Finite-time decentralized H_∞ control for singular large-scale systems [⋆]

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Abstract: In this paper, the definition of finite-time robust H_∞ control for linear continuous-time singular large-scale systems is presented. The main aim of this paper is to design a decentralized state feedback controller which ensures that the closed-loop system is finite-time bounded (FTB), and the effect of the disturbance input on the controller output, meanwhile, is reduced to a prescribed level. A sufficient condition is presented for the solvability of this problem, which can be reduced to a feasibility problem involving linear matrix inequalities (LMIs). A detailed solving method is proposed for the restricted linear matrix inequalities. Finally, examples are given to show the validity of the methodology.

Keywords: Finite-time bounded; Singular large-scale systems; LMIs; Time-varying exogenous disturbances; Finite-time H_∞ control

1. INTRODUCTION

Singular large-scale systems (also known as descriptor large-scale systems, generalized large-scale state-space systems, differential-algebraic large-scale systems) have attracted considerable attention and been applied to many practical situations, such as industrial processes, transportation networks, power systems, and others. High dimensionality, uncertainty, and information structure constraints are well known major motivating features for the development of decentralized control theory. During the past years, the problems of decentralized control for singular large-scale systems have attracted a lot of attention and significant advances have been made on these topics, such as stable and decentralized stabilization (Wo and Zou(2004); Wo et al.(2007); Xie et al.(2006)), decentralized H_∞ control (Jiang et al.(2006); Wo et al.(2010)). It should be pointed out that most of the results in this field relate to stability and performance criteria defined over an infinite-time interval.

However, in many practical applications, the main concerns is the behavior of the system over a fixed finite-time interval, for example, large values of the state are not acceptable in the presence of saturation (Amato et al.(2001)). In this sense, it appears reasonable to define as stable a system whose state, given some initial conditions, remains within prescribed bounds in the fixed finite-time interval, for these purposes finite-time stability(FTS) could be used. The concept of FTS has been revisited in the light of recent results coming from linear matrix inequality (LMI) theory, which has allowed to find computationally appealing conditions guaranteeing FTS of state-space systems. The finite-time control problems for state-space linear continuous-time systems (Amato et al.(2001); Amato et al.(2006)), discrete-time systems (Amato and Ariola(2005); Amato et al.(2010)), linear time-varying continuous systems (Garcia et al.(2009)), nonlinear systems (X. Zhang et al.(2012)) have been considered via state feedback or dynamic output feedback, respectively. The finite-time H_∞ control problems for Markovian jump systems (Wang et al.(2020)), discrete-time Markovian jump nonlinear systems with time-delays (Zhang et al.(2014)) and stochastic systems (Xiang et al.(2012); Fu(2010)) have been considered. Recently, the concept of finite-time control for state-space systems has extended to ones of state-space large systems (Fu(2010);Fu(2011)), singular systems (Feng et al.(2005); Wo and Han(2014); Wo and Li(2018a)), and singular stochastic systems (Y. Zhang et al.(2012)). For singular large-scale systems, Wo et al investigated the finite-time robust control via generalized Lyapunov function approach (Wo et al.(2017)) and the finite-time robust decentralized control for uncertain singular large-scale systems with exogenous disturbances via decentralized state feedback (Wo and Han(2018)), respectively. However, seldom results on the problems of finite-time decentralized H_∞ control were reported so far.

In this paper, we extend the definition of H_∞ control and present a new definition of finite-time H_∞ control for linear continuous singular large-scale systems. Our main propose is to design a decentralized state feedback controller which guarantees that the closed-loop system regular, impulse free, FTB and reduces the effect of the disturbance input on the controlled output to a prescribed level. A sufficient condition is presented for the solvability of this problem, which can be reduced to a feasibility problem involving linear matrix inequalities (LMIs).

Notation. Throughout this paper, matrices, if not explicitly stated, are assumed to have compatible dimensions.

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For real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The notation N^T represents the transpose of the matrix N. $((M)_{ij})$ denotes a $n \times n$ dimensional matrix, which has the form of

$$((M)_{ij}) = \begin{array}{cc} \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & 0 & 0 & \cdots & \vdots \\ \vdots & \cdots & 0 & M & \cdots & \vdots \\ \vdots & \cdots & 0 & 0 & \cdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}_{n\times n} & i \\ j & \end{array}$$

Here $M \in R^{n_i \times n_j}$, 0 are zero matrices with appropriate dimension and $\sum_{i=1}^{N} n_i = n$.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the linear continuous singular large-scale systems(LCSLSS) described by,

$$\begin{cases} E_i\dot{x}_i(t) = A_{ii}x_i(t) + \sum\limits_{j=1, j\neq i}^{N} A_{ij}x_j(t) + G_i\omega_i(t) + B_iu_i(t), \\ z_i(t) = C_ix_i(t) + D_{1i}u_i(t) + D_{2i}(t)\omega_i(t), \\ \qquad (i = 1,\ 2,\ \cdots,\ N) \end{cases} \tag{1}$$

where $x_i(t) \in \Re^{n_i}$ is the state vector, $u_i(t) \in \Re^{m_i}$ is the control input, $\omega_i(t) \in \Re^{q_i}$ is disturbance input, and $z_i(t) \in \Re^{s_i}$ is the controlled output. The matrices $E_i \in \Re^{n_i \times n_i}$ may be singular, we shall assume that $rankE_i = r_i \leq n_i$, $\sum_{i=1}^{N} n_i = n$, $\sum_{i=1}^{N} r_i = r \leq n$. A_{ii}, A_{ij}, B_{1i}, B_{2i}, C_i, D_{1i}, D_{2i} are known constant matrices with appropriate dimensions.

In this paper, the following assumptions, definitions and lemmas play an important role in our later proof.

Assumption 1 The external disturbance $\omega_i(t)$ is time-varying and satisfies the constraint condition,

$$\int_0^T \omega_i^T(t)\omega_i(t)dt < d \tag{2}$$

Assumption 2 There exist two orthogonal matrices U_i and V_i such that E_i has the decomposition as

$$E_i = U_i \begin{bmatrix} \Sigma_i & 0 \\ 0 & 0 \end{bmatrix} V_i^T, \tag{3}$$

where $\Sigma_i = diag(\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{ir_i})$ with $\sigma_{ij} > 0$ for $i = 1, 2, \ldots, r_i$. Partition

$$U_i = [U_{1i} \quad U_{2i}], V = [V_{1i} \quad V_{2i}] \tag{4}$$

conformably with (3). From (3), it can be seen that V_{2i} spans the right null space of E_i, and U_{2i}^T spans the left null space of E, i.e., $E_iV_{2i} = 0$ and $U_{2i}^TE_i = 0$.

For a singular system in the form,

$$\begin{cases} E\dot{x}(t) = Ax(t) + G\omega(t), \\ z(t) = Cx(t) + D\omega(t) \end{cases} \tag{5}$$

we introduce the following definition.

Definition 1. The linear continuous-time singular system(LCTSS)(5) with $u(t) = 0$:

(i)The singular system(5) when $\omega(t) = 0$ is said to be regular if $det(sE - A)$ is not identically zero.

(ii)The singular system(5) when $\omega(t) = 0$ is said to be impulse free if it is $deg(det(sE - A)) = rankE$.

Definition 2. The LCSLSS (1) with $u_i(t) = 0$ is said to be finite-time bounded (FTB) with respect to $(c_1, c_2, T, R_1, R_2, \cdots, R_N, d)$ ($0 < c_1 < c_2$ and $R_1 > 0, R_2 > 0, \cdots, R_N > 0$), if

(i) The LCSLSS (1) is said to be regular and impulse free, when $\omega_i(t) = 0$.

(ii) $\sum_{i=1}^{N} x_{i0}^TE_i^TR_iE_ix_{i0} \leq c_1 \Longrightarrow \sum_{i=1}^{N} x_i^T(t)E_i^TR_iE_ix_i(t) < c_2$, $\forall t \in [0, T]$.

Now consider the following memoryless linear decentralized state feedback controller

$$u_i(t) = K_ix_i(t) \tag{6}$$

Then the resulting closed-loop system form (1) and (6) can be written as

$$\begin{cases} E\dot{x}(t) = (A + BK)x(t) + G\omega(t), \\ z(t) = (C + D_1K)x(t) + D_2\omega(t), \end{cases} \tag{7}$$

where

$$\begin{aligned} & E = block - diag(E_1, E_2, \cdots, E_N), \\ & B = block - diag(B_1, B_2, \cdots, B_N), \\ & G = block - diag(G_1, G_2, \cdots, G_N), \\ & C = block - diag(C_1, C_2, \cdots, C_N), \\ & D_1 = block - diag(D_{11}, D_{12}, \cdots, D_{1N}), \\ & D_2 = block - diag(D_{21}, D_{22}, \cdots, D_{2N}), \\ & K = block - diag(K_1, K_2, \cdots, K_N), \\ & A = \bar{A} + \sum_{i,j=1, j\neq i}^{N} ((A_{ij})_{ij}), \quad \bar{A} = \sum_{i=1}^{N} ((A_{ij})_{ij}), \\ & x = col\{x_1, x_2, \cdots, x_N\}, \quad \omega = col\{\omega_1, \omega_2, \cdots, \omega_N\}, \\ & z = col\{z_1, z_2, \cdots, z_N\}. \end{aligned}$$

Thus, the finite-time decentralized H_∞ control problem we address in this paper can be formulated as determining the memory less decentralized state feedback controller (6) such that, the following requirement is satisfied.

(R1) The closed systems (7) is FTB with respect to $(c_1, c_2, T, R_1, R_2, \cdots, R_N, d)$.

(R2) Under the zero-initial condition, the controlled output $z_i(t)$ satisfies

$$\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt \tag{8}$$

for any nonzero $\omega(t)$ satisfies (2), where $\gamma > 0$ is a prescribed scalar.

In this paper, we study finite-time decentralized H_∞ control problems for LCSLSS (1). First, we study the applicable sufficient conditions for the finite-time decentralized H_∞ bounded of the LCSLSS. Then we further investigate the finite-time decentralized H_∞ control problem to find a memoryless linear decentralized state feedback controller for the given LCSLSS so that the resulting closed-loop satisfies (R1) and (R2).

Lemma 1. (Desoer & Vidyasagar, 1975) The matrix measure $\mu(X)$ of the matrix X has following properties:

(i) $-\|X\| \le Re\lambda(X) \le \mu(X) \le \|X\|$,
(ii) $\mu(X) = \frac{1}{2}\lambda_{max}(X + X^T)$.

Lemma 2. (Zhang et al., 2003) The following items are true.

(i) All P satisfying $E^T P = P^T E \ge 0$ can be parameterized as $P = U_1 W U_1^T E + U_2 S$, where $0 \le W \in \Re^{r\times r}$ and $S \in \Re^{(n-r)\times n}$ are parameter matrices; furthermore, when P is nonsingular, $W > 0$.

(ii) All X satisfying $XE^T = EX^T \ge 0$ can be parameterized as $X = EV_1\widehat{W}V_1^T + \widehat{S}V_2^T$, where $0 \le \widehat{W} \in \Re^{r\times r}$ and $\widehat{S} \in \Re^{n\times(n-r)}$ are parameter matrices; furthermore, when X is nonsingular, $\widehat{W} > 0$.

(iii) If $U_1 W U_1^T E + U_2 S$ is nonsingular with $W > 0$, then there exist $\widehat{W}$ and $\widehat{S}$ such that

$$(U_1 W U_1^T E + U_2 S)^{-T} = EV_1\widehat{W}V_1^T + \widehat{S}V_2^T$$

with $\widehat{W} = \Sigma_r^{-1} W^{-1} \Sigma_r^{-1}$.

3. ANALYSIS OF SYSTEM PERFORMANCE

The following lemma states a sufficient condition for the FTB of system (5) which is a fundament to obtain the main results.

Lemma 3. The singular system (5) is regular and impulse free, if there exists a scalar $\alpha \ge 0$ and an invertible matrix $P \in \Re^{n\times n}$, such that the following conditions (9) and (10) hold.

$$E^T P = P^T E \ge 0 \tag{9}$$

$$A^T P + P^T A < \alpha E^T P \tag{10}$$

Proof. Let $M, N \in \Re^{n\times n}$ be nonsingular matrices such that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{11}$$

New partition $M^{-T}PN$ and MAN conform to MEN, that is

$$M^{-T}PN = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}, MAN = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}. \tag{12}$$

From (9), (11) and (12), it is easy to show that $P_1 = P_1^T \ge 0$ and $P_2 = 0$. By using (10) together with (11) and (12), we have

$$\begin{bmatrix} \Gamma_1 & A_3^T P_4 + P_1^T A_2 + P_3^T A_4 \\ A_2^T P_1 + A_4^T P_3 + P_4^T A_3 & A_4^T P_4 + P_4^T A_4 \end{bmatrix} < 0,$$

where $\Gamma_1 = A_1^T P_1 + P_1^T A_1 + A_3^T P_3 + P_3^T A_3 - \alpha P_1$.
By Lemma 1,

$$Re\lambda(P_4^T A_4) \le \mu(P_4^T A_4) = \frac{1}{2}\lambda_{max}(A_4^T P_4 + P_4^T A_4) < 0.$$

Then it can be easily shown that $P_4^T A_4$ is invertible, which implies that A_4 is invertible, too. Hence, in the light of definition and the results of (Xu & Lam, 2006), we have that the singular system (5) is regular and impulse free. The proof is completed. □

Lemma 4. The LCSLSS (1) ($u_i(t) = 0$) is said to be finite-time bounded(FTB) with respect to $(c_1, c_2, T, R_1, R_2, \cdots, R_N, d)$ if there exist scalars $\varepsilon > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$, $\alpha \ge 0$, invertible matrices $P_i \in \Re^{n_i \times n_i}$ and positive matrices $Q_i > 0$, such that (13)-(16) hold.

$$\begin{bmatrix} \Pi_i & P_i^T G_i & I \\ * & -Q_i & 0 \\ * & * & -\frac{\varepsilon}{N-1} I \end{bmatrix} < 0, \tag{13}$$

$$E_i^T P_i = P_i^T E_i \ge 0, \tag{14}$$

$$\lambda_1(E_i^T P_i) < E_i^T R_i E_i < \lambda_2(E_i^T P_i) \tag{15}$$

$$\lambda_2 e^{\alpha T}[\frac{1}{\lambda_1}c_1 + Nd\lambda_{max}] < c_2 \tag{16}$$

where

$$\Pi_i = A_{ii}^T P_i + P_i^T A_{ii} - \alpha E_i^T P_i + \varepsilon P_i^T(\sum_{i=1, i\ne j}^{N} A_{ij}A_{ij}^T)P_i,$$

$$\lambda_{max} = \max_{1\le i\le N} \lambda(Q_i).$$

Proof. Define

$$X = diag(X_1, X_2, \cdots, X_N), \quad P = diag(P_1, P_2, \cdots, P_N),$$
$$Q = diag(Q_1, Q_2, \cdots, Q_N),$$
$$\bar{A} = diag(A_{11}, A_{22}, \cdots, A_{NN}),$$
$$R = diag(R_1, R_2, \cdots, R_N),$$

Then we have (9) and P is an invertible matrix.
By Schur complements, it is easy to show that (13) is equivalent to

$$\Pi_i + \frac{N-1}{\varepsilon} I + P_i^T G_i Q_i^{-1} G_i^T P_i < 0 \tag{17}$$

It is easy to show that

$$\begin{aligned} &P^T \sum_{i=1, i\ne j}^{N} ((A_{ij})_{ij}) + [\sum_{i=1, i\ne j}^{N} ((A_{ij})_{ij})]^T P \\ &\le \sum_{i=1}^{N} ((\varepsilon P_i^T[\sum_{i=1, i\ne j}^{N} A_{ij}A_{ij}^T]P_i + \frac{N-1}{\varepsilon} I)_{ii}) \end{aligned} \tag{18}$$

Note that (18), we have

$$\begin{aligned} &A^T P + P^T A - \alpha E^T P + P^T G Q^{-1} G^T P \\ =&\bar{A}^T P + P^T \bar{A} - \alpha E^T P + P^T G Q^{-1} G^T P \\ &+ P^T \sum_{i=1, i\ne j}^{N} ((A_{ij})_{ij}) + [\sum_{i=1, i\ne j}^{N} ((A_{ij})_{ij})]^T P \\ \le&\sum_{i=1}^{N} ((\Pi_i + \frac{N-1}{\varepsilon} I + P_i^T G_i Q^{-1} G_i^T P_i)_{ii}) \end{aligned} \tag{19}$$

Noting that (17) and (19), condition (13) imply

$$A^T P + P^T A - \alpha E^T P + P^T G Q^{-1} G^T P < 0 \tag{20}$$

Or equivalently

$$\begin{bmatrix} A^T P + P^T A - \alpha E^T P & P^T G \\ G^T P & -Q \end{bmatrix} < 0, \tag{21}$$

By noting (21) implies that $A^T P + P^T A - \alpha E^T P < 0$, (13) and lemma 3, then the LCSLSS (1)($u_i = 0$) is said to be regular and impulse free when $\omega_i(t) = 0$.

On the other hand, (15) is equivalent to

$$\frac{1}{\lambda_2}E^T RE < E^T P < \frac{1}{\lambda_1}E^T RE \tag{22}$$

Let $V(x(t)) = \sum\limits_{i=1}^{N} x_i^T E_i^T P_i x_i(t) = x^T(t)E^T Px(t) \geq 0$, and denote by $\dot{V}(x(t))$ the derivative of $V(x(t))$ along the solution of LCSLSS $(1)(u_i(t) = 0)$. We have

$$\begin{aligned}\dot{V}(x(t)) =& [Ax(t) + G\omega(t)]^T Px(t) + x^T(t)P^T[Ax(t) \\ &+ G\omega(t)] \\ =& \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} A^T P + P^T A & P^T G \\ G^T P & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}\end{aligned} \tag{23}$$

From (14), (21) and (23), we have

$$\dot{V}(x(t)) < \alpha V(x(t)) + \omega^T(t)Q\omega(t). \tag{24}$$

Multiplying (24) by $e^{-\alpha t}$, we can obtain

$$e^{-\alpha t}\dot{V}(x(t)) - \alpha e^{-\alpha t}V(x(t)) < e^{-\alpha t}\omega^T(t)Q\omega(t).$$

Furthermore,

$$\frac{d}{dt}(e^{-\alpha t}V(x(t))) < e^{-\alpha t}\omega^T(t)Q\omega(t). \tag{25}$$

Integrating (25) from 0 to t, with $t \in [0, T]$, we have

$$e^{-\alpha t}V(x(t)) - V(x(0)) < \int_0^t e^{-\alpha t}\omega^T(\tau)Q\omega(\tau)d\tau.$$

Noting that $\alpha \geq 0$, we can obtain

$$\begin{aligned}V(x(t)) <& e^{\alpha t}[V(x(0)) + \int_0^t e^{-\alpha t}\omega^T(\tau)Q\omega(\tau)d\tau] \\ <& e^{\alpha t}[x^T(0)E^T Px(0) + \int_0^t \omega^T(\tau)Q\omega(\tau)d\tau], \\ & \forall t \in [0, T].\end{aligned} \tag{26}$$

Noting that (22), we have

$$V(x(t)) = x^T(t)E^T Px(t) > \frac{1}{\lambda_2}x^T(t)E^T REx(t) \tag{27}$$

Noting that (26) and assumption 1, from (22) it follows that

$$V(x(t)) < e^{\alpha t}[\frac{1}{\lambda_1}x^T(0)E^T REx(0) + \lambda_{max}dN] \tag{28}$$

Putting together (27) and (28), we have

$$\begin{aligned}x^T(t)E^T REx(t) <& \lambda_2 V(x(t)) \\ <& \lambda_2 e^{\alpha t}[\frac{1}{\lambda_1}x^T(0)E^T REx(0) + \lambda_{max}dN]\end{aligned} \tag{29}$$

Condition (16) implies, when $x_0^T E^T REx_0 \leq c_1$ and for all $\forall t \in [0, T]$, $x^T(t)E^T REx(t) < c_2$.

The proof is completed. □

Theorem 1. The unforced LCSLSS $(1)(u(t) = 0)$ is said to be FTB with respect to $(c_1, c_2, T, R_1,\ R_2, \cdots,$ $R_N, d)$ and (8) is satisfied, if there exist scalars $\varepsilon > 0,\ \lambda_1 > 0,\ \lambda_2 > 0,\ \alpha \geq 0$, invertible matrices P_i, such that (14), (15), (30) and (31) hold.

$$\begin{bmatrix} \Pi_i & P_i^T G_i & I & C_i^T \\ * & -\gamma^2 e^{-\alpha T}I & 0 & D_{2i}^T \\ * & * & -\dfrac{\varepsilon}{N-1}I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{30}$$

$$\lambda_2 e^{\alpha T}[\frac{c_1}{\lambda_1} + Nd\gamma^2 e^{-\alpha T}] < c_2, \tag{31}$$

where $\Pi_i = A_{ii}^T P_i + P_i^T A_{ii} - \alpha E_i^T P_i + \varepsilon P_i^T(\sum\limits_{i=1,i\neq j}^{N} A_{ij}A_{ij}^T)P_i$.

Proof. Note that condition (13) implies that

$$\begin{bmatrix} \Pi_i & P_i^T G_i & I \\ * & -\gamma^2 e^{-\alpha T}I & 0 \\ * & * & -\dfrac{\varepsilon}{N-1}I \end{bmatrix} < 0, \tag{32}$$

From Lemma 4, if let $Q_i = \gamma^2 e^{-\alpha T}I$, conditions (14), (15), (31) and (32) guarantee that the LCSLSS $(1)(u_i(t) = 0)$ is FTB.

Now, we need to prove that (8) holds. Note that

$$\begin{aligned}& A^T P + P^T A - \alpha E^T P + C^T C \\ & + (P^T G + C^T D_2)(\gamma^2 e^{-\alpha T}I - D_2^T D_2)(G^T P + D_2^T C) \\ \leq & \sum_{i=1}^{N}((\Pi_i + \frac{N-1}{\varepsilon}I + C_i^T C_i + (P_i^T G_i + C_i^T D_{2i}) \\ & (\gamma^2 e^{-\alpha T}I - D_{2i}^T D_{2i})^{-1}(G_i^T P_i + D_{2i}^T C_i))_{ii}),\end{aligned} \tag{33}$$

and (30) is equivalent to

$$\begin{aligned}& \Pi_i + \frac{N-1}{\varepsilon}I + C_i^T C_i + (P_i^T G_i + C_i^T D_{2i}) \\ & (\gamma^2 e^{-\alpha T}I - D_{2i}^T D_{2i})^{-1}(G_i^T P_i + D_{2i}^T C_i) < 0\end{aligned} \tag{34}$$

Using Schur complements formula, it is easy to show that (30) implies

$$\begin{bmatrix} A^T P + P^T A - \alpha E^T P & P^T G \\ G^T P & -\gamma^2 e^{-\alpha T}I \end{bmatrix} + \begin{bmatrix} C^T \\ D_2^T \end{bmatrix} [\, C \ \ D_2 \,] < 0 \tag{35}$$

Let $V(x(t)) = x^T(t)E^T Px(t) \geq 0$, we have

$$\begin{aligned}\dot{V}(x(t)) =& [Ax(t) + G\omega(t)]^T Px(t) + x^T(t)P^T[Ax(t) \\ &+ G\omega(t)] \\ =& \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} A^T P + P^T A & P^T G \\ G^T P & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}\end{aligned}$$

From (35), we have

$$\dot{V}(x(t)) < \alpha V(x(t)) + \gamma^2 e^{-\alpha T}\omega^T(t)\omega(t) - z^T(t)z(t) \tag{36}$$

The above equation implies that

$$\frac{d}{dt}(e^{-\alpha T}V(x(t))) < \gamma^2 e^{-\alpha(t+T)}\omega^T(t)\omega(t) - e^{-\alpha T}z^T(t)z(t) \tag{37}$$

Integrating (37) from 0 to T, and noting that $x(0) = 0$, we have

$$\begin{aligned}e^{-\alpha T}V(x(t)) < \int_0^T & [\gamma^2 e^{-\alpha(t+T)}\omega^T(t)\omega(t) \\ & - e^{-\alpha T}z^T(t)z(t)]dt\end{aligned}$$

which implies that

$$\int_0^T e^{-\alpha T}z^T(t)z(t)dt < \gamma^2 e^{-\alpha T}\int_0^T e^{-\alpha t}\omega^T(t)\omega(t)dt \tag{38}$$

Noting that

$$e^{-\alpha T}\int_0^T z^T(t)z(t)dt < \int_0^T e^{-\alpha T}z^T(t)z(t)dt, \tag{39}$$

$$\gamma^2 e^{-\alpha T}\int_0^T e^{-\alpha t}\omega^T(t)\omega(t)dt < \gamma^2 e^{-\alpha T}\int_0^T \omega^T(t)\omega(t)dt \tag{40}$$

From (38)-(40), we can obtain

$$\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt \tag{41}$$

The proof is completed. ☐

Theorem 2. The unforced LCSLSS (1)($u_i(t) = 0$) is said to be FTB with respect to $(c_1, c_2, T, R_1, R_2, \cdots, R_N, d)$ and (8) is satisfied, if there exist scalars $\varepsilon > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$, $\alpha \geq 0$, symmetric positive definite matrix $\hat{W}_i > 0$ and matrix $\hat{S}_i$, such that (42)-(45)

$$\begin{bmatrix} \Phi_i & G_i & X_i & X_iC_i^T \\ * & -\gamma^2 e^{-\alpha T} I & 0 & D_{2i}^T \\ * & * & -\dfrac{\varepsilon}{N-1} I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{42}$$

$$\lambda_1(\Sigma_i U_{1i}^T R_i U_{1i} \Sigma_i)^{-1} < \hat{W}_i < \lambda_2(\Sigma_i U_{1i}^T R_i U_{1i} \Sigma_i)^{-1}, \tag{43}$$

$$\lambda_1 \gamma^2 e^{-\alpha T} < 1, \tag{44}$$

$$\lambda_2 e^{\alpha T}(c_1 + dN) < \lambda_1 c_2 \tag{45}$$

hold, where $\Phi_i = X_iA_{ii}^T + A_{ii}X_i^T - \alpha X_i E_i^T + \varepsilon \sum\limits_{i=1, i\neq j}^{N} A_{ij}A_{ij}^T$, $X_i = E_iV_{1i}\hat{W}_iV_{1i}^T + \hat{S}_iV_{2i}^T$.

Proof. From (42), we can obtain $\Phi_i < 0$ and X_i is invertible. According to Lemma 2, there exist $W_i > 0$ and S_i such that

$$(U_{1i}W_iU_{1i}^TE_i + U_{2i}S_i)^{-T} = E_iV_{1i}\hat{W}_iV_{1i}^T + \hat{S}_iV_{2i}^T$$

with $\hat{W}_i = \Sigma_i^{-1}W_i^{-1}\Sigma_i^{-1}$.

Let $P_i = U_{1i}W_iU_{1i}^TE_i + U_{2i}S_i$, then $P_i^{-T} = E_iV_{1i}\hat{W}_iV_{1i}^T + \hat{S}_iV_{2i}^T = X_i$.

Pre-multiplying (42) by diag(P_i^T, I, I, I) and post-multiply (42) by diag(P_i, I, I, I), we can obtain the equivalent condition (30).

Noting that

$$\begin{aligned} E_i^TP_i = P_i^TE_i &= E_i^TU_{1i}W_iU_{1i}^TE_i \\ &= E_i^TU_{1i}\Sigma_i^{-1}\hat{W}_i^{-1}\Sigma_i^{-1}U_{1i}^TE_i \geq 0 \end{aligned} \tag{46}$$

and (43), we can obtain (14) and (15).

Noting (44) and (45), we have

$$\lambda_2 e^{\alpha T}[\frac{c_1}{\lambda_1} + Nd\gamma^2 e^{-\alpha T}] < \lambda_2 e^{\alpha T}[\frac{c_1}{\lambda_1} + \frac{1}{\lambda_1}Nd < c_2] \tag{47}$$

Hence, The unforced LCSLSS (1)($u(t) = 0$) is FTB with respect to $(c_1, c_2, T, R_1, R_2, \cdots, R_N, d)$ and (8) is satisfied under conditions (42)-(45).

The proof is completed. ☐

Remark 1 Theorem 2 is obtained based on the results in Theorem 1, in which a sufficient condition is given to guarantee the LCSLSS (1) ($u_i(t) = 0$) is said to be FTB with respect to $(c_1, c_2, T, R_1, R_2, \cdots, R_N, d)$ and (8) is satisfied in terms of LMI in (42)-(45) when α is fixed. Therefore, they can be solved efficiently.

4. DESIGN OF CONTROLLER

Theorem 3. There exists a decentralized state feedback controller in the form of (6) such that the closed-loop system (7) is FTB with respect to $(c_1, c_2, T, R_1, R_2, \cdots, R_N, d)$ and (8) is satisfied, if there exist scalars $\varepsilon > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$, $\alpha \geq 0$, symmetric positive definite matrix $\hat{W}_i > 0$ and matrix $\hat{S}_i$, Z_i such that (43)-(45) and (48) hold.

$$\begin{bmatrix} \Upsilon_i & G_i & X_i & X_iC_i^T + Z_iD_{1i}^T \\ * & -\gamma^2 e^{-\alpha T} I & 0 & D_{2i}^T \\ * & * & -\dfrac{\varepsilon}{N-1} I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{48}$$

where $\Upsilon_i = X_iA_{ii}^T + A_{ii}X_i^T + Z_iB_i^T + B_iZ_i^T - \alpha X_iE_i^T + \varepsilon \sum\limits_{i=1, i\neq j}^{N} A_{ij}A_{ij}^T$, $X_i = E_iV_{1i}\hat{W}_iV_{1i}^T + \hat{S}_iV_{2i}^T$.

In this case, a finite-time H_∞ decentralized state feedback controller can be chosen as

$$u_i(t) = Z_i^T(E_iV_{1i}\hat{W}_iV_{2i}^T)^{-T}x_i(t). \tag{49}$$

Proof. From $\hat{W}_i > 0$, we can obtain $X_i = E_iV_{1i}\hat{W}_iV_{1i}^T + \hat{S}_iV_{2i}^T$ is invertible. From the Theorem 2 and noting that $A_{K_i} = A_{ii} + B_iK_i$, and $C_{K_i} = C_i + D_{1i}K_i$. Let $Z_i = X_iK_i^T$ and similar to the proof of Theorem 1 and Theorem 2, we can obtain the conclusion.

The proof is completed. ☐

Remark 2 The sufficient conditions for finite-time stabilization for continuous singular systems with exogenous disturbances in Theorem 3 is given. Noting (42)-(45) and (48), we can see that the conditions in Theorem 3 are not LMIs with respective to c_1, c_2, T, d, γ, ε, α, λ_1, λ_2, $\hat{W}_i$, $\hat{S}_i$, Z_i. However, once we can fix c_1, c_2, T, d, γ and α, they can be turned into LMIs based feasibility problem.

Remark 3 Using LMI method can bring some conservatism, which will increase with the order of LMIs. In the future, we will continue to work hard to reduce the order of LMIs, so as to improve the conservatism of LMIs and the applicability of controller design methods.

5. NUMERICAL EXAMPLE

Example: Consider the singular large-scale (1) systems with

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_{11} = \begin{bmatrix} 0.5 & 0.3 \\ -0.5 & 1.5 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.1 & -0.1 & 0.1 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & -0.1 \\ 0.2 & 0.1 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.3 & -0.5 & 0.2 \\ 0.1 & -0.2 & 1 \end{bmatrix}, G_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.5 & 0.3 \end{bmatrix},$$

$$G_2 = [0.1 \;\; 0.1 \;\; -0.1]^T,\; C_1 = [1 \;\; 1],\; C_2 = [1 \;\; 0.5 \;\; 0.4],$$
$$D_{12} = [0.1 \;\; 0.1],\; D_{11} = 0.1,\; D_{21} = 0.1,\; D_{22} = 0.1$$

In this paper, the finite-time H_∞ decentralized controller can be decided by using the algorithm sketch below, with the aid of Matlab LMI Toolbox:

Step 1: Some fixed values are given for c_1, T, d, γ and R_1, R_2, $, \cdots,$ R_N are given;

Step 2: An initial value for c_2 is given;

Step 3: Starting from stable the index $\alpha = 0$, we kept increasing α until a solution is found or maximum value for α is reached;

Step 4: If no solution is found, then the initial value for c_2 should be increased; Otherwise c_2 can be decreased until its minimum is found.

We chose $R_i = I$, $T = 5$, $c_1 = 1$, $\gamma = 0.5$, $d = 0.01$, $x_{10} = (0.1,\ 0.1)$, $x_{20} = (-0.2,\ 0.1,\ ,0.1)$, $\omega(t) = 0.1 sin(t)$, and the initial value for $c_2 = 10$. By solving the LMIs (43)-(45) and (48), the following finite-time controller is achieved

$$u_1(t) = \left[\, -2.9575 \ \ -1.3143 \,\right] x_1(t),$$

$$u_1(t) = \begin{bmatrix} -3.7444 & -2.7261 & -3.402 \\ 1.5288 & -1.1302 & 0.5909 \end{bmatrix} x_2(t),$$

which guarantees the desired close-loop properties with $c_2 = 10$ and stable index $\alpha = 0.25$.

By applying the controller studied in this paper to the closed-loop plant we can achieve Figures 1-4 from the simulation. Figure 1 and Figure 2 show the states of the two subsystems in closed-loop LCSLSS, and it is obvious that the system is finite-time bounded. Then, Figure 3 denote the input of the two subsystems, and Figure 4 denote the controlled output of the two subsystems. These figures imply that the finite-time H_∞ decentralized controller is effective.

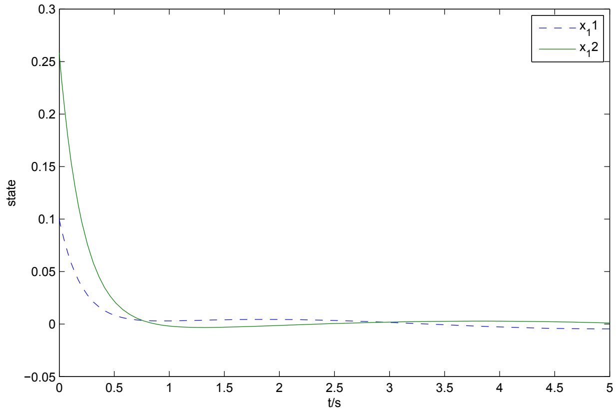

Fig. 1. The state of first subsystem $x_1(t)$

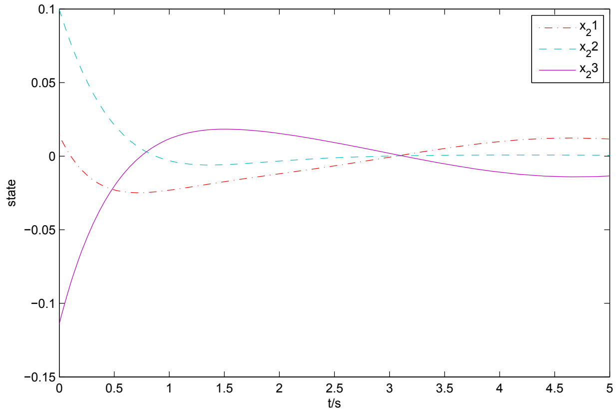

Fig. 2. The state of second subsystem $x_2(t)$

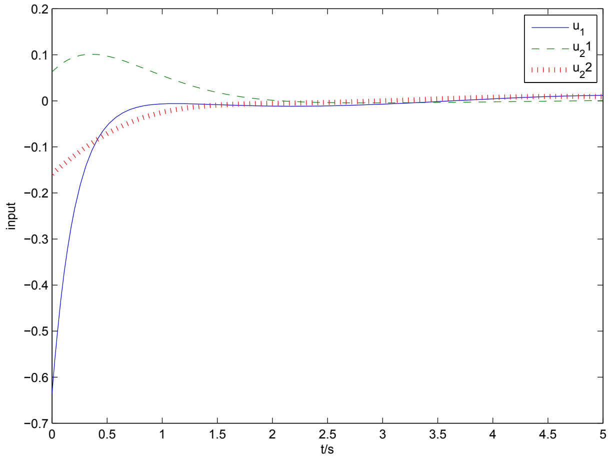

Fig. 3. The input of first subsystem $u_1(t)$ and second subsystem $u_2(t)$

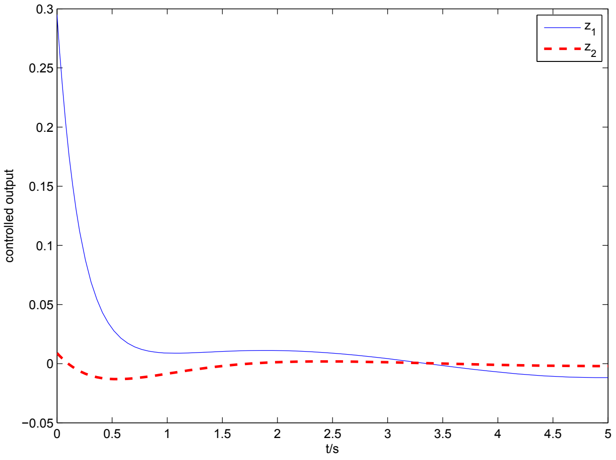

Fig. 4. The controlled output of first subsystem $z_1(t)$ and second subsystem $z_2(t)$

Moreover, we can fix c_2 and find the maximum admissible c_1 to guarantee the desired closed-loop finite-time property.

6. CONCLUSION

In this paper, we extended the definition of H_∞ control to finite-time control H_∞ for LCSLSS. First, new sufficient conditions for FTB are presented, which can decrease conservation. Then, we considered the finite-time H_∞ control problem for LCSLSS via state feedback for a continuous-time system with time-varying norm-bounded exogenous disturbance. The sufficient conditions of the theorems, which ensure the system is FTB and is satisfied (8), are given in terms of linear matrix inequalities, and they can be solved by LMI toolbox. Numerical examples were given to demonstrate the validity of the proposed methodology.

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