Dynamic Pricing for Power Control in Remote State Estimation

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Abstract: This paper considers the remote state estimation with multiple sensors. Each sensor transmits its sensing data to a remote estimator over a shared channel, where simultaneous transmissions are allowed. Regrading the transmission of other sensors as interference signals, the system designer should coordinate the sensors appropriately in order to maximize the overall estimation performance. Motivated by microeconomics, we treat sensors as self-interested power buyers under different unit prices announced by the system designer. Accordingly, the strategic interactions among sensors are formulated in a non-cooperative game, upon which the existence and uniqueness of a pure equilibrium solution are proved. Even if the game admits a conflict of interests among sensors, under well-designed prices, the game outcome aligns with the global optimal solution. We also devise an algorithm to compute these prices with simple iterations, which is given in explicit forms for ease of implementation. Numerical examples are given to demonstrate the developed results.

1. INTRODUCTION

As the next generation control systems, Cyber-physical systems (CPSs) employ communication and information technologies to improve the performance of underlying physical processes, for example the robustness to unexpected disturbance. The incorporation of communication networks, however, poses a number of technical challenges in the control system design. An urgent problem is to tailor the utilization of communication resource to fit control requirements. Generally, the employment of communication networks brings CPSs two inevitable issues: first, sensing, computation and transmission power are limited as replacing or recharging batteries may not always be possible for some off-the-shelf devices (Derler et al., 2012); second, critical signal interference will be encountered in large-scale sensor networks under limited communication bandwidth. These two issues increase packet dropping and further deteriorate the estimation/control performance.

In communication theory, there are several representative methods to improve energy efficiency and alleviate signal interference, such as code division multiple access (CDMA), see (Tse and Viswanath, 2005). Differently, our work focuses on dynamic systems and emphasizes the state estimation performance. In particular, we consider a simple scenario where multiple sensors send their sensing packets to remote estimator through a shared communication channel. This simultaneous transmission may induce data collision and packet dropout. Aiming at high-quality estimation performance at the remote estimator, we look for an efficient allocation of restricted transmission power among these sensors.

A considerable amount of literature has been published on sensor transmission scheduling to achieve accurate estimation under limited energy constraints (Ren et al., 2014, Shi et al., 2011). However, many of them focus on the one-sensor case and model the sensor scheduling as a Markov decision process (MDP) problem. This framework becomes difficult when taking multiple sensors into account due to curse of dimensionality. To avoid signal interference, some preliminary works considering multi-sensor structure only allow one sensor to transmit packet at each time step, e.g., periodic selections of sensors investigated in (Orihuela et al., 2014, Zhao et al., 2014). In contrast with these time-based scheduling, (Gatsis et al., 2015) selected active sensor according to the current channel condition, which is also termed as state-based scheduling. However, these approaches are inapplicable in our problem as at each time step we allow all sensors to send data but with different packet-dropout rates. Moreover, to compute the optimal period or state-based decision condition, the system designer requires a complete knowledge of the system, especially the dynamics parameter or the current state of every sensor. This is nonscalable and too computational costly in large-scale network, and the information acquisition may also violate the privacy of each sensor.

To relieve these troubles, we resort to a distributed solution to our remote estimation problem, which has been studied in the context of game theory. The work (Li et al., 2014) treated each sensor as a self-interested decision maker with estimation performance being its utility, and a noncooperative game among sensors was developed. The distributed power allocation is a Nash equilibrium (NE) solution of the game, which induced an inefficient outcome compared with the centralized solution of the global power-allocation problem. To improve the performance of each player at an equilibrium, a correlated policy, along with the notation of correlation equilibrium (CE) was proposed in (Ding et al., 2017), which still failed to achieve the global optimal performance. An important issue which has not been taken seriously in previous works is that the design of local sensor utilities can be effective tool to reduce equilibrium inefficiency. Motivated by this, here we use system designer to publish a price profile of transmission power for each sensor, under which a power cost is added into the utility of each sensor. Under a well-designed price profile, we found that the resulting game equilibrium achieves the global optimum. Compared with previous works, the main contributions of our work are summarized as follows:

(1) We establish an equivalence between the global power allocation problem in Definition 3.1 and the non-cooperative game played among sensors in Definition 3.2 by the aid of price concept. Moreover, we validate the existence and uniqueness of a pure NE for the game (Theorem 1), and devise a distributed algorithm (Algorithm 1) with a recursive form to design the price profile.

(2) The estimation performance (or utility) of the sensor is heavily coupled with each other since its packet transmission depends on the interference signal launched by others. Hence, the global objective function ($J(\cdot)$ in Definition 3.1) can not be separated directly. By investigating its structure, we build a matrix-form relation between the price profile and Lagrangian multipliers. It is different from most price-based works using primal-dual decomposition (Palomar and Chiang, 2006), in which the prices are exactly the same as Lagrangian multipliers.

Outline. Mathematical models of the system are described in Section 2. We introduce the global power-allocation problem and the multi-player non-cooperative game in Section 3, and conclude it with the problems of interest. In Section 4, we demonstrates the main theoretical results and propose the pricing algorithm. Simulation results are shown in Section 5.

Notations. $\mathbb{N}$ is the set of positive integers. k is the time index. For functions $f, f_1, f_2 : \mathbb{S}^n_+ \to \mathbb{S}^n_+$, $f_1 \circ f_2$ is defined as $f_1 \circ f_2(X) \triangleq f_1\big(f_2(X)\big)$. $\mathbb{1}(\cdot)$ is the indicator function. $\Delta(\cdot)$ represents a set of probability measures and "w.r.t." means "with respect to".

2. MODEL SETUP

As depicted in Fig. 1, the state information of different processes is sent to a remote estimator through one shared channel. The simultaneous data transmission may lead to network collision and packet dropout, which further deteriorate remote estimation performance.

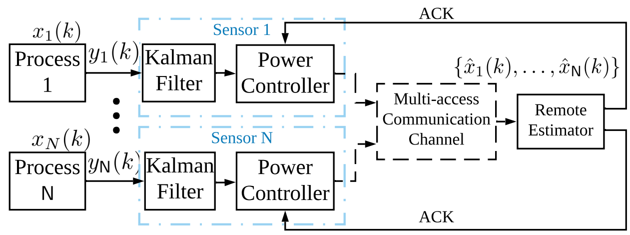


Fig. 1. System Architecture.

2.1 Local Kalman Filter

Consider a networked system containing N sensors, which separately monitor different processes: for $i \in \{1, \ldots, \mathsf{N}\}$ $x_i(k+1) = A_i x_i(k) + w_i(k),\ y_i(k) = C_i x_i(k) + v_i(k)$, where at time k, the state vector of process i is $x_i(k) \in \mathbb{R}^{n_x}$, and the noisy measurement observed by corresponding sensor i is $y_i(k) \in \mathbb{R}^{m_y}$. For each process i, the process noise $w_i(k) \in \mathbb{R}^{n_x}$ and the measurement noise $v_i(k) \in \mathbb{R}^{m_y}$ are zero-mean i.i.d. Gaussian random variables with variances $\mathbb{E}[w_i(k)w_i(j)^\top] = \delta_{kj} Q_i$ with $Q_i \geq 0$, $\mathbb{E}[v_i(k)v_i(j)^\top] = \delta_{kj} R_i$ with $R_i > 0$, and covariance $\mathbb{E}[w_i(k)v_i(j)^\top] = 0,\ \forall j, k$. The initial state $x_i(0)$ is a zero-mean Gaussian random vector with variance $\Sigma_i(0) \geq 0$, and it is uncorrelated with $w_i(k)$ and $v_i(k)$. For each process, we assume the time-invariant pair (A_i, C_i) is detectable and pair $(A_i, \sqrt{Q_i})$ is stabilizable. Moreover, we use sensor i and process i interchangeably hereafter without ambiguity. Here, "smart" sensor i computes the optimal estimate of state $x_i(k)$ via running a Kalman filter locally. The obtained minimum mean-squared error (MMSE) estimate of state $x_i(k)$ is given by $\hat{x}^s_i(k) = \mathbb{E}[x_i(k)|y_i(1), \ldots, y_i(k)]$, with the estimation error covariance defined as $P^s_i(k) \triangleq \mathbb{E}[(x_i(k) - \hat{x}^s_i(k))(x_i(k) - \hat{x}^s_i(k))^\top|y_i(1), \ldots, y_i(k)]$. These terms are computed recursively following the standard Kalman filter equations, and the iteration starts from $\hat{x}^s_i(0) = 0$ and $P^s_i(0) = \Sigma_i(0)$. For notational simplicity, we define the Lyapunov and Riccati operators $\mathfrak{h}_i(\cdot)$ and $\mathfrak{g}_i(\cdot) : \mathbb{S}^{n_x}_+ \to \mathbb{S}^{n_x}_+$ as $\mathfrak{h}_i(X) \triangleq A_i X A_i^\top + Q_i,\ \mathfrak{g}_i(X) \triangleq X - XC_i^\top[C_i X C_i^\top + R_i]^{-1} C_i X$. Due to the detectability of pair (A, C) and stability of $(A, \sqrt{Q})$, the estimation error covariance P^s_k converges exponentially to an unique fixed point $\overline{P}_i$ of $\mathfrak{g}_i \circ \mathfrak{h}_i$ (Anderson and Moore, 1979). Without loss of generality, we ignore the transient periods and assume that the Kalman filter at each sensor has entered steady state; i.e.,

$$P^s_i(k) = \overline{P}_i,\ k \geq 1. \tag{1}$$

As mentioned in (Ding et al., 2017), the steady-state error covariance $\overline{P}_i$ satisfies the monotonic property:

$$\mathtt{Tr}[\overline{P}_i] \leq \mathtt{Tr}[\mathfrak{h}^{t_1}_i(\overline{P}_i)] < \mathtt{Tr}[\mathfrak{h}^{t_2}_i(\overline{P}_i)], \text{ for } 0 \leq t_1 < t_2. \tag{2}$$

2.2 Communication Interference

When sensor i transmits its local estimate $\hat{x}^s_i(k)$ as a data packet to the remote estimator, the communication channel may be occupied by other sensors. This simultaneous transmission may directly affect the information delivery of sensor i. In this work, we assume the shared channel has independent Additive White Gaussian Noise (AWGN), and measure the channel quality for sensor i by the signal-to-interference-and-noise-ratio (SINR) (Tse and

Viswanath, 2005), in which the signals of other sensors are modeled as interfering noises. For sensor $i \in \{1, \ldots, \mathsf{N}\}$, its SINR at time k is defined as:

$$\gamma_i(k) = L \frac{h_i a_i(k)}{\sum\limits_{j \in \{1,\ldots,\mathsf{N}\} \setminus \{i\}} h_j a_j(k) + \sigma^2} \tag{3}$$

in which $a_i(k) \geq 0$ represents the transmission power taken by sensor i. The extra term $\sum\limits_{j \neq i} h_j a_j(k)$ in the denominator of (3) is due to the interference from other sensors, and σ^2 is the channel noise. The parameter $h_i \in (0,1), \forall i \in \{1, \ldots, \mathsf{N}\}$ is the channel gain from sensor i to the remote estimator, and L is the spreading gain of the communication system. Moreover, we characterize the packet-dropout for sensor i by an independent Bernoulli process, denoted by $\eta_i(k)$. Let $\eta_i(k) = 0$ denotes the loss of packet $\hat{x}_i^s(k)$, and $\eta_i(k) = 1$ otherwise. The packet-dropout rate for sensor i depends on its SINR and is defined using a general, continuous, secondly differentiable function as follows: $\mathtt{Pr}(\eta_i(k) = 0) = f(\gamma_i)$, $\forall i \in \{1, \cdots, N\}$, where in general $f(\cdot)$ decreases in γ_i and $f(\gamma_i = 0) = 1$. Its specific formula depends on the channel characteristic and the modulation schemes.

2.3 Remote State Estimation

In regard to process i, let $\hat{x}_i(k)$ denote the MMSE estimate of its state $x_i(k)$ generated by the remote estimator, with the error covariance matrix $P_i(k)$. Similar to (Ding et al., 2017), for each sensor, we define a random variable $\tau_i(k) \in \mathbb{Z}$ as the holding time:

$$\tau_i(k) \triangleq k - \max_{0 \leq l \leq k} \{l : \eta_i(l) = 1\}, \tag{4}$$

which represents the intervals between the present moment k and the most recent time when the data packet $\hat{x}_i^s(k)$ received by the remote estimator successfully. Accordingly, $P_i(k) = \mathfrak{h}_i^{\tau_i(k)}(\overline{P}_i)$, and the iteration of the holding time $\tau_i(k)$ is given by

$$\tau_i(k+1) = (1 - \eta_i(k))(\tau_i(k) + 1). \tag{5}$$

In summary, at time k, under a transmission power profile $\mathbf{a} \triangleq \{a_1, \cdots, a_{\mathsf{N}}\}$ employed by all sensors, the estimation performance for process i is defined by its expected error covariance at time $k+1$:

$$u_i(\mathbf{a}, k) \triangleq -\mathtt{Tr}\{\mathbb{E}[P_i(k+1)]\} = f(\gamma_i) c_i(k) - \mathtt{Tr}[\overline{P}_i], \tag{6}$$

in which $c_i(k) \triangleq \mathtt{Tr}[\overline{P}_i - \mathfrak{h}_i^{\tau_i(k)+1}(\overline{P}_i)]$ is independent of γ_i (or action profile $\mathbf{a}$), and $c_i(k) < 0$ derived from (2). Hence, function $u_i(\mathbf{a}, k)$ is partially increasing in a_i and decreasing in $a_j, \forall j \neq i$. It verifies the intuition that larger inference signals caused by other sensors may deteriorate the estimation performance for process i more greatly.

3. PROBLEM FORMULATION

With the consideration of signal interference introduced above, system designer should allocate the transmission power among sensors wisely in order to maximize the total estimation performance. In this section, we first formulate system designer goal with a constrained optimization problem and propose a pricing mechanism to solve it. Under this, the sensors are treated as selfish players and we elaborate this multi-sensor game afterwards. Last, we summarize this section with two key questions of this work.

3.1 System Designer Goal

We adopt a slotted medium access control (MAC) protocol, where the operation of system is divided into equal time slots. Then, for each time k system designer tries to obtain a proper power allocation by solving the following problem.

Definition 3.1. (Global System Designer Goal). For each time k, system designer copes with the following constrained optimization problem:

$$\max_{\mathbf{a}} \ J = \sum_{i=1}^{\mathsf{N}} u_i(\mathbf{a}, k) \tag{7}$$

$$s.t. \ \sum_{i=1}^{\mathsf{N}} a_i \leq \Theta, \tag{8}$$

$$a_i \geq \theta_i > 0, \forall i \in \mathcal{R}, \tag{9}$$

where the optimal total/global estimation performance is denoted by $J^\star$ with the optimal power allocation $\mathbf{a}^\star$.

More specifically, the constraint (8) suggests that the total power assignment for packet transmission should falls below an upper-bound, denoted by Θ. The constraint (9) shows that the power for each sensor should be positive in order to establish a data transmission connection. Since we focus on solving (7) for each time slot, in the rest of this paper, we will omit the variable k of $\tau_i(k)$, $\gamma_i(k)$, $u_i(\mathbf{a}, k)$ and $c_i(k)$ when the underlying time index k is obvious from the context; otherwise, it will be indicated.

If the feasibility of the optimization problem in Definition 3.1 is guaranteed, the existence of an optimal solution $\mathbf{a}^\star$ is obvious since the constraint is compact and the objective function is continuous. Nevertheless, to solve this problem, the designer requires a complete information of the networked system, for example, the model parameters of the processes, channel parameters and especially the online information, the instantaneous holding times. This not only increases the requirement of storage space, but also the information acquisition may invade the privacy of sensors. Furthermore, note that the estimation performance for sensor i contains a ratio of two linear functions on variable $\mathbf{a}$ (i.e., the SINR term γ_i), and the objective function (7) is a sum of functions of ratios. Solving this optimization problem is, however, NP-hard (Freund and Jarre, 2001). Last, in order to provide a real-time power control service for the multi-sensor system, system designer should resort to distributed optimization techniques instead of solving problem in Definition 3.1 directly.

To cope with above concerns, we motivated by economics (Mas-Colell et al., 1995) propose a pricing mechanism capturing the cost on power usage for each sensor. More specifically, the designer announces the price of transmission power, and the sensors will be charged for a fee depending on its power actions. See Figure 2 for an illustration of the pricing mechanism. Note that each sensor is charged a personalized per-unit price $p_i \geq 0$ for the power resource, and we assume sensors to be price-takers. Hence, informed of the price, each sensor acting as self-interested buyers, will select the best power actions to maximize their benefits, and at last a spontaneous power allocation among sensors is developed.

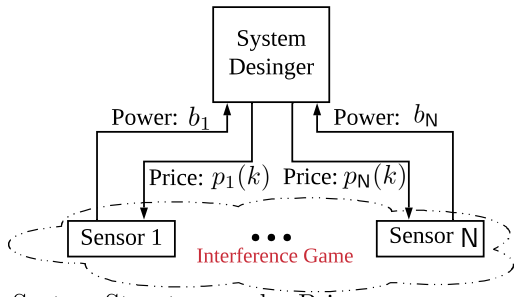


Fig. 2. System Structure under Prices.

3.2 Interference Game

Under the announced price, the sensors will compete with each other for the power resources. Denote by $\mathcal{G}$ the game among sensors and it is characterized by a triplet $< \mathcal{I}, \mathcal{B}, \mathcal{R} >$ with
Players: $\mathcal{I} = \{1, \ldots, \mathsf{N}\}$ is the set of (rational) players, in which $i \in \mathcal{I}$ represents sensor i.
Actions: $\mathcal{B} = \{\mathcal{B}_i, i \in \mathcal{I}\}$ illustrates the set of power actions for each player i with $\mathcal{B}_i = [\theta_i, \Theta]$. Let $b_i \in \mathcal{B}_i$ denote the power action (or pure strategy) taken by player i. Moreover, define $\mathbf{b} = \{b_1, \ldots, b_{\mathsf{N}}\}$ as the action profile played by the overall players. Alternatively, $\mathbf{b} = \{b_i, \mathbf{b}_{-i}\}$, with $\mathbf{b}_{-i}$ the action profile excluding that of player i.
Reward: $\mathcal{R} = \{r_i, i \in \mathcal{I}\}$ is the reward set and r_i represents the reward function for player i with $r_i : \mathcal{B} \to \mathbb{R}$. As discussed previously, each sensor focuses on improving its own estimation accuracy $u_i(\cdot)$ under a charge incurred by packet transmission. It motivates us to formulate the reward function using a concept from microeconomics (Mas-Colell et al., 1995), the *quasilinear utility model*: $r_i(b_i, \mathbf{b}_{-i}) = u_i(\mathbf{b}) - p_i \ln(b_i)$, in which different from conventional linear cost function, the charge is logarithmic in power.

Definition 3.2. (Interference Game). In the multi-sensor game $\mathcal{G}$, each player deals with:

$$\max_{b_i \in \mathcal{B}_i} \ r_i(b_i, \mathbf{b}_{-i}), \quad \forall i \in \mathcal{I}. \tag{10}$$

3.3 Problem of Interest

With the purpose to solve the global optimization problem in Definition 3.1, we introduce the pricing method to coordinate sensors. Under the announced prices, each sensor will make the best decision reacting to the behaviors of others. Hereinafter we seek to answer the two questions: given the prices for each sensor, does there exist a stable solution to the multi-sensor game in Definition 3.2? If so, how to design the prices such that the game outcome $\mathbf{b}$ is in concordance with the global optimal solution $\mathbf{a}\star$.

4. MAIN RESULT

4.1 Best-response Update

In game theory, a common solution concept is Nash equilibrium (NE). For this game $\mathcal{G}$ in Definition 3.2, the action profile $\mathbf{b}^\star = \{b_1^\star, \ldots, b_N^\star\}$ is a (pure) NE if for any player $i \in \mathcal{I}$, $r_i(\mathbf{b} = \mathbf{b}^\star) \geq r_i(\{b_i = b, \mathbf{b}_{-i} = \mathbf{b}^\star_{-i}\})$, $\forall b \in \mathcal{B}_i$. Based on this, we restate the representation of NE in terms of best-response correspondences. For any $\mathbf{b}_{-i} \in \mathcal{B}_{-i}$, the best-response correspondence of player i is given by $\phi_i(\mathbf{b}_{-i}) \triangleq \{b \in \mathcal{B}_i | r_i(b, \mathbf{b}_{-i}) \geq r_i(b', \mathbf{b}_{-i}) \ \forall b' \in \mathcal{B}_i\}$. Consequently, the NE $\mathbf{b}^\star \triangleq \{b_i^\star, b_{-i}^\star\}$ can be reformulated as follows: $b_i^\star \in \phi_i(\mathbf{b}^\star_{-i})$, $\forall i \in \mathcal{I}$. We define the joint correspondence $\Phi : \mathcal{B} \rightrightarrows \mathcal{B}$ such that for all $\mathbf{b} \in \mathcal{B}$, $\Phi(\mathbf{b}) = [\phi_i(\mathbf{b}_{-i})]_{i \in \mathcal{I}}$. The NE is then equivalent to a fixed point of the correspondence $\Phi(\cdot)$.

To prove the existence of NE, we introduce an assumption on the packet drop-out rate $f(\cdot)$, that is, $f(e^x)$ is convex in x. This assumption is milder than that $f(\gamma)$ is convex in γ, e.g., $f(\gamma) = \ln(\gamma) = x$ is convex in x but concave in γ. From communication theory, the packet dropout rate for wireless fast fading channel is $f(\gamma) \propto \gamma^{-c_0}$ with constant $c_0 > 0$, which satisfies the assumption. Recall that $f(\cdot)$ is continuous and secondly differentiable. Based on the form of reward function $r_i(\cdot)$, we can obtain the explicit representation of the best-response for sensor i, given by,

$$\phi_i(\mathbf{b}_{-i}) = \min\{\max\{b_0, \theta_i\}, \Theta\} \tag{11}$$

where $b_0 = \frac{\sum_{j \neq i} h_j b_j + \sigma^2}{L h_i} (\tilde{f})^{-1}(\frac{p_i}{c_i})$ with $\tilde{f}(\gamma) = f'(\gamma)\gamma$. Hence, for any $\mathbf{b} \in \mathcal{B}$, the correspondence Θ only contains one element, and we summarize the NE result:

Theorem 1. The game $\mathcal{G}$ admits an unique pure NE, of which each sensor maximizes its reward by taking power action $b_i^\star = \phi_i(\mathbf{b}^\star_{-i})$ for all $i \in \mathcal{I}$.

Proof. To prove the existence of a NE, we resort to Kakutani's fixed point theorem and verify its conditions orderly. The uniqueness of pure NE is proved as the correspondence $\Phi(\cdot)$ contains one element. We omit the proof details here due to limited space. ■

Note that $f(e^x)$ is convex in x and we can prove that $\tilde{f}^{-1}(\cdot)$ is a non-decreasing function. As $c_i < 0$ and $p_i \geq 0$, we can further conclude that for each sensor a higher price p_i will induce a less purchase of transmission power, which meets the market phenomenon. Consider a special scenario with $p_i = 0$. The corresponding NE leads to an inefficient outcome, called "tragedy of the commons", in which the optimal response for each sensor is to transmit its packet at it maximum power level Θ. This situation is less desirable because each sensor suffers the worst signal interference. To improve the power efficiency and also achieve satisfactory quality of estimation, the price p_i should be designed carefully by system designer, which would be addressed subsequently.

4.2 Price Design

Note that (7) is non-convex in $\mathbf{a}$ even if we assume the packet dropout rate $f(\gamma_i)$ is convex in a_i. Here, we first resort to variable transformation to "convexify" the global problem in Definition 3.1. By applying the strictly increasing and differentiable transformation $a_i = e^{s_i}, \forall i \in \mathcal{I}$, we obtain the transformed global problem as follows:

$$\max_{\mathbf{s}} \ J = \sum_{i=1}^{\mathsf{N}} u_i(\mathbf{s}) \ s.t. \ \sum_{i=1}^{\mathsf{N}} e^{s_i} \leq \Theta, \ s_i \geq \ln(\theta_i), \ \ \forall i \in \mathcal{I},$$

where $\mathbf{s} \triangleq \{s_1, \ldots, s_{\mathsf{N}}\}$. It is sufficient to prove that the object function is concave in s-domain and the feasible set is convex and details are omitted here. Then, we involve new variables $\mathbf{t} = \{t_1, \ldots, t_{\mathsf{N}}\}$ and add new constraints that $t_i = s_i, \forall i \in \mathcal{I}$. Regarding these equality constraints, denote the Lagrangian multiplier as $q_i, i \in \mathcal{I}$ and the Lagrangian function is given by $L_1(\mathbf{s}, \mathbf{t}, \mathbf{q}) = \sum_{i=1}^{\mathsf{N}} u_i(\mathbf{s}) + q_i(t_i - s_i)$. Accordingly, the transformed global optimization problem can be decoupled into two problems in terms of variables $\mathbf{s}$ and $\mathbf{t}$.

Definition 4.1. (Decoupled Problems). The global optimization problem is decoupled into:

$$\max_{\mathbf{t}} \ J_1(\mathbf{t}) \triangleq \sum_{i=1}^{\mathsf{N}} q_i t_i \ \ s.t. \ \sum_{i=1}^{\mathsf{N}} e^{t_i} \leq \Theta. \tag{12}$$

$$\max_{\mathbf{s}, s_i \in [\theta_i, \Theta]} \ J_2(\mathbf{s}) \triangleq \sum_{i=1}^{\mathsf{N}} u_i(\mathbf{s}) - q_i s_i. \tag{13}$$

Note that function $J_1(\mathbf{t})$ in (12) can be regarded as the benefit for system designer by "selling" power resources to sensors and q_i is the per-unit price. The solution of (12) is summarized as follows:

Proposition 4.2. Problem (12) admits an unique optimal solution $\mathbf{t}^\star$ with a closed form given by,

$$t_i^\star = \ln\left(\frac{q_i}{\sum_{i=1}^{\mathsf{N}} q_i}\Theta\right), \ \ \forall in \in \mathcal{I}. \tag{14}$$

It follows the intuition that system designer aiming to gain more monetary benefit would allocate more power to the sensor providing a higher per-unit price. The proof details are omitted here.

On the other hand, the problem (13) cannot be further decoupled into N sub-problems (alike to the game in Definition 3.2) via simply setting $p_i = q_i$. Because the reward of sensor j will be affected by the action of sensor i and the reward function $u_i(\mathbf{s})$ are coupled with each other. However, there exists a decoupling in the fractional structure of SINR, e.g., the numerator of γ_i only depends on b_i (or s_i) and the denominator is a function of $\mathbf{b}_{-i}$. By aid of this, we hope to develop a relationship between multiplier q_i and price p_i. Under a pair of parameters $(p_i, q_i, i \in \mathcal{I})$, it is desirable that the NE $\mathbf{b}^\star$ of game $\mathcal{G}$ is also the optimal solution of (13), i.e., $\mathbf{b}^\star = e^{\mathbf{s}^\star}$. Hence, we derive from the first order derivative of the reward function $\frac{\partial r_i(\mathbf{s})}{\partial s_i} = 0$ that $\frac{\partial u_i(\mathbf{s})}{\partial s_i} = c_i f'(\gamma_i)\frac{\partial \gamma_i}{\partial s_i} = \frac{\partial u_i(\mathbf{s})}{\partial \gamma_i}\gamma_i = p_i$. Also, we compute the first order derivative of objective function $J_2(\mathbf{s})$, given by

$$\begin{aligned}\frac{\partial J_2(\mathbf{s})}{\partial s_i} &= \frac{\partial u_i(\mathbf{s})}{\partial s_i} + \sum_{j \neq i} \frac{\partial u_j(\mathbf{s})}{\partial s_i} - q_i \\ &= p_i - \sum_{j \neq i} \frac{h_i b_i}{I(\mathbf{b}) - h_j b_j} p_j - q_i = 0, \ \ \forall i \in \mathcal{I},\end{aligned}$$

in which $I(\mathbf{b}) \triangleq \sum_{i=1}^{\mathsf{N}} h_i b_i + \sigma^2$. The above equations can be written in a matrix form:

$$\begin{bmatrix} 1 & -\frac{h_1 b_1}{I(\mathbf{b}) - h_2 b_2} & \cdots & -\frac{h_1 b_1}{I(\mathbf{b}) - h_N b_N} \\ -\frac{h_2 b_2}{I(\mathbf{b}) - h_1 b_1} & 1 & \cdots & -\frac{h_2 b_2}{I(\mathbf{b}) - h_N b_N} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{h_N b_N}{I(\mathbf{b}) - h_1 b_1} & -\frac{h_N b_N}{I(\mathbf{b}) - h_2 b_2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix}$$

Denote the constructed matrix on the left hand side to be $\Pi(\mathbf{b})$, and then we have $\mathbf{p} = \Pi^{-1}(\mathbf{b})\mathbf{q}$.

In summary, we start with the global problem in Definition 3.1 and decompose it into two sub-problems, which can be

Algorithm 1 Pricing Design and Optimal Power Allocation

Input: Channel parameters h_i, L and σ^2, power constraints θ_i and Θ.
Output: Optimal power allocation $\mathbf{a}^\star(k)$ of the global problem in Definition 3.1 for each time slot k.
Initialization:
$k = 0$ and initialize the holding time to be zero for each sensor.
While time $k \geq 0$
Set the initial iteration $n = 0$, and the centralized platform announces initial price $p_i(n)$ to each sensor.
While $||[p_i(n+1)] - [p_i(n)]|| \leq \epsilon$
For sensor, it will repeatedly observe the current channel state, the announced price $p_i(n)$, the holding time via the feedback acknowledgment (ACK) packet, and then report to system designer its optimal request $b_i^\star(n+1)$ obtained by (11).
For system designer, it first reacts the multiplier $q_i(n)$ with the optimal allocation $e^{t_i^\star}(n+1)$ according to (14). Then, based on the collected power request, it updates the multiplier and price according to

$$\mathbf{q}(n+1) = \mathbf{q}(n) - \alpha(n)[\mathbf{t}^\star(n+1) - \ln(\mathbf{b}^\star(n+1))],$$

$$\mathbf{p}(n+1) = [\Pi(\mathbf{b}^\star(n+1))]^{-1}\mathbf{q}(n+1).$$

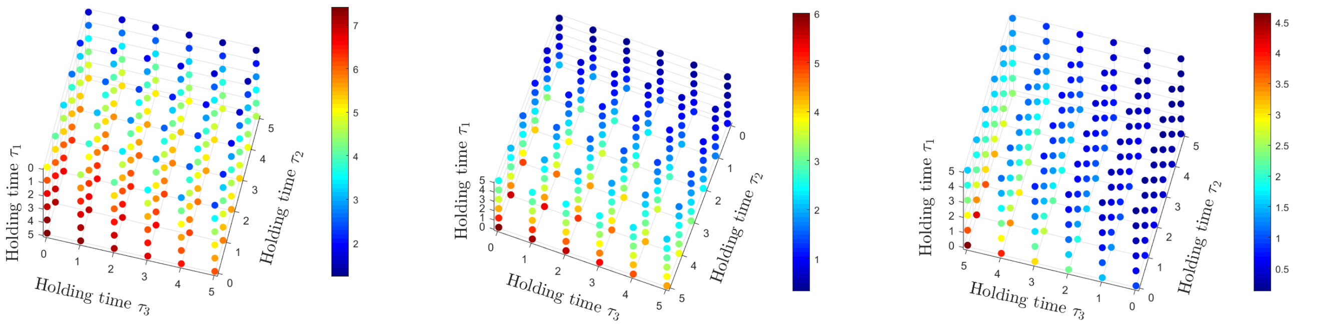
and spreads $p_i(n+1)$ to sensors.
$n := n+1$ until **end**
Each sensor transmits estimation packets following the optimal power allocation, and then updates their holding times according to (5).
$k := k+1$ until **end**

Sensor	A_i	Q_i	C_i	R_i
$i=1$	$\begin{pmatrix} 1.1 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \end{pmatrix}$	1
$i=2$	$\begin{pmatrix} 1.2 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \end{pmatrix}$	1
$i=3$	$\begin{pmatrix} 1.1 & 1 \\ 0 & 1.3 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

implemented by system designer and the sensors, respectively. We summarize in Algorithm 1 the pricing design in an explicit recursive form and the power allocation for sensors, in which $||\cdot||$ is the induced norm and ϵ represents the termination condition. First, as the algorithm is based on the dual-primal decomposition and the best-response $\Phi(\cdot)$ admits an unique fixed point, it is obvious that if the step size $\alpha(k)$ is sufficient small, Algorithm 1 will converge to a power allocation $\mathbf{a}^\star = \mathbf{b}^\star$ solving the global problem. Second, the updates of prices $\mathbf{p}(n)$ only depend on the iterations of $\mathbf{q}(n)$ and the collected power request $\mathbf{b}^\star(n)$ from the sensors. Hence, there is no need for system designer to know the current holing time and process parameters. This protects the information privacy for each sensor.

5. SIMULATION

We consider an example with three processes and the dynamics parameters are shown in above table. Assume that the channels are wireless fast-fading channels with packet dropout rate $f(\gamma) = \frac{1}{4}(\gamma)^{-1}$. The background noise for the channel is $\sigma^2 = 0.2$, and channel gains $[\, h_1 \ h_2 \ h_3 \,] = [\, 0.3 \ 0.6 \ 0.9 \,]$. Some parameter setting for constrains, the step size and convergence condition of algorithm are listed



(a) Power Allocation a_1. (b) Power Allocation a_2. (c) Power Allocation a_3.

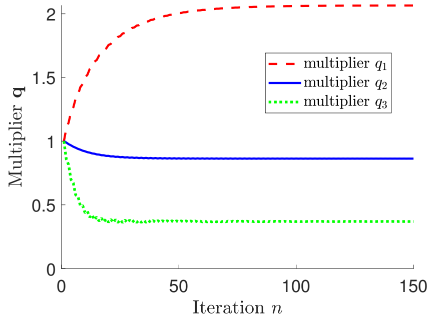


Fig. 4. Convergence result of $\mathbf{q}$ iterations.

aa follows: $\Theta = 8$, $\theta_i = \frac{1}{40}$, $\forall i$, $\alpha = 0.1$ and $\epsilon = 10^{-5}$. Take the triple of holding times $\tau_i = 1, \forall i$ as an example, and we have $[\, c_1 \; c_2 \; c_3 \,] = [\, -42.88 \; -28.78 \; -11.70 \,]$ accordingly. The evolution of multipliers $\mathbf{q}$ is depicted in Figure 4 and the algorithm converges within 50 iterations. The optimal prices, the resulting power allocation and the packet dropout rates for sensors are as follows: $[\, p_1 \; p_2 \; p_3 \,] = [\, 16.13 \; 14.38 \; 10.73 \,]$, $[\, a_1 \; a_2 \; a_3 \,] = [\, 5.01 \; 2.09 \; 0.90 \,]$, $[\, f(\gamma_1) \; f(\gamma_2) \; f(\gamma_3) \,] = [\, 0.38 \; 0.50 \; 0.91 \,]$. Intuitively, as c_i measures the current estimation performance of sensor i and c_1 is the minimum one, the transmission task w.r.t. sensor 1 is most emergent for system designer in order to improve the overall performance. The resulting optimal power allocation verifies this intuition with $a_1 > a_2, a_1 > a_3$. Moreover, we plot the optimal power allocation $\mathbf{a}$ for different triples of holding times in Figures 3(a), 3(b) and 3(c). As depicted in the color bar, the gradation of color represents the amount of transmission power. From Fig. 3(a), we find that a_1 is partially increasing in τ_1, which meets the intuition that current local estimate $\hat{x}_i(k)$ containing more valuable information requires a larger power resource. Furthermore, a_1 is partially deceasing in (τ_2, τ_3). It reveals that in order to achieve higher total estimation performance, it is preferable for system designer to sacrifice sensor 1 since the current data packets for sensors 2 and 3 are more important.

6. CONCLUSION

This work investigates a case where multiple sensors transmit their data packets to a remote estimator over a shared communication channel. Considering alleviating the signal interference, we propose a pricing method to coordinate the sensors so as to achieve the optimal estimation performance, and provide an algorithm to design the prices for each sensor. Even though the convergence of algorithm is analyzed in this work, further investigation is required to speed it up.

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