordinates increments, due to the presence of elastostatic effects; T_i is a Denavit-Hartenberg matrix.

The model of IM elastostatic described by the following expressions, Dumas et al. (2011):

$$\boldsymbol{\zeta} = \boldsymbol{K}_\zeta J_\zeta{}^T F, \quad J_\zeta = \frac{\partial \boldsymbol{t}(\boldsymbol{\Phi},\boldsymbol{Q},\boldsymbol{\zeta})}{\partial \boldsymbol{\zeta}}, \quad (3)$$

where $t = [\delta_x, \delta_y, \delta_z, \delta_A, \delta_B, \delta_C]^T$ is a vector of flange Cartesian coordinates and orientations increments due to the presence of elastostatic effects; $\boldsymbol{K}_\zeta = diag(k_{\zeta1},\cdots,k_{\zeta6}) \in R^{6\times6}$ is a diagonal matrix of compliance coefficients; F is a vector of external forces and torques applied to IM flange.

IM motion control traditionally based on its kinematic model only, excluding additives defined by elastostatic model (3).

Matrix $\tilde{\boldsymbol{\Phi}}$ is consist of manipulators kinematic parameters, which corresponds to their nominal geometric parameters given in the technical documentation.

However, exact kinematic parameters $\boldsymbol{\Phi}$ of a particular IM may vary from its nominal parameters $\tilde{\boldsymbol{\Phi}}$ due to inaccuracy in the manufacture and connection of its mechanical elements by a small values:

$$\boldsymbol{\Phi} = \tilde{\boldsymbol{\Phi}} + \Delta. \quad (4)$$

The IM kinematic parameters are used not only to calculate current flange position and orientation, but also to evaluate reference joints angles vector $\boldsymbol{Q}^*$ that provides WT location in desired position with given orientation. Using $\tilde{\boldsymbol{\Phi}}$ will lead to the evaluation of $\tilde{\boldsymbol{Q}}^*$ and then deviation of flange from a given reference position. The same effect will be observed if elastostatic effects will not be taken into account. That is, the deviation of the parameters of the kinematic model used by the controller from the real IM parameters and neglecting elastostatic effects leads to flange positioning errors when using heavy WT. This is especially important for cases when IM trajectory is generated automatically based on information from various vision systems (laser or optical scanners, cameras, etc.).

The enhance of the kinematic and elastostatic parameters can be done with the help of special measuring systems, which allow to determine linear and angular coordinates of robot tools with high accuracy. However, the use of such systems is often impossible due to their extremely high cost. Moreover, IM already has a high-precision joint angles measuring system which can be used to calculate its parameters.

Thus, in this paper the following problem will be solved. Consider PUMA-type IM with kinematic and elastostatic parameters described by $\boldsymbol{\Phi}$ and $\boldsymbol{K}_\zeta$, respectively. The robot controller solves direct and inverse kinematic problems using the matrix of nominal parameters $\tilde{\boldsymbol{\Phi}}$ and neglecting $\boldsymbol{K}_\zeta$. This will cause an error ε of IM flange positioning in Cartesian coordinate system. To reduce this error, it is necessary to develop a method for estimating IM parameters based on a series of its generalized coordinates measurements.

3. METHOD OF ESTIMATION OF KINEMATIC AND ELASTOSTATIC PARAMETERS OF INDUSTRIAL MANIPULATOR

To estimate IM parameters, we will modify the method proposed in Gubankov et al. (2018). The procedure of reference data obtaining is similar to tool center point (TCP) calculation for typical IM: Cartesian position is fixed and all degrees of freedom are used to change the configuration (joint) of robot. That is there will be n series of measurement of vectors $\boldsymbol{Q}$. Each i-th series of measurements consists of m_i vectors $\boldsymbol{Q}$, which correspond to the position of the TCP at the same point of space X^i with different orientation in base frame. The coordinates of X^i are unknown. As a rule, for such measurements a sharp probe is used (see Fig. 1).

As a result of measurements, a data array is formed:

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\Psi}_1 \\ \vdots \\ \boldsymbol{\Psi}_n \end{bmatrix}, \boldsymbol{\Psi}_i = \left(\boldsymbol{Q}_1^i, ..., \boldsymbol{Q}_{m_i}^i\right)^T, i = \left(\overline{1,n}\right). \quad (5)$$

For each vector $\boldsymbol{Q}_j^i, i = \left(\overline{1,n}\right), j = \left(\overline{1,m_i}\right)$ one can assign

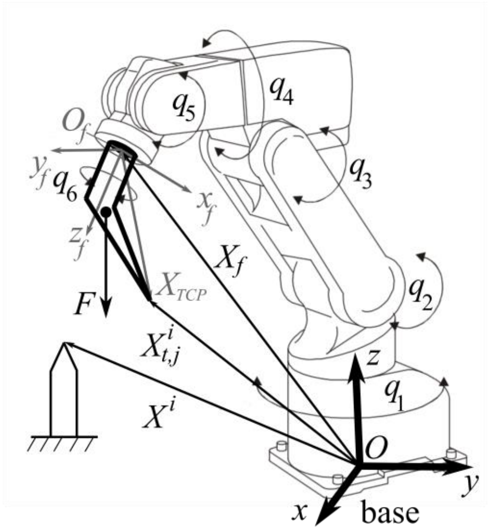

Fig.1. Measurement data obtaining

vector $\hat{X}_{t,j}^i$ of coordinate of TCP in the base system $Oxyz$ (see Fig. 1). This vector is calculated based on (1) and (2) using the matrix $\hat{\boldsymbol{\Phi}}$ of estimation of kinematic parameters and diagonal matrix $\hat{\boldsymbol{K}}_\zeta$ of compliances coefficients estimations. The expression to calculate $\hat{X}_{t,j}^i$ will be as follows:

$$\boldsymbol{T}_j^i(\hat{\boldsymbol{\Phi}},\boldsymbol{Q}_j^i,\hat{\boldsymbol{\zeta}}) = \begin{bmatrix} \hat{R}_{f,j}^i & \hat{X}_{t,j}^i \\ 0 & 1 \end{bmatrix} = (\prod_{k=1}^{6} T_{k,j}^i(\varphi_k, q_{k,j}^i, \hat{\zeta}_k))\hat{T}_{TCP} , \quad (6)$$

where $\hat{T}_{TCP} = \begin{bmatrix} I & \hat{X}_{TCP} \\ 0 & 1 \end{bmatrix}$, $I \in R^{3\times 3}$ is an identity matrix; $\hat{X}_{TCP}$ is a estimation of vector of coordinates of the TCP in the flange coordinate system $O_f x_f y_f z_f$.

The coordinates of the points $\hat{X}_{t,j}^i$ evaluated according to (6) will differ from the real TCP position due to the differs of IM parameters from their real values. However, since the WT in each series of measurements is located in the same point X^i with unknown coordinates, the real coordinates of the TCP in one series of measurements will be the same. This fact can be used to identify the parameters of the manipulator.

The estimation of matrix $\hat{\boldsymbol{\Phi}}$ of kinematic parameters and matrix $\hat{\boldsymbol{K}}_\zeta$ of compliance coefficients can be done by choosing the specified parameters so that the $\hat{X}_{t,j}^i$, evaluated according to (6) with help of $\hat{\boldsymbol{\Phi}}$ and $\hat{\boldsymbol{K}}_\zeta$, for a separate series of measurements become closer to the minimum distance. That is, the measure of the quality of parameters identification can be made according to the following criterion:

$$\Im(\boldsymbol{\Xi},\hat{\boldsymbol{\Phi}},\hat{k}_\zeta) = \sum_{i=1}^{n}\sum_{j=1}^{m_i-1}\sum_{k=j+1}^{m_i}\left(\hat{X}_{t,j}^i - \hat{X}_{t,k}^i\right)^2 . \quad (7)$$

As one can see, expression (7) does not contain the real coordinates of points X^i, so it not needs the high accuracy measurement systems for estimation of manipulator parameters. Therefore, the task of manipulator parameters estimation is formulated as follows:

$$\hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{K}}_\zeta = \arg\min_{\boldsymbol{\Phi},\boldsymbol{K}_\zeta} \Im(\boldsymbol{\Xi},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{K}}_\zeta) . \quad (8)$$

The numerical optimization method of Levenberg-Marquardt will be used for estimation of manipulator parameters. For this purpose, the initial measurement data needs to be presented in following view:

$$r_p^i = \hat{X}_{t,j}^i - \hat{X}_{t,k}^i,\ i = \left(\overline{1,n}\right),\ j = \left(\overline{1,m_i-1}\right),\ k = \left(\overline{2,m_i}\right),\ p = \left(\overline{1,l_i}\right),$$

$$R_i = \begin{bmatrix} r_1^i \\ \vdots \\ r_{l_i}^i \end{bmatrix}, l_i = \sum_{j=1}^{m_i-1} j,\ P = \begin{bmatrix} R_1 \\ \vdots \\ R_n \end{bmatrix} \in R^{3L},\ L = \sum_{i=1}^{n} l_i . \quad (9)$$

Criteria (7) considering (9) can be rewritten as follows:

$$\Im = \boldsymbol{P}^T \boldsymbol{P} . \quad (10)$$

Therewith the matrix $\boldsymbol{\Phi}$ of manipulator parameters and matrix $\boldsymbol{K}_\zeta$ of compliance coefficients can be presented as follows:

$$\vartheta = \left[\varphi_1,\ \cdots, \varphi_6,\ k_{\zeta 1}, \cdots, k_{\zeta 6}, X_{TCP}\right]^T = \left[\vartheta_\Phi \quad \vartheta_\zeta \quad \vartheta_{TCP}\right]^T . \quad (11)$$

From (11) one can see that 33 manipulator parameters will be estimated: 24 parameters describe manipulator kinematic, 6 parameters describe compliance coefficients and 3 parameters describe position of TCP in flange coordinate system.

Also from (9) the proposed method of forming the criterion (10) allows to increase the amount of data for identification with a limited number of measurements. For example, when the probe is moved to one point 20 times with different orientations, 190 different pairs of measurements are obtained to use in (10).

Before starting identification, the parameter vector ϑ must be initialized with initial values. For this purpose one can use the following values:

$$\hat{\boldsymbol{\Phi}}_0 = \tilde{\boldsymbol{\Phi}},\ \hat{\boldsymbol{K}}_{\zeta 0} = 0 . \quad (12)$$

Initial estimates of the vector $\hat{X}_{TCP}$ can be obtained using the built-in IM software or using the method described in Gubankov (2018), which allows for the evaluation of this vector to use all the measurements obtained for the identification procedure.

To carry out the identification procedure using the Levenberg-Marquardt method, it is necessary to calculate the derivative of the matrix $\boldsymbol{P}$ from the identifiable parameters of the IM:

$$G = \frac{\partial \boldsymbol{P}(\boldsymbol{\Xi}, \hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{K}}_\zeta)}{\partial \vartheta} \in R^{L\times 33}. \tag{13}$$

For evaluating G model (6), considering (1), should be rewritten as follows:

$$\hat{X}^i_{t,j} = X^i_{f,j}(\hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{K}}_\zeta, \boldsymbol{Q}^i_j) + R^i_{f,j}(\hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{K}}_\zeta, \boldsymbol{Q}^i_j)\hat{X}_{TCP}, \tag{14}$$

where $X^i_{f,j} \in R^{3\times 1}$ and $R^i_{f,j} \in R^{3\times 3}$ are coordinate vector and orientation matrix of the flange in base coordinate system, which calculated based on $\boldsymbol{Q}^i_j$.

Based on the smallness of $\boldsymbol{\zeta}$ due to the elastostatic effects of IM and considering (3), the first term of expression (14) can be written as:

$$\begin{gathered} X^i_{f,j} = X^{Kin,i}_{f,j}(\hat{\boldsymbol{\Phi}}, \boldsymbol{Q}^i_j) + \Delta^i_{p,j}(\boldsymbol{Q}^i_j, \boldsymbol{K}_\zeta, \widetilde{\boldsymbol{\Phi}}), \\ \begin{bmatrix} \Delta^i_{p,j} \\ \Delta^i_{A,j} \end{bmatrix} = J_\zeta(\boldsymbol{Q}^i_j, \widetilde{\boldsymbol{\Phi}})\boldsymbol{K}_\zeta J_\zeta(\boldsymbol{Q}^i_j, \widetilde{\boldsymbol{\Phi}})^T F, \end{gathered} \tag{15}$$

and the second term as:

$$\begin{gathered} R^i_{f,j}(\hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{K}}_\zeta, \boldsymbol{Q}^i_j)\hat{X}_{TCP} = (R^{Kin,i}_{f,j}(\hat{\boldsymbol{\Phi}}, \boldsymbol{Q}^i_j) + \Delta^i_{R,j}(\boldsymbol{Q}^i_j, \hat{\boldsymbol{K}}_\zeta))\hat{X}_{TCP}, \\ R^{Kin,i}_{f,j}(\hat{\boldsymbol{\Phi}}, \boldsymbol{Q}^i_j) = [I \quad J \quad K], \\ \Delta^i_{R,j}(\boldsymbol{Q}^i_j, \hat{\boldsymbol{K}}_\zeta) = [\frac{\partial I}{\partial \boldsymbol{Q}}\zeta^i_j \quad \frac{\partial J}{\partial \boldsymbol{Q}}\zeta^i_j \quad \frac{\partial K}{\partial \boldsymbol{Q}}\zeta^i_j]. \end{gathered} \tag{16}$$

where $X^{Kin,i}_{f,j}$, $R^{Kin,i}_{f,j}$ are position and orientation matrix of flange in base system, obtained without taking into account the elastostatic effects.

Using (14)-(16), we write the expression for $\partial X^i_{t,j}/\partial \vartheta$. In this case, we assume that the expressions describing the elastostatic effects depend on the initial values of the kinematic parameters $\widetilde{\boldsymbol{\Phi}}$. This is true because of the small differences between the real kinematic parameters of the IM from their nominal values.

For parameters describing the kinematic model of the manipulator, one can write:

$$\frac{\partial X^i_{t,j}}{\partial \vartheta_{\boldsymbol{\Phi},k}} = \frac{\partial X^i_{f,j}(\hat{\boldsymbol{\Phi}}, \boldsymbol{Q}^i_j)}{\partial \vartheta_{\boldsymbol{\Phi},k}} + \frac{\partial R^{Kin,i}_{f,j}(\hat{\boldsymbol{\Phi}}, \boldsymbol{Q}^i_j)}{\partial \vartheta_{\boldsymbol{\Phi},k}}\hat{X}_{TCP}, k = (\overline{1,24}).$$

$$\frac{\partial X^i_{t,j}}{\partial \vartheta_{\boldsymbol{\Phi}}} = \begin{bmatrix} \frac{\partial X^i_{t,j}}{\partial \vartheta_{\boldsymbol{\Phi},1}} & \cdots & \frac{\partial X^i_{t,j}}{\partial \vartheta_{\boldsymbol{\Phi},24}} \end{bmatrix}. \tag{17}$$

For elastostatic parameters one can write:

$$\begin{gathered} \frac{\partial X^i_{t,j}}{\partial \vartheta_\zeta} = \frac{\partial \Delta^i_{p,j}}{\partial \vartheta_\zeta} + \frac{\partial \Delta^i_{R,j}}{\partial \vartheta_\zeta}, \\ \begin{bmatrix} \frac{\partial \Delta^i_{p,j}}{\partial \vartheta_\zeta} \\ \frac{\partial \Delta^i_{A,j}}{\partial \vartheta_\zeta} \end{bmatrix} = J_\zeta(\boldsymbol{Q}^i_j, \widetilde{\boldsymbol{\Phi}})J_\zeta{}^T(\boldsymbol{Q}^i_j, \widetilde{\boldsymbol{\Phi}})F, \\ \frac{\partial \Delta^i_{R,j}}{\partial \vartheta_\zeta} = \begin{bmatrix} \frac{\partial \Delta^i_{R,j}}{\partial \vartheta_{\zeta 1}} & \cdots & \frac{\partial \Delta^i_{R,j}}{\partial \vartheta_{\zeta 6}} \end{bmatrix}, \\ \frac{\partial \Delta^i_{R,j}}{\partial \vartheta_{\zeta k}} = \begin{bmatrix} I^i_{\boldsymbol{Q},j,k}F^i_{\zeta,j,k} & J^i_{\boldsymbol{Q},j,k}F^i_{\zeta,j,k} & K^i_{\boldsymbol{Q},j,k}F^i_{\zeta,j,k} \end{bmatrix}\hat{X}_{TCP}, \end{gathered} \tag{18}$$

where $I^i_{\boldsymbol{Q},j,k}$, $J^i_{\boldsymbol{Q},j,k}$, $K^i_{\boldsymbol{Q},j,k}$ are the k-th columns of matrixes $\partial I^i_j/\partial \boldsymbol{Q}$, $\partial J^i_j/\partial \boldsymbol{Q}$ и $\partial K^i_j/\partial \boldsymbol{Q}$, respectively; $F^i_{\zeta,j,k}$ is the k-th entry of vector $J_\zeta(\boldsymbol{Q}^i_j, \widetilde{\boldsymbol{\Phi}})F$.

For parameters describing the coordinate vector of the TCP, one can write:

$$\frac{\partial X^i_{t,j}}{\partial \vartheta_{TCP}} = R^{Kin,i}_{f,j}(\hat{\boldsymbol{\Phi}}, \boldsymbol{Q}^i_j). \tag{19}$$

Thus, considering (17)-(19) one finally can right:

$$\begin{gathered} \frac{\partial X^i_{t,j}}{\partial \vartheta} = \begin{bmatrix} \frac{\partial X^i_{t,j}}{\partial \vartheta_\Phi} & \frac{\partial X^i_{t,j}}{\partial \vartheta_\zeta} & \frac{\partial X^i_{t,j}}{\partial \vartheta_{TCP}} \end{bmatrix}, \\ \frac{\partial r^i_l}{\partial \vartheta} = \left(\frac{\partial X^i_{t,j}}{\partial \vartheta} - \frac{\partial X^i_{t,k}}{\partial \vartheta}\right), \frac{\partial R_i}{\partial \vartheta} = \begin{bmatrix} \frac{\partial r^i_1}{\partial \vartheta} \\ \vdots \\ \frac{\partial r^i_{l_i}}{\partial \vartheta} \end{bmatrix}, G = \frac{\partial P}{\partial \vartheta} = \begin{bmatrix} \frac{\partial R_1}{\partial \vartheta} \\ \vdots \\ \frac{\partial R_n}{\partial \vartheta} \end{bmatrix}, \\ i = (\overline{1,n}),\ j = (\overline{1,m_i}),\ k = (\overline{2,m_i}),\ s = (\overline{1,m_j - 1}),\ l_i = \sum_{j=1}^{m_i - 1} j. \end{gathered} \tag{20}$$

The expression (20) is used to tune the parameter vector ϑ using the method of Levenberg-Marquardt as follows:

$$\begin{gathered} \Delta\vartheta(k) = -\left[G^T G + \mu(k)E\right]^{-1} G^T P \\ \hat{\vartheta}(k+1) = \begin{cases} \hat{\vartheta}(k) + \Delta\hat{\vartheta}(k), \mu(k+1) = \dfrac{\mu(k)}{\eta}, \\ \qquad if\ \Im(\hat{\vartheta}(k) + \Delta\hat{\vartheta}(k)) < \Im(\hat{\vartheta}(k)) \\ \hat{\vartheta}(k), \mu(k+1) = \mu(k)\eta, \\ \qquad if\ \Im(\hat{\vartheta}(k) + \Delta\hat{\vartheta}(k)) \geq \Im(\hat{\vartheta}(k)) \end{cases}, \end{gathered} \tag{21}$$

where $E \in R^{33\times 33}$ is the unity diagonal matrix; $\mu(k)$ is the step of parameter tuning; $0<\eta<1$ is the coefficient of step changing.

Herewith on each iteration the vector P is updated on the base of (7) with using of current estimation of vector $\hat{\vartheta}(k)$.

As shown in Gubankov (2018) the proposed method does not allow to identify the parameters d_1 and θ_1. Therewith for identification of parameter $k_{\zeta 1}$ it is necessary to form the lateral force applied to a flange.

As the result of algorithm (21) working the estimation of vector $\hat{\vartheta}$ is formed. This estimation provides the attraction of points $\hat{X}^i_{l,j}$ to minimal distance between each other in i-th measurement set. Using of calculated kinematic and elastostatic parameters in manipulator controller allow to increase accuracy of manipulator tool positioning in Cartesian coordinate system.

4. RESULTS OF MATHEMATICAL SIMULATION

To verify the efficiency of the proposed method of identifying the parameters of the IM, mathematical simulation in MATLAB was carried out. In this simulation IM Kuka KR60 was considered, which nominal kinematic parameters are:

$$\tilde{\boldsymbol{\Phi}} = \begin{bmatrix} \theta,\circ & a, mm & d, mm & \alpha,\circ \\ 0 & 350 & 815 & -90 \\ 0 & 850 & 0 & 0 \\ -90 & 145 & 0 & 90 \\ 0 & 0 & -820 & -90 \\ 0 & 0 & 0 & 90 \\ 0 & 0 & -170 & 180 \end{bmatrix}.$$

To test the proposed method, a data array Ξ was generated. This array contains 3 series of 18 measurements each correspond to the working tool $X_{TCP} = [100,0,150]^T$ location with different orientation to the points with coordinates $X^1 = [-50,1300,900]$, $X^2 = [-300,-1500,1200]$, $X^3 = [1400,700,1450]$. Thus, it was considered that parameters of IM differed from nominal on size Δ:

$$\Delta = \begin{bmatrix} \theta,\circ & a, mm & d, mm & \alpha,\circ \\ 0 & 0.4 & 0 & 0.057 \\ 0 & 0.3 & 0.4 & -0.114 \\ 0.0573 & 0.3 & 0 & 0.1146 \\ -0.114 & 0.2 & 0.3 & 0.0573 \\ 0.1719 & 0.3 & 0.2 & -0.1719 \\ 0 & 0.2 & 0.3 & 0 \end{bmatrix},$$

and compliance coefficients of IM have following values:

$$k_\zeta = [1.0 \quad 0.7 \quad 1.0 \quad 1.2 \quad 1.2 \quad 1.5]10^{-8}(Nm)^{-1}.$$

For generating the array Ξ for each measurement, the inverse kinematics problem was solved based on the model (1). The result of it solving are vectors $\boldsymbol{Q}^i_j$ which provide the location of the TCP to the corresponding point X^i with a given orientation of the IM flange, which was set randomly.

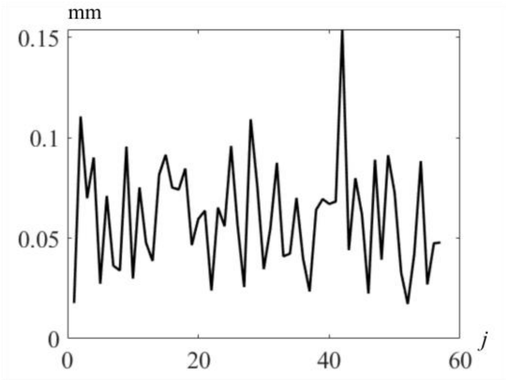

Fig. 2. The deviation of tool form point X^i

Also during the generation of the array, the error of coming the tool in point X^i was simulated. The value of this error for each measurement is shown in Fig.2.

As a result of identification of the parameters of the IM model the following results were obtained. The value $\Im$ changed from 2146 to 3.29 in 499 iterations.

The deviations of estimated parameters from their true values were as follows ($\boldsymbol{\Phi}$ is known only in simulation case):

$$\boldsymbol{\Phi} - \hat{\boldsymbol{\Phi}} = \begin{bmatrix} \theta,\circ & a, mm & d, mm & \alpha,\circ \\ 0 & -0.001 & 0 & -0.001 \\ 0.005 & 0.035 & 0.204 & -0.001 \\ 0.003 & -0.0183 & -0.196 & -0.001 \\ 0.0 & 0.0018 & 0.011 & -0.002 \\ -0.004 & -0.0231 & 0.023 & -0.006 \\ 0.001 & 0.295 & 0.46 & -0.035 \end{bmatrix},$$

the estimates of the compliance coefficients are: $\hat{k}_\zeta$ = [0.0 0.72 1.04 1.15 1.28 1.79]10^{-8}, and $\hat{X}_{TCP}$ = [100.24 0.076 149.36]^T.

From the presented results one can see the errors in determining the linear kinematic parameters of IM do not exceed 0.025 mm, angular 0.006° and the error in determining the values of the compliance coefficients generally does not exceed 5% (the exception is the coefficient $k_{\zeta 6}$, the error of estimation of which is 20%). It can be seen that the coefficients d_2 and d_3 are estimated together and the initial deviation in the parameter d_2 is distributed between these parameters. This means that their sum is important, but not the terms values. It is also should be noted that the parameters of the last link are estimated together with $\hat{X}_{TCP}$, which leads to deviations of these parameters from their true values, but allows one to accurately determine the position of the TCP in Cartesian space. Also, should be noted that the identification of the parameter $k_{\zeta 1}$ is possible only if you provide the action of a known external force that creates a torque along the z axis of the base frame. However, in the proposed method of identification, it is technically difficult to do this. Therefore, the development of such method for identifying this coefficient is a separate task that requires further solutions.

To verify the accuracy of the IM with using the obtained parameters an array of $j = \overline{1,60}$ Q vectors was formed, which do not coincide with the data used for this identification. These vectors were used to calculate the position of the working point of the IM tool using the initial (nominal) kinematic model, model (1) with identified parameters and model (1) with exact parameter values.

Errors in determining the coordinates of the working point of the tool with using the nominal kinematic model IM and the identified parameters are presented in Fig. 3.

As can be seen from the Fig. 3, the use of nominal parameters and non-accounting of elastostatic effects leads to large errors in determining the coordinates of the working point of the tool. This error is particularly large in z, due to the action of an external force directed down the z axis. The resulting parameters allow to determine the working point position with an error not exceeding 0.06 mm, for the individual coordinates that is sufficient to run most manufacturing operations, such as Filaretov et al. (2015, 2019). At the same time, increasing accuracy of the tool moving to the specified point at the first step of proposed method (for generating array Ξ), will increase accuracy of parameter identification.

CONCLUSIONS

Proposed method includes two stages. On the first stage the operator manually moves the manipulator tool with different orientation to same fixed point and save data about rotation angle of manipulator joints. On the second stage the estimation is made by means of Levenberg-Marquardt method. This estimation is made to provide decreasing the distance between position of manipulator tool calculated by means of model (6). The simulation results confirmed the efficiency of the proposed method and the possibility of simultaneous identification of the parameters of the kinematic model of IM and compliance coefficients without the use of external measuring devices.

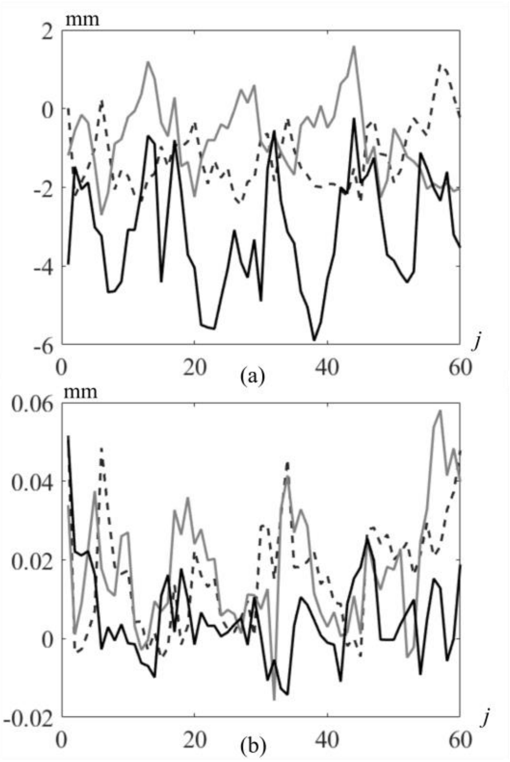

Fig. 3. Errors of determining of tool working point position with using nominal (a) and identified (b) IM parameters (error at x-coordinate is a dotted line, y-coordinate is a gray line, z-coordinate is a black solid line correspondingly)

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