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Modified Backstepping Algorithm with Disturbances Compensation for Nonlinear MIMO Systems ⋆

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Abstract: The paper deals with a novel modified backstepping algorithm to ensure the stability of the nonlinear multiple-input multiple-output plant under unknown bounded parametric uncertainties, Lipschitz nonlinear disturbances, and cross-couplings. The modification is based on the auxiliary loop method that permits to estimate undesired dynamics as disturbances and suppress them. High robustness of the closed-loop system without using high-gain components is achieved. The ractical contribution of the results is demonstrated using laboratory platform "Twin Rotor MIMO System".

Keywords: Backstepping, recurcive procedure, auxiliary loop method, robust control, nonlinear system, MIMO systems, output control, parametric uncertainties, external disturbances, cross reactions

1. INTRODUCTION

Backstepping is one of the methods specially designed for nonlinear systems control when nonlinearities may not be constrained by linear bounds. The method was thoroughly described in Kokotovic (1992) to demonstrate a way of "peaking processes" problem resolving in nonlinear systems, including not feedback linearizable systems. Also, it was persuasively discussed that high-gain linear feedback control has a number of weaknesses and limitations while dealing with nonlinear systems.

Among others, two important and interesting features of backstepping were shown in Kokotovic (1992). First, a system should be presented by "strict feedback" form to ensure global stability while the more general "pure feedback" (lower triangular) form guaranties nonvanishing stability region, which may not be global. Some generalizations of backstepping deals with system transformation approach using a set of filters to provide the right form Marino and Tomei (1993a), Marino (1990), Marino and Tomei (1992). One of the basic results that describes a transformation for nonlinear systems to obtain a partially linear form (which can have a strict feedback linear part) is Krener and Isidori (1983). In Furtat (2009), Furtat et al. (2015), Furtat and Tupichin (2016), Vrazhevsky (2018), the only one filter is used for a system to achieve the "strict feedback" form. The result of Vrazhevsky (2018) is partially used in current research during the control system synthesis stage as one of the more simple and effective approaches in practice.

Second, a robust backstepping algorithm was presented to control a system with interval parametric uncertainty and nonlinear disturbances. More precisely the robust backstepping is introduced in Freeman and Kokotovic (1992), Freeman and Kokotovic (1993), Marino and Tomei (1993b). Further research enlarge the class of nonlinear systems for which backstepping can be utilized Polycarpou and Ioannou (1993), Krstic and Deng (1998), Pan and Basar (1999), Freeman and Praly (1998), Jiang and Nijmeijer (1997), Zhou and Zhang (2004), Serrani and Isidori (2000).

In Kokotovic and Arcak (2001) two restrictions of the robust backstepping procedure are discussed. Nonlinearities should be bounded by known functions in lower triangular form depending only on state variables of the system. Another restriction is that disturbances, wherever they present in strict (or pure) feedback form of system description, should be matched with state variables, which are considered as a virtual control.

Mismatched disturbances presence is a standalone problem in control theory. In Swaroop et al. (2000) a similar to the backstepping procedure is designed to ensure semi-global stability of the nonlinear system in the strict-feedback form with mismatched disturbances presence (both Lipschitz and non-Lipschitz). In Sun et al. (2015) the modification of backstepping algorithm is designed

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for a class of nonlinear systems with unknown mismatch disturbances.

Some backstepping algorithms provide high accuracy of functioning without high gain components in control system Miroshnik et al. (2013), Wang and Lin (2012), Khalil (2002). In Nikiforov (2003) a not high-gain adaptive and robust controller for linear systems output control is designed. A set of generalizations for different classes of nonlinear systems is considered in Freeman and Kokotovic (1993), Marino and Tomei (1993b), Sun et al. (2015), Qu (1993), Zheng and Yang (2007). Control systems based on those results require a large set of filers need to be included witch makes practical implementation difficult.

Within more recent works, there are many results that combine robust and adaptive backstepping approaches with neural networks and fuzzy logic. In Wen et al. (2011) backstepping became a base for two controllers for systems with input saturation without assumptions on the uncertain parameters within a known compact set and an a priori knowledge on the bound of the external disturbance.

The proposed approach is a modification of the backstepping method and leads to control of a nonlinear plant under unknown bounded disturbances. The modification is based on the auxiliary loop method that was first proposed in Tsykunov (2011). The auxiliary loop method is a model-based robust control approach that permits to estimate unknown disturbances and suppress them. There are a set of solutions in different areas which are preferable than analogs due to the simplicity of the implementation and high accuracy of signals in a steady-state, see Belyaev et al. (2013), Furtat (2014), Furtat and Chugina (2016), Furtat (2013), Fradkov and Furtat (2013), Furtat et al. (2013),Vrazevsky et al. (2016). The auxiliary loop technique was combined with the backstepping method in Furtat (2009). In Furtat et al. (2015) it was shown that the proposed algorithm compensates mismatched unknown bounded disturbances in linear systems. In Furtat et al. (2016) the stability of modified backstepping algorithm from Furtat (2009) was proved for a linear case with delays in the state vector. In current work, the result is generalized in the area of nonlinear MIMO systems with cross-couplings.

This research provides a backstepping based method to control nonlinear MIMO systems with couplings and external disturbances. Model transformation to provide a strict feedback form of the system is given and only one filter is used to obtain this transformation. The backstepping method is combined with an auxiliary loop technique to ensure the convergence of all signals in the closed-loop system to a small enough set of attraction, which can be regulated by a set of tunable parameters. High robustness of the closed-loop system is demonstrated.

2. PROBLEM STATEMENT

Consider a nonlinear multi-agent system defined by

$$\begin{aligned} \dot{x}_i &= \sum_{j=1}^{m} f_j^i(x_j,\varepsilon_j) + b_i(x_i,\varepsilon_i)(u_i+\varphi_i(x_i,t)), \\ y_i &= h_i(x_i), i=\overline{1,m}, x_i(0)=x_{i0}, t_0=0, \end{aligned} \tag{1}$$

where $f_j^i(x_j,\varepsilon_j)\big|_{j=i} + b_i(x_i,\varepsilon_i)(u_i+\varphi_i(x_i,t))$ is a self-dynamics of i-th agent; $\sum\limits_{j=1}^{m} f_j^i(x_j,\varepsilon_j), j \neq i$ are cross-coupling functions; $[\varepsilon_1,...,\varepsilon_m]^{\mathrm{T}} \in \varepsilon$ are vectors of unknown parameters and ε is a known bounded set; $x_i \in R^{n_i}$ are state vectors; $y_i \in R$ are estimated scalar outputs; f_i, b_i and h_i are sufficiently smooth functions; $u_i \in R$ are scalar control signals; $\varphi_i \in R$ are unknown disturbance functions which are bounded for all x and t (or bounded on t and Lipschitz on x).

Define a control goal in the form

$$\|e\| < \delta, t > t_f, \tag{2}$$

where $e = y(t) - y_{ref}(t)$, $e = [\overline{e}_1,...,\overline{e_m}]^{\mathrm{T}}$, $y = [y_1,...,y_m]^{\mathrm{T}}$, $y_{ref} = [y_{ref,1},...,y_{ref,m}]^{\mathrm{T}}$. Each $\overline{e}_i = y_i(t) - y_{ref,i}(t), i = \overline{1,m}$, is defined as a tracking error of ith agent, δ is a small enough number represents a desired tracking accuracy, $t_f > 0$ is a transient time, $y_{ref,i}$ are reference signals which are bounded with derivatives.

3. MODEL TRANSFORMATION

The plant (1) can be rewritten in generalized form

$$\begin{aligned} \dot{x} &= f(x,\varepsilon) + b(x,\varepsilon)(u+\varphi(x,t)) \\ y &= h(x), \end{aligned} \tag{3}$$

where $f(x,\varepsilon) = \left[\sum\limits_{j=1}^{m} f_j^1(x_j,\varepsilon_j),\ ...,\ \sum\limits_{j=1}^{m} f_j^i(x_j,\varepsilon_j)\right]^{\mathrm{T}}$,

$x = [x_1,\ ...,\ x_m]^{\mathrm{T}} \in R^n$, $n = \sum\limits_{i=1}^{m} n_i$, $\varepsilon = [\varepsilon_1,\ ...,\ \varepsilon_m]^{\mathrm{T}}$, $u = [u_1,\ ...,\ u_m)]^{\mathrm{T}}$, $\varphi(x,t) = [\varphi_1(x_1,t),\ ...,\ \varphi_m(x_m,t)]^{\mathrm{T}}$, $b(x,\varepsilon) = diag\,[b_1(x_1,\varepsilon_1),\ ...,\ b_m(x_m,\varepsilon_m)]$,

$h(x) = [h_1(x_1),\ ...,\ h_m(x_m)]^{\mathrm{T}}$.

To construct a control system, the plant (3) need to be transformed using known transformation methodology provided in Isidori (1989). For this purpose, a relative dynamic degree of the plant should be found. Following the classical definition of relative dynamic degree for a nonlinear system by Khalil (2002), Miroshnik et al. (2013), the derivative of y defined by $\dot{y} = L_f h(x,\varepsilon) + L_b h(x,\varepsilon)(u+\varphi(x,t))$, where $L_f h(x,\varepsilon) = \frac{\partial h(x)}{\partial x} f(x,\varepsilon)$, $L_b h(x,\varepsilon) = \frac{\partial h(x)}{\partial x} b(x,\varepsilon)$ are called Lie derivatives of h along f and b respectively. In case $L_b h(x,\varepsilon) = 0$, the function $\dot{y}$ becomes independent of u and so $\dot{y} = L_f h(x,\varepsilon)$. Continuing calculations for higher derivatives $y^{(i)}$, $2 \le i \le \gamma$, the following notations are appropriated

$$\frac{\partial(L_f h)}{\partial x} b(x,\varepsilon) = L_b L_f h(x,\varepsilon), \frac{\partial(L_f h)}{\partial x} f(x,\varepsilon) = L_f^2 h(x,\varepsilon),$$

$$\vdots$$

$$\frac{\partial(L_f^{\gamma-1} h)}{\partial x} b(x,\varepsilon) = L_b L_f^{\gamma-1} h(x,\varepsilon), \frac{\partial(L_f^{\gamma-1} h)}{\partial x} f(x,\varepsilon) = L_f^{\gamma} h(x,\varepsilon).$$

Thus, the second derivative of y defined by $\ddot{y} = \frac{\partial(L_f h)}{\partial x}\dot{x} = L_f^2 h(x,\varepsilon) + L_b L_f h(x,\varepsilon)(u+\varphi(x,t))$. If $L_b L_f h(x,\varepsilon) = 0$, then $\ddot{y} = L_f^2 h(x,\varepsilon)$ independent of u. Repeating calculations until the equation with the dependence of control signal will be found, we obtain on some step γ the representation

$$y^{(\gamma)} = L_f^{\gamma} h(x,\varepsilon) + L_b L_f^{\gamma-1} h(x,\varepsilon)(u + \varphi(x,t)). \quad (4)$$

The control system synthesis in the next Section deals with representation (4) of the plant (3) under the following assumptions:

(1) The relative degree γ of plant (3) is known or can be estimated and function $c(x,\varepsilon) = L_f^{\gamma} h(x,\varepsilon)$ is bounded (or bounded on ε and Lipschitz on x).
(2) The sing of function $\beta(x,\varepsilon) = L_b L_f^{\gamma-1} h(x,\varepsilon)$ is known. Let, without loss of generality, $\beta(x,\varepsilon) > 0$.
(3) There exists function $\phi^{-1}(x)$ such that

$$\bar{x}(t) = \phi(x(t)) = [y(t), \dot{y}(t), ..., y^{(\gamma-1)}(t)]^{\mathrm{T}} = = [h(x), L_f^1 h(x), ..., L_f^{\gamma-1} h(x)]^{\mathrm{T}}.$$

Using representation (4), error equation $e = y - y_{ref}$ can be transformed into $p^{\gamma} e = c(x,\varepsilon) + \beta(x,\varepsilon)(u + \varphi(x)) - p^{\gamma} y_{ref}$, where $p = \frac{d}{dt}$ is a differential operator. Consider operator $Q(p^{\gamma-1}) = \sum_{g=1}^{\gamma} k_g p^{\gamma-g}$ such that $Q(p^{\gamma}) = p^{\gamma} + Q(p^{\gamma-1})$ is a Hurwitz polynomial. Using $Q(p^{\gamma-1})$, rewrite $p^{\gamma} e$ in the form $Q(p^{\gamma})e = u + \psi(x,u,\varepsilon,y_{ref})$, where $\psi = c + \beta\varphi + (\beta - 1)u - p^{\gamma} y_{ref} + Q(p^{\gamma-1})e$ is an augmented disturbance function. Thus, error model takes the form

$$e(t) = Q(p^{\gamma})^{-1} u(t) + Q(p^{\gamma})^{-1} \psi(x,u,\varepsilon,y_{ref},t). \quad (5)$$

4. MAIN RESULT

For term $Q(p^{\gamma})^{-1} u(t)$ in (5) consider the following filter

$$\dot{v}(t) = A_0 v(t) + l u(t),$$
$$v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_{\gamma}(t) \end{bmatrix}, l = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, A_0 = \begin{bmatrix} -k_1 & & \\ -k_2 & I_{\gamma-1} & \\ \vdots & & \\ -k_{\gamma} & 0 \cdots & 0 \end{bmatrix}. \quad (6)$$

Rewrite (5) as $e(t) = v_1(t) + Q(p^{\gamma})^{-1} \psi(x,u,\varepsilon,y_{ref},t)$. For the convenience of further calculations let denote $e(t) = e_1(t)$. The derivative of (5) takes the form

$$\dot{e}_1(t) = -c_1 e_1 - k_1 v_1(t) + v_2(t) + f_1, \quad (7)$$

where $f_1 = c_1 e_1 + pQ(p^{\gamma})^{-1} \psi(x,u,\varepsilon,y_{ref},t)$ are new disturbance functions.

According to the backstepping approach (see Kokotovic (1992)), there exists an iterative procedure of control scheme synthesis by calculating the set of virtual control laws that leads to real control law formulation on the last step. Construct additionally a set of auxiliary loops to evaluate and compensate unknown disturbances as in Tsykunov (2011). Such modification increases robustness in a closed-loop system without using high-gain feedback components.

4.1 Step 1

Consider auxiliary loops in the form ϑ

$$\dot{e}_1^a = -c_1 e_1^a - k_1 v_1 + v_2, \quad (8)$$

and define mismatch errors $\xi_1 = e_1 - e_1^a$ with their derivatives

$$\dot{\xi}_1 = \dot{e}_1 - \dot{e}_1^a = -c_1 \xi_1 + f_1. \quad (9)$$

It follows from (9) that disturbances can be estimated by the function $\hat{f}_1 = \dot{\xi}_1 + c_1 \xi_1$. Rewrite (7) in the form

$$\dot{e}_1 = -c_1 e_1 - k_1 v_1 + v_2 + \dot{\xi}_1 + c_1 \xi_1. \quad (10)$$

Let the function v_2 is a virtual control signal in (10). Construct for each agent a virtual control law

$$\nu_1 = k_1 v_1 - \hat{\dot{\xi}}_1 - c_1 \xi_1, \quad (11)$$

where $\hat{\dot{\xi}}_1$ are evaluations of $\dot{\xi}_1$. The dirty differential filter (DDF) is used as an observer to estimate unknown derivatives

$$\mu \dot{\hat{y}} + \hat{y} = \dot{y}, \quad (12)$$

where $\mu > 0$ is a sufficiently small positive number, y is a measured input, $\hat{y}$ is an evaluation of $\dot{y}$. A high-gain observer is used in order to simplify theoretical proof analysis. It also should be noted that high-gain observer usage does not make the control approach to be a high-gain, except the small enough time period in the begin of system functioning till the observer transients are over. Denoting $v_2 = \nu_1$, we get

$$\dot{e}_{1i} = -c_1 e_1 + e_2 + \eta_1, \quad (13)$$

where $\eta_1 = \hat{\dot{\xi}}_1 - \dot{\xi}_1$ is an estimation error, e_2 is a virtual control error

$$e_2 = v_2 - \nu_1. \quad (14)$$

4.2 Step 2

In accordance with (6), derivatives of (14) are defined by

$$\dot{e}_2 = -k_2 v_1 + v_3 - \dot{\nu}_1 \pm c_2 e_2. \quad (15)$$

Consider auxiliary loops in the form

$$\dot{e}_2^a = -c_2 e_2^a - k_2 v_1 + v_3, \quad (16)$$

and define a second set of mismatch errors

$$\xi_2 = e_2 - e_2^a, \quad (17)$$

with their derivatives

$$\dot{\xi}_2 = \dot{e}_2 - \dot{e}_2^a = -c_2 \xi_2 + f_2, \quad (18)$$

where each function $f_2 = -\dot{\nu}_1 + c_2 e_2$ is denoted as disturbance functions for the system (17) and, respectively, (15). From (18) it follows that disturbances can be estimated by $\hat{f}_2(t) = \dot{\xi}_2 + c_2 \xi_2$. Rewrite (15)

$$\dot{e}_2 = -c_2 e_2 - k_2 v_1 + v_3 + \dot{\xi}_2 + c_2 \xi_2. \quad (19)$$

As it was done on the first step, suppose that functions v_3 can be used as control signals for the system 19. Construct virtual control laws

$$\nu_2 = k_2 v_1 - \hat{\dot{\xi}}_2 - c_2 \xi_2, \quad (20)$$

where $\hat{\dot{\xi}}_2$ are evaluations of $\dot{\xi}_2$. Denoting $v_3 = \nu_2$, we get

$$\dot{e}_2 = -c_2 e_2 + e_3 + \eta_2, \quad (21)$$

where $\eta_2 = \hat{\dot{\xi}}_2 - \dot{\xi}_2$ are estimation errors, e_3 are virtual control errors defined by $e_3 = v_3 - \nu_2$.

4.3 Step $r, r = \overline{3, \gamma - 1}$

The same calculations as on step 2 are correct for the consecutive analysis of errors $e_r = v_r - \nu_{(r-1)}, r = \overline{3, \gamma - 1}$. As a result, the following auxiliary loops should be formed

$$\dot{e}_r^a = -c_r e_r^a - k_r v_1 + v_{(r+1)}, r = \overline{3, \gamma - 1}. \quad (22)$$

Therefore, the mismatch errors $\xi_r = e_r - e_r^a, r = \overline{3, \gamma - 1}$ can be found. Introduce virtual control laws in the form

$$\nu_r = k_r v_1 - \hat{\dot{\xi}}_r - c_r \xi_r, r = \overline{3, \gamma - 1}. \quad (23)$$

Equation (23) provide error dynamic equations

$$\dot{e}_r = -c_r e_r + e_{(r+1)} + \eta_r, \quad (24)$$

where $e_{(r+1)} = v_{(r+1)} - \nu_r$ and $\eta_r = \hat{\dot{\xi}}_r - \dot{\xi}_r$.

4.4 Step γ

Taking a derivative of the error e_γ which is calculated on the previous step, we get

$$\dot{e}_\gamma = -k_\gamma v_1 + u - \dot{\nu}_{(\gamma-1)} \pm c_\gamma e_\gamma. \quad (25)$$

Consider auxiliary loops

$$\dot{e}_\gamma^a = -c_\gamma e_2^a - k_\gamma v_1 + u, \quad (26)$$

and define a final set of mismatch errors $\xi_\gamma = e_\gamma - e_\gamma^a$ with their derivatives

$$\dot{\xi}_\gamma = \dot{e}_\gamma - \dot{e}_\gamma = -c_\gamma \xi_\gamma + f_\gamma, \quad (27)$$

where $f_\gamma = -\dot{\nu}_{(\gamma-1)} + c_\gamma e_\gamma$ is denoted as disturbance functions for the system (25). From (27) it follows that disturbance functions for the system (25) can be evaluated by $\hat{f}_\gamma = \dot{\xi}_\gamma + c_\gamma \xi_\gamma$ and the system (25) can be rewritten as follows

$$\dot{e}_\gamma = -c_\gamma e_\gamma - k_\gamma v_\gamma + u + \dot{\xi}_\gamma + c_\gamma \xi_\gamma. \quad (28)$$

Let us define a control law for each agent of the system (28) by

$$u = k_\gamma v_\gamma - \hat{\dot{\xi}}_\gamma - c_\gamma \xi_\gamma, \quad (29)$$

where $\hat{\dot{\xi}}_\gamma$ are evaluations of $\dot{\xi}_\gamma$. Substituting (29) into (28), we get

$$\dot{e}_\gamma = -c_\gamma e_\gamma + \eta_\gamma, \quad (30)$$

where $\eta_\gamma = \hat{\dot{\xi}}_\gamma - \dot{\xi}_\gamma$ are estimation errors.

Theorem 1. Let the assumptions hold. There exist constants $c_r > 0$, $\mu_{r0} > 0$, $r = \overline{1, \gamma}$, such that for any $\mu_r \in (0; \mu_{r0}]$ the control system that consists in in the filters (6), the set of auxiliary loops (8), (16), (22), (26), observers (12), virtual control laws (11), (20), (23) and real control laws (29) provides the control goal (2) for plant (1).

5. EXPERIMENTAL RESULTS

To demonstrate the applicability of the algorithm, we use the platform "Twin Rotor MIMO System" (TRMS). The platform is intended for analyzing the efficiency of control approaches on highly nonlinear systems with cross-couplings and disturbances presence. It represents a simplified copter dynamics in two planes: vertical and horizontal. According to the flight dynamics definitions, the vertical angle of the plant is denoted as a pitch angle and the horizontal angle is denoted as a yaw angle. General view of the system is shown in Fig. 1

Aerodynamic forces produced by rotation of main and tail rotors control the position of the platform in both planes. In turn, the torque of corresponding DC-motor drives each rotor. The voltages applied to each DC-motor are control inputs. Angular positions of the main beam are plant outputs. Thus, the system can be described as

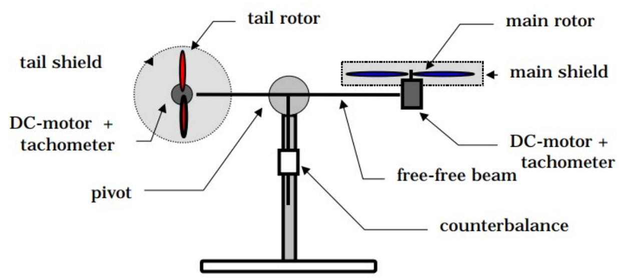

Fig. 1. Twin Rotor MIMO Syatem (TRMS)

two dynamically related subsystems, each one has its own single input and single output channel.

TRMS dynamics is defined by

$$\begin{aligned} J_1\ddot{\alpha} &= -k_{f11}\dot{\alpha} - k_{f12}\,\mathrm{sign}\,(\dot{\alpha}) + gN_1 cos\alpha - \\ &- gN_2 \sin\alpha - N_3\dot{\beta}^2 \sin\alpha\cos\alpha + F_1\,(u_1)\,, \\ J_2\ddot{\beta} &= -k_{f21}\dot{\beta} - k_{f22}\,\mathrm{sign}(\dot{\beta}) + F_2\,(u_2)\,, \end{aligned} \quad (31)$$

where α is a pitch angle of the plant, β is a yaw angle of the plant, $J_1 = \text{const}$, $J_2 = \chi(\alpha)$ are inertia moments for pitch and yaw dynamics consistently, $\chi(\alpha)$ is a smooth nonlinear function, $k_{f_{ij}}, i, j = \overline{1, 2}$ are friction forces coefficients, $N_{\overline{1,3}}$ are known numerical coefficients depend on weight and size parameters, g is the acceleration of gravity, F_1, F_2 are nonlinear functions that combine a DC-motor and aerodynamic forces dynamics. All numerical parameters needed to be known to build a model of the system (31) are presented in documentation Ltd. (1998). In our case, the only information we need to build a control system is its relative degree. In the documentation, the DC motor dynamics and aerodynamical forces are approximated by polynomials with high orders. Using this approximated model, the plant represents two subsystems with equal relative degrees $\gamma_1 = \gamma_2 = \gamma = 2$. The control system parameters are chosen as follows

$$A_{0_{pitch,yaw}} = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}, c_{pitch} = \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix},$$

$$c_{yaw} = \begin{bmatrix} 0.075 \\ 0.075 \end{bmatrix}, \mu_{pitch,yaw} = \begin{bmatrix} 0.06 \\ 0.06 \end{bmatrix}.$$

Experimental results are shown in Fig. 2 - 7 and demonstrates a high quality of the closed-loop system in both stabilisation and tracking modes. Considering the laboratory bench used and experimental result performance, the proposed method is able to be successfully used in various industrial, scientific and modern educational practical oriented programs like in Čech et al. (2019).

6. CONCLUSION

A robust modified backstepping output control algorithm for nonlinear MIMO systems with cross-couplings is provided. The control system demonstrates a high quality of transients and high accuracy in steady-state. The proposed algorithm provides robustness with respect to external disturbances and cross-couplings. Practical applicability is verified by experiments on laboratory bench both in tracking mode and stabilization mode. Proposed control method refers to decentralized approaches but it can be easily transformed into the centralized one. Control goal

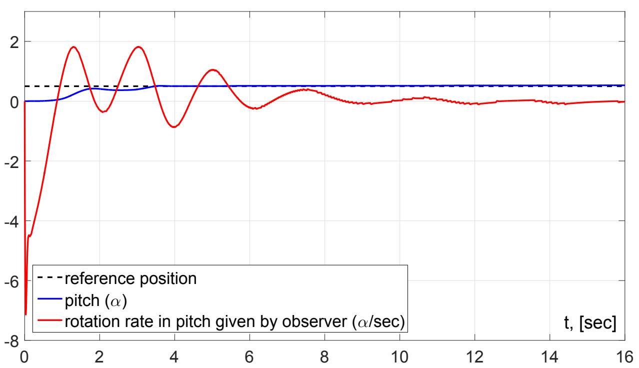

Fig. 2. Transients and steady state of pitch angle stabilization

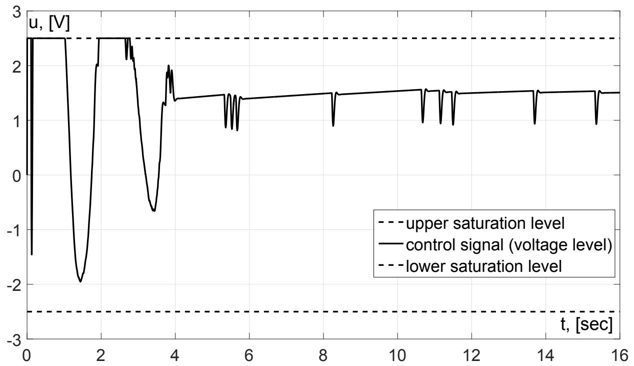

Fig. 3. Control signal of TRMS pitch stabilization system

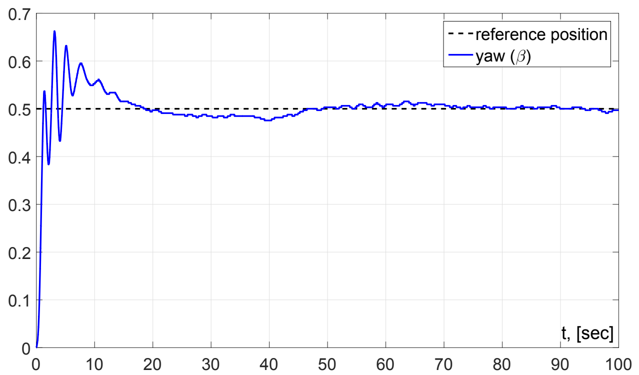

Fig. 4. Transients and steady state of TRMS yaw angle stabilization

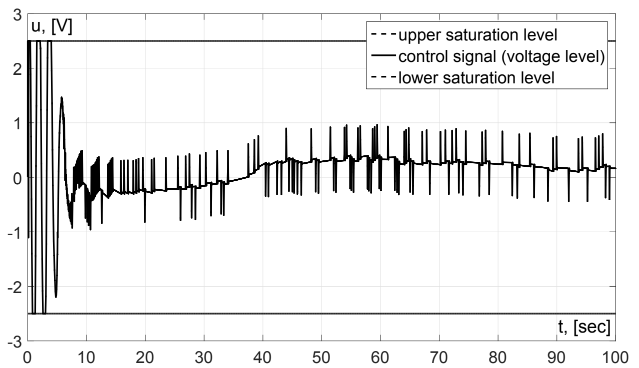

Fig. 5. Control signal of TRMS yaw stabilization system.

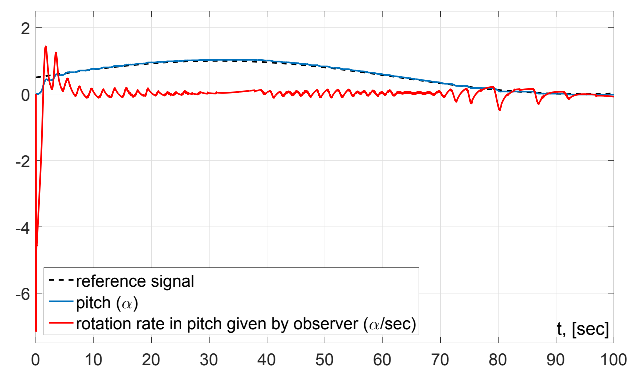

Fig. 6. Transients of TRMS pitch angle in tracking mode

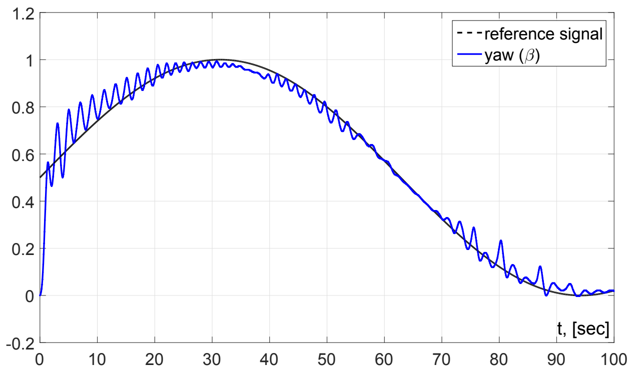

Fig. 7. Transients of TRMS yaw angle in tracking mode

(2) represents a synchronization process for the system (1) in case of $y_{ref,i}(t) = y_{ref}(t), i = \overline{1,n}$:

$$\|e\| < \delta, t > t_f,$$
$$e = [e_{11}, e_{12}, \ldots, e_{1n}]^{\mathrm{T}}, e_{1i} = y_i(t) - y_{ref}(t), i = \overline{1,n},$$

without additional changes required. This goal leads to a centralized control algorithm in a tracking task with one reference trajectory defined for all subsystems (agents). Proposed control method refers to decentralized approaches but it can be easily transformed into the centralized one by choosing the same parameter values for each subsystem. It may lead to declining of attainable quality of the closed-loop system because of the choice of tuning parameters is based on the most "uncomfortable" agent dynamics. During the Lyapunov function analysis, quite a rough inequalities were used. It means that obtained parameters limitations could be "softer" in practical cases which confirms by experiment.

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Appendix A. PROOF OF THE THEOREM 1

Consider an error dynamic system by combining (12), (13), (21), (24), (30)

$$\begin{aligned} &\dot{e}_j(t) = -c_j e_j(t) + e_{(j+1)} + \eta_j(t), j = \overline{1, \gamma - 1}, \\ &\dot{e}_\gamma(t) = -c_\gamma e_\gamma(t) + \eta_\gamma(t), \\ &\mu_{j1}\dot{\eta}_j(t) = -\eta_j(t) - \mu_{j2}\ddot{\xi}_j(t), j = \overline{1, \gamma}. \end{aligned} \tag{A.1}$$

Taking into account (A.1) and by choosing $c_j > 0$, $\mu_{j1} = \mu_{j2} = \mu_j > 0$ it follows that each error e_j is bounded if corresponding observation errors η_j are bounded. From the last equation of (A.1) it follows that all observation errors are bounded if signals $\ddot{\xi}_j$ are bounded. To analyze the boundedness property of $\ddot{\xi}_j$ use the following Lemma Furtat (2014).

Lemma 2. Consider the system

$$\dot{x} = f(x, \mu_1, \mu_2, t), \tag{A.2}$$

where $x(t) \in R^{S_1}$, $\mu = col(\mu_1, \mu_2) \in R^{S_2}$, $f(x, \mu_1, \mu_2, t)$ is a Lipschitz function continuous in x, bounded on t and has a bounded closed set of attraction for $\mu_2 = 0$ in the form $\Omega = \{x : P(x) \leq C\}$, where $P(x) \in R^{S_1}$ is a continuous positive-defined function. Let there exist some numbers $C_1 > 0$ and $\bar{\mu}_1 > 0$ such that the following condition holds

$$\sup_{|\mu_1| \leq \bar{\mu}_1} \left[\left\langle [\nabla P(x)]^{\mathrm{T}}, f(x, \mu_1, 0, t) \right\rangle \Big| \, P(x) = C \right] \leq -C_1.$$

Then there exist $\mu_0 > 0$ such that for $\mu_2 \in (0; \mu_0]$ the system (A.2) has the same set of attraction Ω.

The asymptotic stability of the system (A.1) in case $\mu_{j2} = 0$ should be shown to satisfy Lemma conditions. Then the set of attraction of the original system (A.1) with $\mu_{j2} > 0$ is the same as in the case with $\mu_{j2} = 0$.

Consider a Lyapunov function for the system (A.1)

$$V = 0.5\sum_{j=1}^{\gamma} e_{ji}^2 + 0.5\sum_{j=1}^{\gamma} \eta_{ji}^2. \tag{A.3}$$

Derivative of V along the trajectories (A.1) for $\mu_{j2} = 0$ takes the form

$$\dot{V} = -\sum_{j=1}^{\gamma} c_j e_j^2 + \sum_{j=1}^{\gamma-1} e_j e_{(j+1)} + \sum_{j=1}^{\gamma} e_j \eta_j - \sum_{j=1}^{\gamma} \frac{\eta_j^2}{\mu_{j1}}. \tag{A.4}$$

The following bounds are satisfied for (A.4)

$$e_j e_{(j+1)} \leq \frac{1}{2}e_j^2 + \frac{1}{2}e_{(j+1)}^2, \; e_j\eta_j \leq \frac{1}{2}e_j^2 + \frac{1}{2}\eta_j^2. \tag{A.5}$$

Using (A.5), rewrite (A.4) in the form $\dot{V} \leq (1 - c_1)e_1^2 + \sum_{j=2}^{\gamma-1}(1.5 - c_j)e_j^2 + (1 - c_\gamma)e_\gamma^2 + \sum_{j=2}^{\gamma}(0.5 - \mu_{j1}^{-1})\eta_j^2$. There exist constants $c_{1,\gamma} > 1$, $c_j|_{j=\overline{2,\gamma-1}} > 1.5$, $\mu_{j1}^{-1}\big|_{j=\overline{1,\gamma}} > 0.5$ such that Lyapunov function derivative is strictly negative and inequality (A.4) can be rewritten by

$$\dot{V} \leq -\bar{c}\sum_{j=1}^{\gamma} e_j^2 - \bar{\mu}\sum_{j=1}^{\gamma} \eta_j^2 \leq -\alpha V, \tag{A.6}$$

where $\bar{c} = \sup_{j=\overline{1,\gamma}} c_j$, $\bar{\mu} = \sup_{j=\overline{1,\gamma}} \mu_{j1}^{-1}$, $\alpha = \sup(\bar{c}, \bar{\mu})$. From (A.6) it follows that there exists a positive defined function $V(t)$ such that its derivative is a strictly negative function along the trajectories (A.1) and solutions of the system (A.1) exponentially tend to zero. This property guarantees boundedness of all signals in (A.1). Thus, Lemma conditions hold for case of $\mu_{j2} = 0$ and there exists a parameter $\mu_0 > 0$ such that for any $\mu_{j2} \in (0; \mu_0]$ the system (A.1) has the same set of attraction as in case if $\mu_{j2} = 0$. The asymptotic stability does not apply for non-simplified case but it is possible to find a small enough attraction set for $\mu_{j2} > 0$. Consider a Lyapunov function (A.3) for the system (A.1) with $\mu_{j2} > 0$ and its derivative takes the form $\dot{V} = -\sum_{j=1}^{\gamma} c_j e_j^2 + \sum_{j=1}^{\gamma-1} e_j e_{(j+1)} + \sum_{j=1}^{\gamma} e_j\eta_j - \sum_{j=1}^{\gamma}((\mu_{j2}/\mu_{j1})\ddot{\xi}_j\eta_j + \mu_{j1}^{-1}\eta_j)$. Using bounds (A.5) and taking into account $\mu_{j1} = \mu_{j2} = \mu_j|_{j=\overline{1,\gamma}} > 0$, $\dot{V}$ can be bounded by $\dot{V} \leq (1 - c_j)e_j^2\big|_{j=\{1,\gamma\}} + \sum_{j=2}^{\gamma-1}(1.5 - c_j)e_j^2 + \sum_{j=1}^{\gamma}(0.5 - \mu_j^{-1})\eta_j^2 - \sum_{j=1}^{\gamma}\ddot{\xi}_j\eta_j$. There exist constants $\mu_j^{-1} > 0.5$, $c_j|_{j=\{1,\gamma\}} > 1$, $c_j|_{j=\overline{2,\gamma-1}} > 1.5$, such that Lyapunov function satisfies the inequality

$$\dot{V} \leq -\alpha V - \sum_{j=1}^{\gamma} \ddot{\xi}_j\eta_j, \tag{A.7}$$

$\alpha = \sup_{j=\overline{1,\gamma}} \left\{(2c_j|_{j=\{1,\gamma\}} - 2); (2c_j|_{j=\overline{2,\gamma-1}} - 3); (2\mu_j^{-1} - 1)\right\}$. The term $\sum_{j=1}^{\gamma}\ddot{\xi}_j\eta_j$ can be represented by $\sum_{j=1}^{\gamma}\ddot{\xi}_j\eta_j = \sum_{j=1}^{\gamma}\frac{\mu_j p^4}{\mu_j p+1}\xi_j$ and, thus, satisfy the condition

$$\lim_{\mu_j \to 0}\left[\sum_{j=1}^{\gamma}\ddot{\xi}_j\eta_j\right] = 0, j = \overline{1, \gamma}. \tag{A.8}$$

From (A.7), (A.8) it follows that

$$V \leq -e^{-\alpha t}V(0) - \mu_j(1 - e^{-\alpha t})\alpha^{-1}\bar{\xi},$$

where $\bar{\xi} = \sum_{j=1}^{\gamma}\frac{p^4}{\mu_j p+1}\xi_j$. A set of attraction for the system (A.1) can be can be evaluated by

$$\|e\| \leq \sqrt{-e^{-\alpha t}2V(0) - \mu_j(1 - e^{-\alpha t})\bar{\xi}\alpha^{-1}}, j = \overline{1, \gamma},$$

From the last inequality it follows that the accuracy of the closed-loop system depends on parameters α and μ_j while the parameter α is defined by choosing parameters μ_j, c_j.