

## Vibration reduction control for redundant flexible robot manipulators

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**Abstract:** In this paper, both vibration reduction and task accuracy improvement for flexible redundant manipulators, using the resolution of redundancy, is addressed. The exciting force of flexural motion, which is induced by the motion of the manipulator, causes undesired deflection at the end-effector. The null space inherent in redundancy feature is exploited to damp out the vibration while maintaining the end-effector posture. In order to exploit all the degrees of freedom in the null space, a feedback control design based on torque optimization has been introduced using analytical method for the redundancy resolution. The simulation analysis is presented to test the effectiveness of the proposed approach.

*Keywords:* Vibration Suppression, Redundancy Resolution, Flexible Robot Manipulators, Feedback Control Design.

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### 1. INTRODUCTION

The robotic vision of the advanced industry pays more attention to flexible manipulators, which have high load to mass ratio. However, the presence of flexible substructures, joints and/or links, may induce oscillator behavior which deteriorates the accuracy of the end-effector position. In addition, the industrial application including robot manipulation process are in need of robot manipulators which are able to operate in various constrained environments. In this context, the kinematic redundancy of redundant robots is highly investigated. The fact that it is offering more degrees of freedom (DOFs) regarding to those which are required to execute a given task, leads to several possible configurations in joints space to hold the same Cartesian posture of the end-effector. In literature, this internal motion is referred to as self-motion or null-space. The redundancy is used to deal with different kinematic and dynamic issues related to robot manipulators, in order to enhance their performances. For instance, in Xinyu (2015) and Khatib (1987), the redundancy is used to deal with obstacle avoidance and joints constraints.

The control of redundant flexible robot manipulator is an interesting research topic, since a vast investigating is in progress to date. In order to achieve a given optimal performance, in Benosman (2004), the dynamic of flexible elements is included in the generalised dynamic model of the flexible manipulator, and used to design various controllers algorithms. The input shaping technique, which is based on pre-estimated frequencies of the vibration modes, generates a smooth trajectory so that the flexural dynamic will be reduced, see Sahinkaya (2001). Since, different shaping filters are developed, such as exponential filter in Biagiotti (2015), and damped jerk-limited profile in Béarée (2014). On the other hand, as cited in Benosman (2004), the active control schemes are established similarly to those found in nonlinear and linear control theory for dynamic system, such

as PID control, singular perturbation control, and so on. The mentioned control designs are developed for both redundant and non-redundant manipulators. For example in Nakanishi (2008), the operational space control design is proposed for the redundant manipulators, in velocity and acceleration levels. Although, a little attention has been paid to take advantage of the redundancy in the controller design and a few works have introduced this ability, for instance in Sadeghian (2013), a control design in dynamic level for multi-priority control in redundant space is presented.

The joint motion is capable to affect the vibrations since the dynamic of the rigid body and the flexible parts of the manipulator are highly coupled. This ability may be used to achieve efficiently smooth motion without exciting the vibration dominant modes. The redundancy ability to affect the manipulator flexibility has been introduced and examined in some works. In Yue (2002) and Gao (2017), the null-space of the joint acceleration has been used to achieve an optimal joints motion so that the vibration are reduced. Nevertheless, the optimization algorithm consumes a considerable time, and even in some cases the optimal solution for joints motion is irregular and difficult to execute on-line. In Nguyen (1992) and Bian (2011), authors have established a control design based on the Jacobian null-space, which is used explicitly to damp out the vibration modes. It has been proved that the self-motion gets effectively rid of the manipulator flexibility effects only when the number of redundancy degrees is the same as the number of selected modes to be controlled. Similarly, in Hirano (2014), a flexural state estimator using force/torque sensor is added this time to the control design to reduced the induced vibration of flexible appendage in space robot. In Sasaki (2007), the resonance ratio control is designed to suppress the vibrations. The work space observer is used to estimate the flexural reactive force, which

is injected afterward in the Jacobian null-space in addition to the joint local control, by assuming that the joints velocities are low. In Hishinuma (2006), the vibration reduction is guaranteed by resolving the kinematic redundancy with the help of an additional constraint which is obtained from vibration dynamics, this approach is known as the task-space augmentation approach which is based on the augmented Jacobian. In this case, the joints motion solution may become unfeasible around algorithm singularity.

However, in the methods mentioned above, all the redundant DOFs were devoted to damp out the vibration modes in aim to ameliorate the accuracy of the end-effector. Nevertheless, realizing more than one performance in same time via redundancy resolution, would be highly appreciated, specially in case of hyper-redundant flexible manipulator. Thus, in the proposed control algorithm, we exploit all redundancy DOFs to compensate for the nonlinear dynamics in motion, and to damp out the induced vibration as well. These two stages control approach which is investigated in the following analysis, has not yet presented in literature.

In this paper, we analyze the ability of using the redundancy to optimize and carry out multiple performance criteria for flexible redundant manipulators. A control algorithm including the local redundancy resolution scheme for flexible serial manipulators is proposed. The motion planning of the manipulator is generated so that the nonlinear dynamics are compensated, while the dominating vibratory modes are damped out. Numerical simulations are conducted on a 5 degree of freedom planar manipulator to prove the effectiveness of the proposed approach.

## 2. GENERALIZED DYNAMIC MODEL

### 2.1 Dynamic analysis

The dynamic structure of the flexible manipulator is the combination of both the joints motion and the flexural motion, i.e the elastic deformation in the flexible elements, which is depicted by  $n_r$  rigid DOFs and  $n_f$  flexural DOFs respectively. Based on Lagrange's formulation and assumed mode method, the dynamic model of the flexible manipulator with flexible links is generally described by  $n$  dynamics equations of motion (for more details see Gao (2017) and Bian (2013))

$$M(\Theta) \ddot{\Theta} + D(\dot{\Theta}, \Theta) \dot{\Theta} + K(\Theta) \Theta = \Gamma + \tilde{\Gamma} \quad (1)$$

where  $n = n_r + n_f$  is the total DOFs of the system,  $\Theta = [q_r^T \ q_f^T]^T \in \mathbb{R}^n$  is the generalized coordinates vector,  $q_r \in \mathbb{R}^{n_r}$  and  $q_f \in \mathbb{R}^{n_f}$  are respectively the joint coordinates vector and the elastic displacement coordinates vector.  $M(\Theta) \in \mathbb{R}^{n \times n}$  is the global mass matrix,  $D(\dot{\Theta}, \Theta) \in \mathbb{R}^{n \times n}$  denotes the global damping matrix,  $K(\Theta) \in \mathbb{R}^{n \times n}$  is the global stiffness matrix.  $\Gamma \in \mathbb{R}^n$  is the generalized torques vector, which depends on the control torques.  $\tilde{\Gamma} \in \mathbb{R}^n$  is the vector which aggregates the Coriolis, gravitational and centripetal forces in addition to other nonlinear dynamics including the externally applied forces and the non-conservative forces such as friction forces. One may note that, the flexible links of the flexible manipulator are modeled as clamped base beams, see Bian (2013). The dynamic model of the flexible manipulator is supposed to be estimated by an observer or practically identified, see for instance Sasaki (2007).

The dynamics of both rigid and flexural motion may be described separately from (1) as follows

$$M_{rr} \ddot{q}_r + \tilde{\Gamma}_r = \tau \quad (2)$$

$$M_{fr} \ddot{q}_r + M_{ff} \ddot{q}_f + D_f \dot{q}_f + K_f q_f = \tilde{\Gamma}_f \quad (3)$$

where  $M_{rr} \in \mathbb{R}^{n_r \times n_r}$ ,  $M_{fr} \in \mathbb{R}^{n_r \times n_r}$  and  $M_{ff} \in \mathbb{R}^{n_f \times n_f}$  are block matrices of the global mass matrix,  $\tau \in \mathbb{R}^{n_r}$  is the control torques vector applied to the manipulator joints,  $\tilde{\Gamma}_r \in \mathbb{R}^{n_r}$  is the non-linear dynamics in the rigid motion.  $\tilde{\Gamma}_f \in \mathbb{R}^{n_f}$  is nonlinear force which is applied to the flexural dynamics.  $D_f \in \mathbb{R}^{n_f \times n_f}$  and  $K_f \in \mathbb{R}^{n_f \times n_f}$  are respectively the damping and the stiffness matrix of the flexural motion.

### 2.2 Redundancy resolution

The end-effector position of the flexible manipulator, which is operating in  $n_t$  dimensional work space, is function of both joints angles and elastic displacement coordinates as follows

$$x = f(q_r, q_f) \quad (4)$$

where  $x \in \mathbb{R}^{n_t}$ .

The kinematic model can be derived from (4) by time differentiation

$$\dot{x} = J_r \dot{q}_r + J_f \dot{q}_f \quad (5)$$

$$\ddot{x} = J_r \ddot{q}_r + \dot{J}_r \dot{q}_r + J_f \ddot{q}_f + \dot{J}_f \dot{q}_f \quad (6)$$

where  $J_r \in \mathbb{R}^{n_t \times n_r}$  and  $J_f \in \mathbb{R}^{n_t \times n_f}$  are the rigid and the flexible Jacobian matrix respectively.

From (3), one may notice that the flexural displacement can not be controlled directly by the joints torques. Usually, in the motion planning process only the nominal joints motion is considered to generate the desired trajectory for the flexible robot manipulators, see Sasaki (2007) and Gao (2017). In this case, the term  $(J_f \ddot{q}_f + \dot{J}_f \dot{q}_f)$  is not considered in redundancy resolution. The dynamical resolution of redundancy which is presented in (6) becomes

$$\ddot{x} = J_r \ddot{q}_r + \dot{J}_r \dot{q}_r \quad (7)$$

The flexible manipulator is kinematically redundant, i.e. the number of actuated (rigid) joints  $n_r$  is greater than the dimension of the operational work-space  $n_t$ . The degree of redundancy  $m$  is defined as  $m = n_r - n_t$ .

The resolution of redundancy based on the pseudo-inverse of the Jacobian matrix is usually used to determinate the joint acceleration. In Gao (2017), the projected joints dynamic in the Jacobian null-space is only devoted to damp out the vibration. In this case, having extra degrees of redundancy will be used to damp out effectively the vibration but not to fulfil additional tasks. In this study, we investigate the use of the extra number of redundancy DOFs in aim to perform more than one second task simultaneously. Therefore, an other method for resolution of redundancy, which is based on the manipulator dynamic analysis, has been considered in the development of the proposed strategy to exploit all the available degrees of redundancy, see Ghosal (1993). The joint torques are used in redundancy resolution, which is advantageous since they are the main input variables to the manipulator controller.

Substituting (2) into (7), the end-effector acceleration can be rewritten as

$$\ddot{x} = J_r M_{rr}^{-1} (\tau - \tilde{\Gamma}_r) + \dot{J}_r \dot{q}_r \quad (8)$$

The following terms are defined for the convenience of the further study development,

$$\vartheta = J_r M_{rr}^{-1} \quad (9)$$

$$\chi = J_r \dot{q}_r - J_r M_{rr}^{-1} \tilde{\Gamma}_r \quad (10)$$

The equation (8) then becomes

$$\ddot{x} = \vartheta_{\tau_i} \tau_i + \vartheta_{\tau_r} \tau_r + \chi \quad (11)$$

where the joint torques are decomposed to two sets,  $\tau_i \in \mathbb{R}^{n_t}$  the task torques to perform a given task, and  $\tau_r \in \mathbb{R}^m$  the redundant torques which are extra torques regarding the task.  $\vartheta_{\tau_i} \in \mathbb{R}^{n_t \times n_t}$  and  $\vartheta_{\tau_r} \in \mathbb{R}^{n_t \times m}$  are matrices that form the matrix  $\vartheta$ , where the  $i$ th task torque  $\tau_{r,i}$  (respectively the  $j$ th redundant torque  $\tau_{r,j}$ ) is associated with the  $i$ th column of the matrix  $\vartheta_{\tau_i}$  (respectively  $j$ th column of the matrix  $\vartheta_{\tau_r}$ ), ( $i = 1, 2, \dots, n_t$ ) and ( $j = 1, 2, \dots, m$ ). The term  $\chi \in \mathbb{R}^{n_t}$  aggregates all non-linear dynamics.

At time instant  $t^*$ , the joint coordinates  $(\dot{q}_r(t^*), q_r(t^*))$  are generally known, as a result the vector  $\chi$  can be determined, as a consequence, the mapping  $h$  given by

$$h : \tau \mapsto \ddot{x} \quad (12)$$

is affine and the admissible acceleration in this case are  $\ddot{x}_{ac} \in D_{\tau}$ , where  $D_{\tau} = \{\ddot{x} = h(\tau), \text{ s.t. } \tau_{min,i} \leq \tau_i \leq \tau_{max,i}, i = 1, 2, \dots, n_r\}$  where  $\tau_{min,i}$  and  $\tau_{max,i}$  are, respectively, the lower and the upper limit of the  $i$ th joint torque.

The analytical redundancy resolution at acceleration-level is used to set up the proposed approach. The algorithm combines both an alternative control design to feedback linearization and a method for reducing the vibrations, in aim to improve the end-effector accuracy.

### 3. CONTROL DESIGN VIA REDUNDANCY RESOLUTION

The feedback linearization control approach is the combination of two imbricated control loops. First, the inner-loop is constructed to ensure the system linearization. Then, the outer-loop is designed to guarantee the control of the resulted linear system. This control strategy is one of the robust schema for the manipulator control. The free  $m$  redundant joint torques  $\tau_{r,i}$ , ( $i = 1, 2, \dots, m$ ) in the mapping (11) are used to cancel the nonlinear dynamics, unlike the traditional feedback linearization control which exploits all the manipulator torques for the same process. Analytically, that can be reached by setting

$$\vartheta_{\tau_r} \tau_r + \chi = 0 \quad (13)$$

Since the vector  $\chi$  is in  $\mathbb{R}^{n_t}$ , only  $n_t$  torques from  $m$  available redundant torques are sufficient to cancel completely the nonlinear term  $\chi$ . But if the redundant DOFs number is less than the task dimension, i.e. ( $m < n_t$ ), we can cancel  $\chi$  in only  $m$  preferential directions. Furthermore, in the cases where there are excess of redundant degrees of freedom, i.e. ( $n_t < m$ ), an infinity of the redundant torques combination can cancel completely the nonlinear term  $\chi$ . In this paper, the last case is deeply analyzed. The forward analysis have demonstrated that the additional  $(m - n_t)$  redundant degrees are useful to affect the flexural motion and eventually damp out the induced vibrations.

Considering the last case, the general solution for redundant torques that satisfies the cancellation of the non-linear dynamics  $\chi$  is given by

$$\tau_r = -\vartheta_{\tau_r}^+ \chi + (I_m - \vartheta_{\tau_r}^+ \vartheta_{\tau_r}) \tau_{\epsilon} \quad (14)$$

where  $\vartheta_{\tau_r}^+ \in \mathbb{R}^{m \times n_t}$  is the generalized inverse matrix of  $\vartheta_{\tau_r}$ , the resulting vector  $(I_m - \vartheta_{\tau_r}^+ \vartheta_{\tau_r}) \tau_{\epsilon} \in N(\vartheta_{\tau_r})$ , where  $N(\vartheta_{\tau_r})$  is the null-space of  $\vartheta_{\tau_r}$ ,  $\tau_{\epsilon} \in \mathbb{R}^m$  is an arbitrary vector,  $I_m \in \mathbb{R}^{m \times m}$  is the unit matrix.

After the linearization process has been achieved, the joint torques  $\tau_i$  and  $\tau_r$ , which guarantee the system linearization, are determined by solving the equations in (11) and (14) for a given end-effector acceleration  $\ddot{x}^d$ , providing that the matrix  $\vartheta_{\tau_i}$  is not singular. One may note that, the choice of torque sets  $\tau_i$  and  $\tau_r$  depends on various factors, such as torques magnitude limits, for more details see Ghosal (1993). After that, a conventional linear controller can be used to guarantee the end-effector tracking. In the following analysis, we investigate the possibility of the additional self-motion to reduce the deformations due to the flexural motion. The diagram of the proposed control method is presented in the Fig.1.

## 4. METHOD FOR VIBRATIONS REDUCTION

### 4.1 Analysis of the flexural motion

The presence of the arbitrary vector  $\tau_{\epsilon}$  in (14) may generate an infinite number of solutions for the redundant torques  $\tau_r$ . This self-motion ability, which doesn't influence the end-effector motion, is used to adjust the joint motions and the flexural motion.

Substituting the joints acceleration  $\dot{q}_r$  value from (2) into (3) yields to

$$M_{ff} \ddot{q}_f + D_f \dot{q}_f + K_f q_f = F_f \quad (15)$$

$$F_f = \tilde{\Gamma}_f + M_{fr} M_{rr}^{-1} \tilde{\Gamma}_r - \mu_{\tau_i} \tau_i - \mu_{\tau_r} \tau_r \quad (16)$$

where  $F_f \in \mathbb{R}^{n_f}$  is the exciting force of the flexural motion,  $\mu_{\tau_i} \in \mathbb{R}^{n_f \times n_t}$  and  $\mu_{\tau_r} \in \mathbb{R}^{n_f \times m}$  are blocks matrices of the matrix  $\mu \in \mathbb{R}^{n_f \times n_r}$ , where  $\mu = M_{fr} M_{rr}^{-1}$ .

The extended expression of the exciting force  $F_f$ , that includes the arbitrary vector  $\tau_{\epsilon}$ , is obtained by combining the two equations (14) and (16),

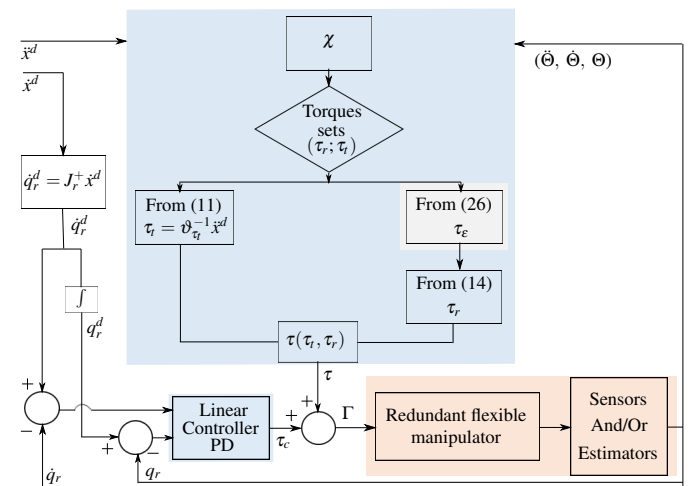


Fig. 1. Diagram of the proposed feedback control (blue zone) with vibration reduction action (gray zone) for flexible redundant manipulator (red zone). Where  $\tau(\tau_i, \tau_r)$  is a torque vector which is composed by components combination of  $\tau_i$  and  $\tau_r$ .  $\tau_c$  is the control torque for the linearized system.

$$F_f = -\mu_{\tau_r}(I - \vartheta_{\tau_r}^+ \vartheta_{\tau_r})\tau_{\varepsilon} + \tilde{\Gamma}_f + M_{f_r} M_{r_r}^{-1} \tilde{\Gamma}_r - \mu_{\tau_r} \tau_r + \mu_{\tau_r} \vartheta_{\tau_r}^+ \chi \quad (17)$$

Let's consider that the damping matrix  $D_f$  is diagonalizable, then the modal analysis of (15) will provide  $n_f$  independent equations as follows

$$\ddot{\phi}_f + D_{\phi} \dot{\phi}_f + K_{\phi} \phi_f = F_M \quad (18)$$

where  $D_{\phi} = \text{diag}(2\xi_i \omega_i) \in \mathbb{R}^{n_f \times n_f}$ ,  $K_{\phi} = \text{diag}(\omega_i^2) \in \mathbb{R}^{n_f \times n_f}$ ,  $\xi_i$  and  $\omega_i$  are the  $i$ th modal damping term and the  $i$ th natural frequency for ( $i = 1, 2, \dots, n_f$ ).  $\phi_f = M_m^{-1} q_f \in \mathbb{R}^{n_f}$  is a vector that contains the mode coordinates, where  $M_m \in \mathbb{R}^{n_f \times n_f}$  is the mode shape matrix.  $F_M = M_{\phi}^{-1} M_m^T F_f$ ,  $F_M \in \mathbb{R}^{n_f}$  is the modal generalized force, where  $M_{\phi} \in \mathbb{R}^{n_f \times n_f}$  is the modal mass matrix,  $M_{\phi} = M_m^T M_{f_f} M_m$ .

The arbitrary vector  $\tau_{\varepsilon}$  is considered as a control optimization term that can be used to damp out the flexural motion. The modal generalized force is a controllable force via self-motion and it contains two terms: nonlinear term  $F_{M,n}$  and control term  $F_{M,c}$  which depends on the arbitrary vector  $\tau_{\varepsilon}$

$$F_{M,n} = M_{\phi}^{-1} M_m^T \cdot [\tilde{\Gamma}_f + M_{f_r} M_{r_r}^{-1} \tilde{\Gamma}_r - \mu_{\tau_r} \tau_r + \mu_{\tau_r} \vartheta_{\tau_r}^+ \chi] \quad (19)$$

$$F_{M,c} = -M_{\phi}^{-1} M_m^T \mu_{\tau_r} (I - \vartheta_{\tau_r}^+ \vartheta_{\tau_r}) \tau_{\varepsilon} \quad (20)$$

#### 4.2 Flexural motion damping

To damp out the vibrations of oscillatory system, the exciting force should be eliminated, and more than this, the system damping should be increased as well. Since the self-motion can directly affect the vibration responses of the flexible redundant manipulator as shown in (20), the arbitrary vector  $\tau_{\varepsilon}$  could be chosen properly to increase the damping of the manipulator vibration modes and suppress the exciting force, while the desired end-effector position is tracked. The value of the control term  $F_{M,c}$  in the modal generalized force which satisfies the vibrations damping is given by

$$F_{M,c} = -D_{\phi}^d \dot{\phi}_f + D_{\phi} \dot{\phi}_f - F_{M,n} \quad (21)$$

substituting this chosen term into the equation (18), the flexural motion system becomes

$$\ddot{\phi}_f + D_{\phi}^d \dot{\phi}_f + K_{\phi} \phi_f = 0_{n_f} \quad (22)$$

where,  $D_{\phi}^d = \text{diag}(2\xi_i^d \omega_i) \in \mathbb{R}^{n_f \times n_f}$  is the desired modal damping matrix,  $\xi_i^d$  is the  $i$ th desired modal damping, ( $i = 1, 2, \dots, n_f$ ). In practice, the damping ratio  $D_{\phi}$  is negligible, a numerical estimator using torque/force sensors may be used to obtain the flexural reactive force. Furthermore, the numerical differential value of  $\dot{\phi}_f$ ,  $\ddot{\phi}_f$ , may be determined as well, see Hirano (2014).

The value of the arbitrary vector  $\tau_{\varepsilon}$  which satisfies the vibration reduction is the solution of the following equation and that by substituting (19) and (20) into (21)

$$A \tau_{\varepsilon} = B \quad (23)$$

where,

$$A = -M_{\phi}^{-1} M_m^T \mu_{\tau_r} (I - \vartheta_{\tau_r}^+ \vartheta_{\tau_r}) \quad (24)$$

$$B = -M_{\phi}^{-1} M_m^T \cdot [\tilde{\Gamma}_f + M_{f_r} M_{r_r}^{-1} \tilde{\Gamma}_r - \mu_{\tau_r} \tau_r + \mu_{\tau_r} \vartheta_{\tau_r}^+ \chi] + (D_{\phi} - D_{\phi}^d) \dot{\phi}_f \quad (25)$$

#### 4.3 Analysis of self-motion solution

In the relation (23), there are  $m$  unknown components of  $\tau_{\varepsilon}$  with  $n_f$  equations. The solution of this system of equations depends

on the rank ( $\rho$ ) of the matrix  $A$  and the augmented matrix  $[A \ B]$ . When the manipulator is not in singular configuration,  $\rho((I - \vartheta_{\tau_r}^+ \vartheta_{\tau_r})) = m - n_r$ . In general, the matrix  $\mu_{\tau_r}$  has the full rank,  $\rho(\mu_{\tau_r}) = \min(n_f, m)$ . Since the matrix  $M_m^T$  has the full rank,  $\rho(M_m^T \mu_{\tau_r}) = \rho(\mu_{\tau_r}) = \min(n_f, m)$ . The rank of matrix  $A$  depends on the number of both the flexural degrees  $n_f$  and the difference between the redundant degrees and the task dimension  $m - n_r$ . The first case considers ( $m - n_r$ )  $\geq n_f$ , the redundant degrees are more than the flexural DOFs and  $\rho([A \ B]) = n_f$ , there are infinity of solution  $\tau_{\varepsilon}$  that can damp out the vibration effectively. The second case considers ( $m - n_r$ )  $< n_f$ , either  $m < n_f$  or  $m > n_f$ , we have  $\rho(A) = m - n_r$ , if  $\rho([A \ B]) > \rho(A)$  there are no solutions to the system of equations. But, on the other hand, there are an infinity of solutions that minimize  $\|A \tau_{\varepsilon} - B\|_2$  when  $\rho([A \ B]) = \rho(A)$ , the general solution  $\tau_{\varepsilon}^*$  is given by

$$\tau_{\varepsilon}^* = A^+ B + (I - A^+ A) \tau_{\varepsilon,0} \quad (26)$$

where  $\tau_{\varepsilon,0} \in \mathbb{R}^m$  is arbitrary vector,  $A^+ \in \mathbb{R}^{m \times n_f}$  is the generalized inverse matrix of  $A$ . The self-motion solution  $\tau_{\varepsilon}^*$  can damp out effectively the flexural motion. The equation (26) shows that the solution of  $\tau_{\varepsilon}^*$  still depends on another arbitrary vector  $\tau_{\varepsilon,0}$ , which means that there are many solutions that satisfy the vibration reduction. Thus, the arbitrary vector  $\tau_{\varepsilon,0}$  may be exploited to design secondary task for ensuring additional performance criteria. In this case, generally, the additional ability can be used to avoid singularities or obstacles, on promise of accuracy improvement for the flexible redundant manipulator.

## 5. SIMULATIONS AND DISCUSSION

To test and demonstrate the effectiveness of control strategy presented in this paper, 5R planar flexible manipulator is used in numerical simulations, see Fig. 2. The first 4 links are supposed to be rigid and the last link is flexible. Since the planar manipulator has five rigid DOFs  $n_r = 5$  and  $n_t = 2$  (since the end-effector rotation is considered to be free), three degrees of redundancy are available,  $m = 3$ . The motion of the end-effector is expressed in the origin reference frame (XY). In practice, the first mode has the most contribution to the flexural motion than the other vibration modes. In the following analysis, only the two first vibration modes of the flexural motion are retained,  $n_f = 2$ . The parameters of 5R planar manipulator are given as follows: the length of each 5 links is 1m. The rigid links have

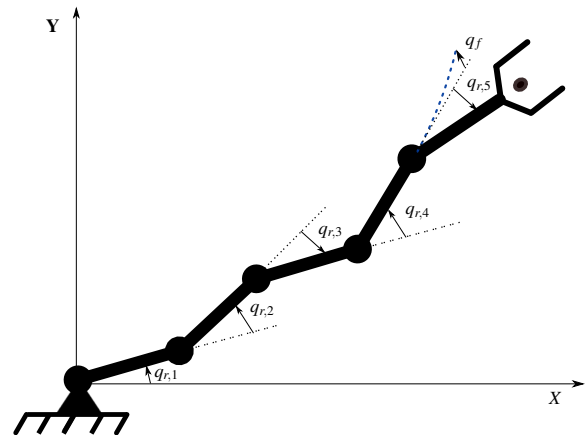


Fig. 2. Redundant 5R planar flexible manipulator.

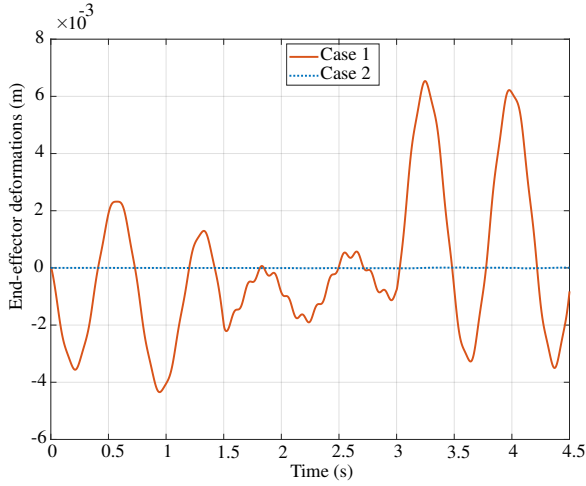


Fig. 3. End-effector deformations for both cases.

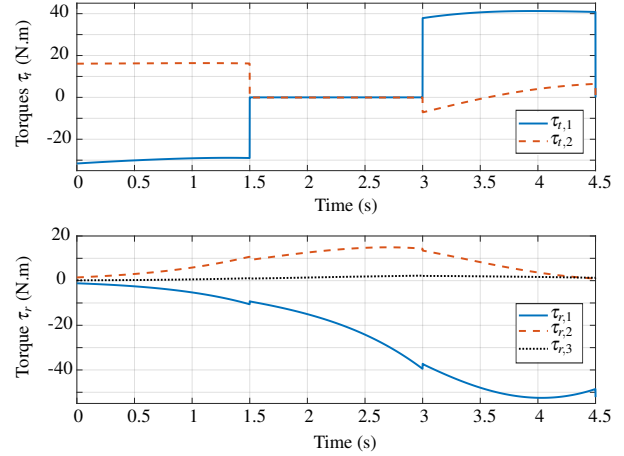


Fig. 5. Control torque profiles- case 2.

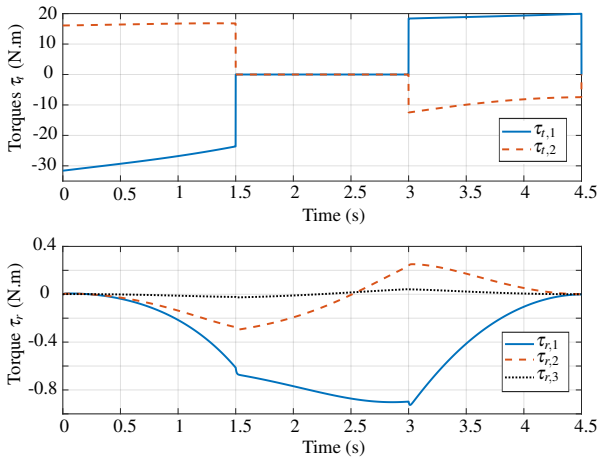


Fig. 4. Control torque profiles- case 1.

square cross-section of  $0.00045m^2$  and density of  $8000Kg/m^3$ . The flexible link has square cross-section of  $0.000225m^2$  with elastic modulus of  $69GPa$  and density of  $2700Kg/m^3$ . All terms due to gravity in the dynamical equations are null as long as the manipulator moves in a horizontal plane (XY). Without considering any external forces, only Coriolis force is retained in the non-linear term,  $\tilde{\Gamma}_r = C(\Theta, \dot{\Theta})$ . In this example, we set the two first actuators as task torques  $\tau_{r,1} = \tau_1$  and  $\tau_{r,2} = \tau_2$ . The last 3 torques are considered to be the redundant torques ( $\tau_{r,1} = \tau_3$ ,  $\tau_{r,2} = \tau_4$ ,  $\tau_{r,3} = \tau_5$ ). In addition, no limits on the joint torques have been taken into account. The initial joint angles are  $q_r(0) = [80 \ -145 \ 120 \ -60 \ -30]^T$  degrees, with null initial velocities  $\dot{q}_r(0) = [0 \ 0 \ 0 \ 0 \ 0]^T rad/s$ . The desired velocity of the end-effector is set to be null at the beginning and the end of the motion. The end-effector was controlled to follow a desired accelerations which has Bang-bang profile as follows

$$\ddot{x}_d(t) = \begin{cases} [0.1 \ 0.45]^T m/s^2 & 0s \leq t \leq 1.5s \\ [0 \ 0]^T m/s^2 & 1.5s < t \leq 3s \\ [-0.1 \ -0.45]^T m/s^2 & 3s < t \leq 4.5s \end{cases} \quad (27)$$

The results have shown the responses of the system to the proposed control approach for the two cases, i.e. with and

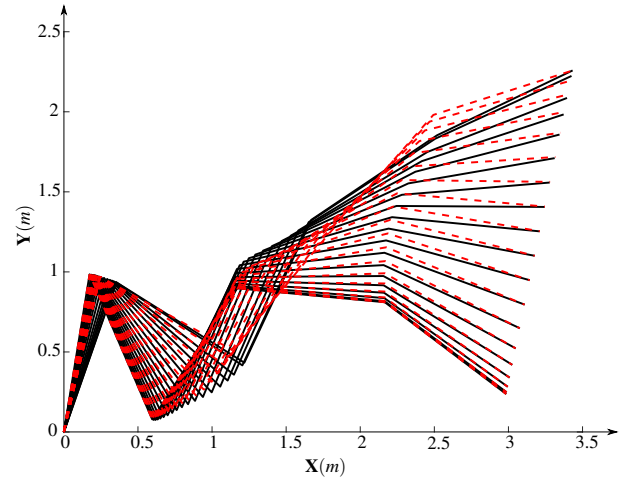


Fig. 6. Motion configurations of the planar manipulator (solid black line: case1, dashed red line: case2).

without the utilization of torques self-motion to damp out the induced vibration.

*Case 1 ( $\tau_e$  is null)* : In this case, the self-motion is only calculated to cancel the system non-linearity in (13). It was possible to apply the proposed control algorithm because the degree of the redundancy is greater than the work-space dimension. Besides, a conventional PD controller at joint level is designed and then applied to the resulting linear system for maintaining the end-effector position tracking. On the other hand, it is shown in Fig. 3 that the end-effector deformations are non-negligible, since the end-effector vibrations have not been reduced. The end-effector follows the desired trajectory with vibrations of maximum amplitude equals to  $6.6mm$ .

*Case 2 ( $\tau_e$  is not null)* : In this case,  $\tau_e$  is calculated from (26). Taking into account the relation (26), the self-motion is recalculated to get rid of the non-linearity, and to damp out the vibration modes of the resulting flexural motion simultaneously. One may note that, the torques magnitude of  $\tau_r$  may become very large as the end-effector approaches singularity. As shown in Fig. 3, the deformations of the end-effector are well reduced via self-motion. Compared the Fig.4 with Fig.5, it can be clearly seen that the self-motion has been changed to

take into account the additional task of reducing the vibration deformations, see Fig. 6. The end-effector follows the desired trajectory with small deformations (around 0.02mm of maximum amplitude).

The torque profiles are shown in Fig. 4 and Fig. 5. The actuator torques of the two first actuators ( $\tau_1$  and  $\tau_2$ ) are significantly larger than the other actuator torques. This depends on which actuator torques are chosen as  $\tau_i$  or  $\tau_r$ . Therefore, it is preferably to have a large limit for the first two torques to develop a sufficient energy during the manipulator motion. The resulted torques in actuators  $\tau_3$ ,  $\tau_4$  and  $\tau_5$  in Fig. 4 are slightly different to those in Fig. 5. This is due to the changing in the self-motion solution in purpose to damp out the vibrations.

The real time implementation of the proposed algorithm is feasible. Since the robot state such as joint coordinates are generally measurable via implemented sensors. The main challenge is to measure the flexural motion states. Nevertheless, that may be resolved using information derived from torque/force sensors to estimate both the flexural reactive force and the modal coordinates following the estimation approach in Hirano (2014). Thus, the discretization and the synchronization problem of data is avoided. Afterwards, the robot model can be simultaneously linearized since the calculation can be factorized.

In the all previous methods presented in Gao (2017); Nguyen (1992); Bian (2011); Hirano (2014); Sasaki (2007), the redundancy resolution is used for vibration reduction only and none see the advantage of having more redundant degrees specially in case of the hyper-redundant flexible manipulator. Although, we proposed algorithm that takes advantage of all redundant DOFs, first for controlling the manipulator (feedback linearisation via redundancy resolution), and secondly to suppress the induced vibrations of the flexible links. Moreover, no model simplifications are needed for control design which ensures the reliability of the proposed algorithm. The vibration behaviour is well treated compared to the previous articles, through the analysis, one can see clearly how the vibration are damped. The simulation result shows the effectiveness of the proposed approach.

## 6. CONCLUSION

In this paper, the self-motions of redundant flexible manipulators are exploited for both designing a control algorithm and establishing a novel method for vibrations reduction of its end-effector while tracking a given trajectory. The effectiveness of this approach has been validated by the results of numerical simulations. It has been shown that the self-motions have cancelled the undesirable dynamics, besides, through the dynamic coupling between the rigid and the flexural motions the vibrations of the end-effector have been reduced as well. The analysis of the self-motions satisfying the vibration reduction revealed an additional ability which may be used on promise of improving the kinematic and/or dynamic performances of flexible redundant manipulators.

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## REFERENCES

- Xinyu, wang, et al. (2015). Obstacle avoidance for kinematically redundant robot. *IFAC-PapersOnLine*, 48.28, 490–495.
- Khatib, Oussama. (1987). A unified approach for motion and force control of robot manipulators: The operational space formulation. *IEEE Journal on Robotics and Automation*, 3.53, 43–53.
- Benosman, Mouhacine and Le Vey, G. (2004). Control of flexible manipulators: A survey. *Robotica*, 22.5, 533–545.
- Sahinkaya, MN. (2001). Input shaping for vibration-free positioning of flexible systems. *Journal of Systems and Control Engineering*, 215.5, 467–481.
- Biagiotti, Luigi, et al. (2015). Optimal trajectories for vibration reduction based on exponential filters. *IEEE Transactions on Control Systems Technology*, 24.2, 609–622.
- Béarée, Richard. (2014). New Damped-Jerk trajectory for vibration reduction. *Control Engineering Practice*, 28, 112–120.
- Nakanishi, Jun, et al. (2008). Operational space control: A theoretical and empirical comparison. *The International Journal of Robotics Research*, 27.6, 737–757.
- Sadeghian, Hamid, et al. (2013). Dynamic multi-priority control in redundant robotic systems. *Robotica*, 31.7, 1155–1167.
- Yue, Shigang, et al. (2002). Point-to-point trajectory planning of flexible redundant robot manipulators using genetic algorithms. *Robotica*, 20.3, 269–280.
- Gao, Xin, et al. (2017). Research on Joint Torque Optimization Method of Redundant Space Manipulators with Vibration Suppression. *ITM Web of Conferences*, 12, 1004.
- Nguyen, Luong A, et al. (1992). Dynamic control of flexible, kinematically redundant robot manipulators. *IEEE Transactions on Robotics and Automation*, 8.6, 759–767.
- Bian, Yushu, et al. (2011). Study on vibration reduction and mobility improvement for the flexible manipulator via redundancy resolution. *Nonlinear Dynamics*, 65.4, 359–368.
- Hirano, Daichi, et al. (2014). Simultaneous control for endpoint motion and vibration suppression of a space robot based on simple dynamic model. *2014 IEEE International Conference on Robotics and Automation (ICRA)*, 6631–6637.
- Sasaki, Koji and Murakami, Toshiyuki. (2007). Vibration suppression control of redundant manipulator with flexible structure by considering nullspace motion. *IEEE Transactions on Industry Applications*, 127, 1043–1049.
- Hishinuma, Toshimitsu and Nanchev, Dragomir N. (2006). Singularity-consistent vibration suppression control with a redundant manipulator mounted on a flexible base. *2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 3237–3242.
- Bian, Yushu and Gao, Zhihui. (2013). Impact vibration attenuation for a flexible robotic manipulator through transfer and dissipation of energy. *Shock and Vibration*, 24.4, 665–680.
- Ghosal, A., and Desa, S. (1993). Dynamical Resolution of Redundancy for Robot Manipulators. *Transactions of the ASME, Journal of Mechanical Design*, 115.3, 592–598.