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Auto-Tuning of PID Controller with Gain Margin Specification for Digital Voltage-Mode Buck Converter

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Abstract: This paper investigates application of an auto-tuning of a digital PID controller for a DC-DC buck converter, based on the modified relay feedback test (MRFT). Amplitude and frequency of the MRFT oscillations are used as input to PID controller tuning rules that are proposed in this paper. These rules are coordinated with the MRFT through a certain parameter in order to allow for the specification of gain margin – for which a mathematical proof is provided in the paper. Another contribution of this work is the development of the implementation of the MRFT auto-tuning method on a digitally-controlled DC-DC buck converter. A PID controller is auto-tuned and tested on a digitally-controlled buck converter prototype, and its performance is compared to that of an optimal but non-auto-tunable controller. Results show good performance of the proposed method. A final contribution is the discussion of important practical considerations regarding the application of the MRFT-based auto-tuning to switching converters.

Keywords: Auto-tuning, relay-feedback test, DC-DC converter, digital control, buck converter

1. INTRODUCTION

The last few decades have seen wide application of digital controllers in power electronic converters, especially in high-power converters where cost of the digital controller is a small fraction of the total system cost. But in low-cost low-power DC-DC converter applications such as switched-mode power supplies (SMPS), the less expensive analog controller is still mainly preferred. However, recent advancements in CMOS technology have led to cost-efficient microcontrollers offering good performance. This has encouraged the use of digital controllers in low-power DC-DC converters, and consequently increased the implementation of auto-tuning algorithms (Maksimovic et al, 2004). This paper deals with the application of a PID controller auto-tuning on a digitally-controlled DC-DC power-electronic buck converter.

Auto-tuning methods may be classified into parametric and non-parametric. While both methods begin with a test stage, parametric auto-tuning involves online system identification, based on which an appropriate controller is designed. But in non-parametric auto-tuning, the test results are directly used to update the controller – for example through an iterative tuning procedure or using pre-programmed tuning rules. In what follows, selected works from literature related to auto-tuning methods applied to DC-DC converters are reviewed.

In (Stefanutti et al, 2007), a non-parametric PID controller auto-tuning method based on the relay feedback test (RFT) is applied to a DC-DC buck converter. The method involves three stages of test and tuning. However, the method has some limitations. For example, in the first stage of tuning, RFT oscillations are excited at the LC resonant frequency in order to identify it (by including an integrator in the loop along with the relay. But since large oscillations may result at the resonant frequency, the test is carried out at a reduced voltage during the ramp startup of the converter. This somewhat limits the method's practicality, since auto-tuning may be required during regular operation when it is not possible to reduce the operating voltage. Similar work is reported in (Corradini et al, 2007), but measurement of the amplitude of oscillations is avoided in order to reduce measurement errors, and only time-based measurements of the oscillations are used. Even though this improves the robustness of the method in comparison to (Stefanutti et al, 2007), the tuning part still consists of several stages/iterations, which extends the tuning time.

A parametric auto-tuning approach is adopted in (Faraji-Niri and Shaheydari, 2016). The system is first identified using Fourier analysis of the RFT measurements, the tuning rules developed offline are used to calculate controller parameters. This method, however, requires measurement of the time from the application of RFT until oscillations start. This is difficult to obtain with reasonable accuracy, and the work has not been verified experimentally. Another approach of exciting sustained oscillations is used in (Zhao and Prodic, 2007) where the resolution of the digital PWM is temporarily reduced (as opposed to using a relay). This allows for continued regulation of the system even during the test stage – though with poorer tracking. Measurements from the test are used to fetch controller coefficients from a stored lookup table. The digital control hardware must therefore have enough memory to store the lookup table. The controller may also need to have the capability to perform interpolation operations if the lookup table has limited data points stored.

Auto-tuned controllers using the injection of pseudo-random binary sequence (PRBS) perturbations are also reported in (Miao et al, 2005), (Shirazi et al, 2007), (Shirazi et al, 2009), and (Serrano and Tsakalis, 2016). In these works the test stage is followed by a system identification stage, and a controller is then tuned online using different methods. But the test time for PRBS-based methods is longer compared to RFT-based ones, and the tuning approaches are iterative and/or require stages.

In (Shehada et al, 2019), simple auto-tuning of a DC-DC buck converter consisting of a basic RFT test followed by the use of the Ziegler and Nichols (Z-N) PID tuning rules is reported. While use of the Z-N PID tuning rules does not guarantee stability nor a certain performance, the work in (Shehada et al, 2019) promotes the concept of simple test and tuning given its practicality and robustness. The present paper builds on the concept in (Shehada et al, 2019), but presents a much improved auto-tuning method based on the modified relay feedback test (MRFT) (Boiko, 2012). The method guarantees stability with a specified gain margin. The method consists of two stages. In the first stage, the MRFT is performed on the DC-DC buck converter, from which measurements of the amplitude and frequency of the oscillations are taken. In the second stage, the measurements are used to compute the controller parameter values using tuning rules proposed in this paper. The whole tuning approach involves the concept of *coordinated test and tuning*, which guarantees a specified gain margin, regardless of the component values of the converter used. Experimental verification of the auto-tuning method on a buck converter prototype is provided.

In section 2 the MRFT is explained as an evolution of the conventional RFT, and its mathematical analysis is provided. Section 3 describes tuning rules used in the MRFT auto-tuning method. Mathematical proof is provided to show that the tuning rules indeed guarantee the specified gain margin if: 1) some pre-defined coefficients in the tuning rules are designed to satisfy certain constraints, and 2) the MRFT test parameter is coordinated with the tuning rules. Section 4 describes the experimental buck converter setup for which the auto-tuning method is tested. An optimal controller used for comparison against the auto-tuned controller is also described in section 4. Experimental results and some practical considerations are given in section 5, while conclusions are given in section 6.

2. THE MODIFED RLEAY FEEDBACK TEST

Limit-cycle oscillations (LCO) used for tuning purposes are typically excited at the phase cross-over frequency (ω_π), as knowledge of ω_π and the corresponding magnitude provides information that is valuable for controller tuning. Methods of driving a system into LCO include the Ziegler-Nichols closed-loop test (Ziegler and Nichols, 1942) and the relay feedback test (Liu et al, 2013). We further review the RFT as it provides the background for presenting the MRFT.

2.1 The Conventional RFT

The block diagram of the RFT is shown in Fig. 1. The relay function is described by the equation below, where $u(t)$ is the control command produced by the relay, $e(t)$ is the error signal given by the difference between the reference $r(t)$ and the output $y(t)$, and h is the magnitude of the relay command. $W_p(s)$ is the transfer function of the plant to be controlled.

$$u(t) = \begin{cases} h, & e(t) > 0 \\ -h, & e(t) < 0 \end{cases} \tag{1}$$

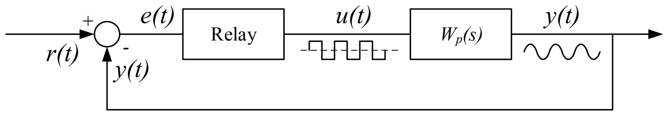


Fig. 1. Block Diagram of the Relay Feedback Test

In most practical situations, the relay operates around a certain non-zero steady-state value, so that all signals in Fig. 1 represent increments to the respective system values in a steady state. The measured frequency of oscillations is called the ultimate frequency (Ω_0 in *rad/s*). The time period of these oscillations, given by $2\pi/\Omega_0$, is called the ultimate time period (T_u). The amplitude of the oscillations is denoted by a_0. The ultimate gain, K_u, refers to the gain from the fundamental component of $e(t)$ to the fundamental component of $u(t)$. In other words, it is the ratio of the amplitudes of the first harmonics of $u(t)$ and $e(t)$. The expression for K_u, derived using a relay model based on describing function (DF) theory (Atherton, 2006), is given below.

$$K_u = \frac{4h}{\pi a_0} \tag{2}$$

The Z-N tuning rules may be used to calculate acceptable controller parameters based on the measured Ω_0 and K_u. However, the Z-N tuning rules guarantee stability with a gain margin of 2 only when a proportional-only controller is used. For a different controller such as PI, PD, or PID, not only a gain margin of 2 may not be attained, rather stability itself is not guaranteed. Nevertheless, Z-N tuning for PID controllers rules usually provide satisfactory performance, and are commonly used in practice. But it would be more beneficial in terms of controller design to force oscillations at what would be ω_π of the open-loop system. The MRFT, first proposed in (Boiko, 2012), was developed in order to address this issue.

2.2 The Modified Relay Feedback Test

In the MRFT, an algorithm given in (3) is used instead of the relay of Fig. 1. $u(t-)$ is the control output immediately prior to the time t. b_1 and b_2 define the switching conditions; they are calculated using $b_1 = -\beta e_{min}$ and $b_2 = \beta e_{max}$, where β is a constant between -1 and 1, and $e_{max} > 0$ and $e_{min} < 0$ represent the last (positive) maximum and last (negative) minimum of the error signal $e(t)$, respectively.

$$u(t) = \begin{cases} h, & \{e(t) \geq b_1\} \; or \; \{e(t) \geq -b_2 \; \& \; u(t-) = h\} \\ -h, & \{e(t) \leq -b_2\} \; or \; \{e(t) \leq b_1 \; \& \; u(t-) = -h\} \end{cases} \tag{3}$$

e_{max} and e_{min} are initially set to zero, making $b_1 = b_2 = 0$. The test is started with the loop closed around the modified relay and the plant, and oscillations start to develop in $y(t)$ and consequently in $e(t)$. Every time a maximum (e_{max}) or minimum (e_{min}) is recorded, the switching conditions (b_1 or b_2) for the upcoming half-cycle are updated, as outlined in (Boiko, 2013). After a few transient cycles, oscillations stabilize such that $|e_{max}| = |e_{min}|$. Similar to RFT, measurements of frequency (Ω_0) and amplitude of oscillations ($a_0 = |e_{max}| = |e_{min}|$) are taken. K_u is still calculated using (2). When sustained oscillations are established with $b = b_1 = b_2 = \beta e_{max} = -\beta e_{min} = \beta a_0$, the MRFT acts, in a sense, as a hysteretic relay with hysteresis value depending on the amplitude of oscillations. The DF for this relay is given below:

$$N(a) = \frac{4h}{\pi a}\sqrt{1-\left(\frac{b}{a}\right)^2} - j\frac{4h}{\pi a}\left(\frac{b}{a}\right) \quad (4)$$

$$\Rightarrow \; N(a) = \frac{4h}{\pi a}\left(\sqrt{1-\beta^2} - j\beta\right) \quad (5)$$

The following harmonic balance equation should be satisfied at Ω_0.

$$W_p(j\Omega_0)N(a_0) = -1 \;\Rightarrow\; W_p(j\Omega_0) = -\frac{1}{N(a_0)} \quad (6)$$

The expressions for $-\frac{1}{N(a_0)}$, and its magnitude and argument, are given below (with a_0 replaced by a for generality).

$$-\frac{1}{N(a)} = -\frac{\pi a}{4h}\left(\sqrt{1-\beta^2} + j\beta\right)$$

$$\left|\frac{1}{N(a)}\right| = \left|-\frac{\pi a}{4h}\right|\sqrt{\left(\sqrt{1-\beta^2}\right)^2 + (\beta)^2} = \frac{\pi a}{4h} = \frac{1}{K_u} \quad (7)$$

$$arg\left(-\frac{1}{N(a)}\right) = arg\left(-\frac{\pi}{4}\frac{a}{h}\right) + arg\left(\left(\sqrt{1-\beta^2} + j\beta\right)\right)$$

$$\Rightarrow \; arg\left(-\frac{1}{N(a)}\right) = -\pi + \tan^{-1}\frac{\beta}{\sqrt{1-\beta^2}} = -\pi + \sin^{-1}\beta \quad (8)$$

It is noted that the magnitude (7) is only a function of the amplitude of oscillations, a, while the argument is only a function of β (8). The Nyquist plot of $-\frac{1}{N(a)}$ is given by a straight line (a ray) starting from the origin and extending in proportion with a. The line forms an angle $\psi = \sin^{-1}\beta$ with the negative real axis, as illustrated in Fig. 2, which also shows the Nyquist plot of $W_p(j\omega)$. The state of sustained oscillations is thus represented by the intersection of the two plots, occurring at $\omega = \Omega_0$, and expressed mathematically in (9) and (10) below.

$$\left|W_p(j\Omega_0)\right| = \left|-\frac{1}{N(a_0)}\right| = \frac{\pi a_0}{4h} = \frac{1}{K_u} \quad (9)$$

$$arg\left(W_p(\mathrm{j}\Omega_0)\right) = arg\left(-\frac{1}{N(a_0)}\right) = -\pi + \sin^{-1}\beta \quad (10)$$

A deficiency of using the conventional RFT followed by the Z-N PID tuning rules is that the gain margin cannot be predicted. The objective of the MRFT-based tuning method is to design a PID controller that results in ω_π of the open-loop system being equal to Ω_0 of the test. Since $|W_p(j\Omega_0)|$ would then be equal to $|W_p(j\omega_\pi)|$, designing tuning rules that allow for the specification of $|W_p(j\Omega_0)|$ is then equivalent to the specification of gain margin.

3. RULES FOR TUNING WITH SPECIFICATION ON GAIN MARGIN

Consider a PID controller of the following form, where K_c is the proportional gain, T_i the integral time, and T_d the derivative time.

$$W_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right) \quad (11)$$

Tuning rules of *homogenous* form, taken from (Boiko, 2012), are used in this paper. The tuning rules are given below, where coefficients c_1, c_2, and c_3 are positive constants.

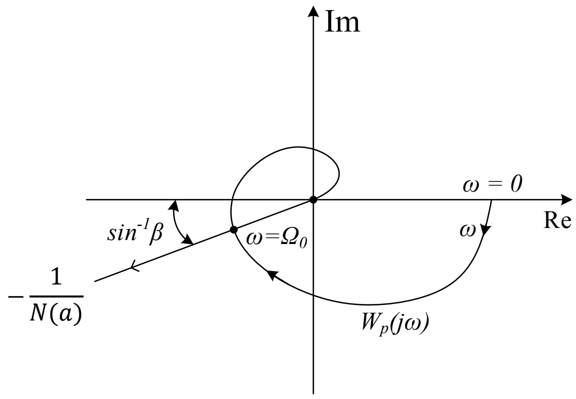


Fig. 2. Nyquist plot of the plant and the negative reciprocal of the relay's DF

$$K_c = c_1 K_u = c_1\frac{4h}{\pi a_0},\;\; T_i = c_2 T_u = c_2\frac{2\pi}{\Omega_0},\;\; T_d = c_3 T_u = c_3\frac{2\pi}{\Omega_0} \quad (12)$$

The similarity in form to the Z-N tuning rules is to be noted. However, for the tuning rules in (12), the ability of MRFT to excite oscillations at any desired frequency from a certain range through using a respective value of β in the test, along with the appropriate choice of c_1, c_2, and c_3, will allow for the specification of a gain margin, γ_m (Boiko, 2012). To ensure this, the coefficients c_1, c_2, and c_3, and the parameter β of the MRFT must be selected in coordination with each other as per the constraints below (Boiko, 2012).

$$\gamma_m = \frac{1}{c_1\sqrt{1+\xi^2}}, \quad \beta = -\frac{\xi}{\sqrt{1-\xi^2}} \qquad \text{where } \xi = 2\pi c_3 - \frac{1}{2\pi c_2} \quad (13)$$

In other words, the use of a set (β, c_1, c_2, c_3) that satisfies (13) will yield the specified γ_m. It should be noted that although the constraints in (13) were proposed in (Boiko, 2012), the rigorous proof was not provided. It is given below.

3.1 Proof that Constraints in (13) Guarantee the Specified Gain Margin

$$G_{ol}(\mathrm{j}\Omega_0) = W_c(\mathrm{j}\Omega_0)\cdot W_p(\mathrm{j}\Omega_0)$$

$$\Rightarrow \; G_{ol}(\mathrm{j}\Omega_0) = c_1 K_u\left(1 + j\Omega_0 c_3\frac{2\pi}{\Omega_0} + \frac{1}{j\Omega_0}\frac{\Omega_0}{2\pi c_2}\right)W_p(\mathrm{j}\Omega_0)$$

$$\Rightarrow \; G_{ol}(\mathrm{j}\Omega_0) = \frac{c_1}{|W_p(\mathrm{j}\Omega_0)|}(1 + j\xi)\cdot\left|W_p(\mathrm{j}\Omega_0)\right|\cdot e^{j arg\left(W_p(\mathrm{j}\Omega_0)\right)}$$

$$\Rightarrow \; G_{ol}(\mathrm{j}\Omega_0) = c_1(1 + j\xi)e^{j arg(W_p(\mathrm{j}\Omega_0))}$$

$$\Rightarrow \; |G_{ol}(\mathrm{j}\Omega_0)| = c_1\sqrt{1+\xi^2}$$

$$\Rightarrow \; \gamma_m = \frac{1}{|G_{ol}(\mathrm{j}\Omega_0)|} = \frac{1}{c_1\sqrt{1+\xi^2}}$$

This proves that the first constraint in (13) is a required condition for the specified γ_m to be achieved. Also, if $\Omega_0 = \omega_\pi$, the argument $G_{ol}(\mathrm{j}\Omega_0)$ is equal to $-\pi$.

$$\arg\left(G_{ol}(j\Omega_0)\right) = -\pi = \arg\left(W_c(j\Omega_0)\right) + \arg\left(W_p(j\Omega_0)\right)$$

$$\Rightarrow \; -\pi = \tan^{-1}\xi + \arg\left(W_p(j\Omega_0)\right)$$

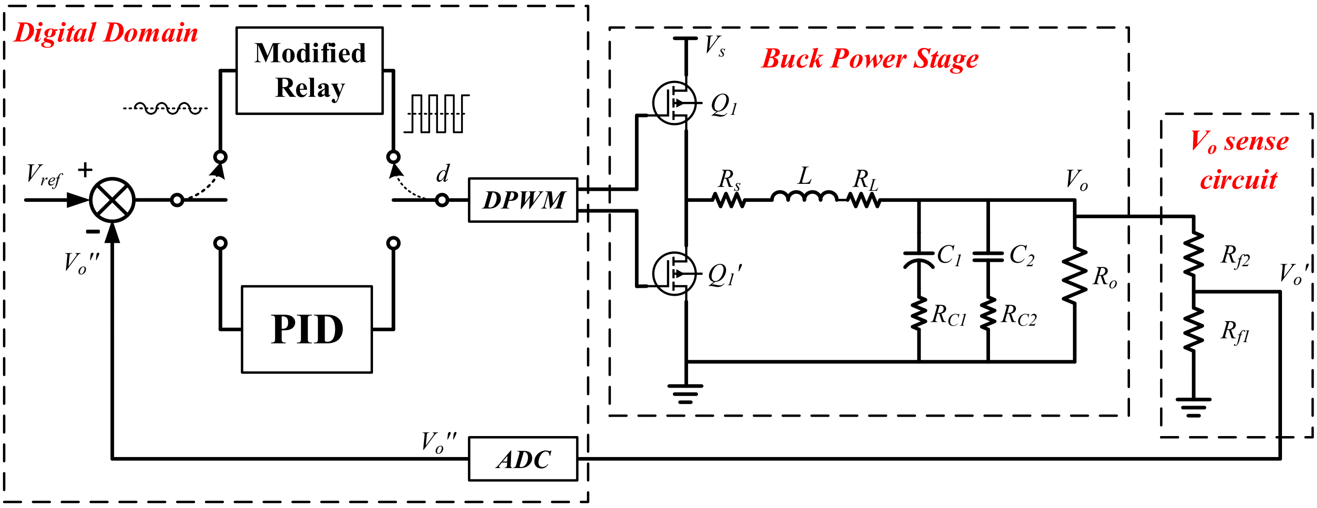


Fig. 3. Schematic of Digitally-Controlled Buck Converter System

➔ $\arg\left(W_p(j\Omega_0)\right) = -\pi - \tan^{-1}\xi$

Using (10) to substitute for the left-hand side of the last result above gives the following.

$-\pi + \sin^{-1}\beta = -\pi - \tan^{-1}\xi$

➔ $\sin^{-1}\beta = -\tan^{-1}\xi$

➔ $\beta = -\sin(\tan^{-1}\xi) = -\frac{\xi}{\sqrt{1-\xi^2}}$

This proves that the second constraint in (13) is also required for achieving $\Omega_0 = \omega_\pi$ and thus ensuring that the γ_m specification is valid. Next, it is noted that the constraints in (13) constitute two equations with four unknowns – assuming that γ_m is specified. This gives a great deal of freedom in choosing the set (β, c_1, c_2, c_3), which for example may be utilized to meet certain quality performance criteria.

For the system under study, the desired gain margin is taken as $\gamma_m = 3$. A digitally-controlled buck converter is used as model system for calculating a specific set (β, c_1, c_2, c_3) that is to be tested on the buck converter prototype. Following multiple trials of simulation and experiments of the model system, the following set satisfying the constraints in (13) for $\gamma_m = 3$ is found.

$$\beta = -0.3,\ c_1 = 0.318,\ c_2 = 3.171,\ c_3 = 0.058 \qquad (16)$$

A negative β means that the relay would lead the error signal, resulting in an oscillation frequency Ω_0 that is larger than ω_π of the plant, and that lies in the second quadrant of the Nyquist plot. It was found that the use of a negative β yields a PID that gives a slightly better transient response than a PID based on MRFT run with a positive β. However, the derived PID controller is not optimal either, and that task of optimizing the set (β, c_1, c_2, c_3) for say a given performance criterion is left as a future work. However, as explained above, the test and tuning rules do guarantee stability with $\gamma_m = 3$, and that is true regardless of the specific choice of the converter's component values.

4. EXPERIMENTAL SETUP

A digitally controlled synchronous buck converter is used for the experimental evaluation of the designed controller. A simplified schematic of the experimental setup is given in Fig. 3. A 150 MHz Texas Instruments fixed-point microcontroller TMS320F28335 is used as the digital controller. The output voltage (V_o) is stepped down by the sensing circuit. The A/D converter (ADC) converts V_o' to a digital value V_o''. The buck converter Pulse-Width Modulation (PWM) switching frequency is 200 KHz. The sampling rate is 200 KHz and is synchronized with the PWM.

Voltage-mode control only is considered, where only the output voltage is controlled; current in the inductor (L) is not controlled. However, a small resistor (R_s) is used to sense inductor current for protection purposes. The output filter capacitance consists of a parallel combination of two larger Aluminum electrolytic capacitors (together as C_1) and three smaller ceramic capacitors (together as C_2). C_1 filters the switching frequency component and reduces voltage ripple, while C_2 limits higher frequency noise. Such arrangement also lowers the equivalent series resistance (ESR) of the capacitors, which reduces ripple in V_o and improves the stability of the control loop. R_L, R_{C1}, and R_{C2} are the parasitic resistances of L, C_1, and C_2, respectively. The parameters of the experimental converter are given in Table 1. Results of the MRFT and the transient response using the controller based on (16) are given in the next section.

Table 1. Buck Converter Parameters

Parameter	Value
V_s	9 V
$V_{o\text{-}ref}$	2 V
L	10 μH
C_1	2 paralleled 330 μF (Electrolytic)
C_2	3 paralleled 22 μF (Ceramic)
R_o	1.57 Ω
R_s	0.03 Ω

For the purpose of validating and benchmarking the MRFT-based controller's performance, another optimal controller is designed as follows. Experimentally measured FR data of the buck converter system is obtained using the Texas Instruments SFRA (software) tool, which works by injecting small sine perturbations in the digital duty-cycle command d and measuring V_o'' from the ADC. A transfer function (TF) of high order is fitted to the obtained FR data, and the transient response of this TF is simulated for a step in the $V_{o\text{-}ref}$. The "integral of time weighted absolute error" (ITAE) is computed in the simulation using the equation below, where $\boldsymbol{X}$ is the vector $[K_p\ T_i\ T_d]$, t_{init} is the time instant at which the $V_{o\text{-}ref}$ step is applied, and t_s is the settling time.

$$ITAE(X) = \int_{t_{init}}^{t_s} (t - t_{init})|e(t - t_{init}, \boldsymbol{X})|dt \quad (17)$$

A least-squares based algorithm is used for optimizing $\boldsymbol{X}$ to give the lowest ITAE. The parameters of the ITAE-optimized PID controller are given in the next section. It should be noted, however, that while this optimal controller is indeed expected to perform better than the MRFT auto-tuned controller, the optimal controller is not suitable for online auto-tuning. Furthermore, an optimized MRFT auto-tuned controller can be developed, which as mentioned earlier is left as a future work.

5. EXPERIMENTAL RESULTS

The first stage of the MRFT auto-tuning method is to run the system with the modified relay in place of the PID controller. Fig. 4 shows an oscilloscope waveform of the MRFT-induced oscillations in V_o, which appear around the 2 V steady state operating point. Fig. 5 shows a digital sample of the error signal with oscillations, which was stored in a buffer in the microcontroller. It is seen that the error signal is highly almost noise free, which is achieved by timing the sampling to occur in between the converter's switching events. Also shown in Fig. 5 is the relay status, where +1 indicates that h is added to the steady-state controller output, while −1 indicates that h is subtracted from the steady-state controller output. The auto-tuning controller automatically detects the frequency and amplitude of the MRFT oscillations in order to calculate the PID controller coefficients using (12) and the c_1, c_2, and c_3 in (16). Based on that, the resulting K_u was 23.12, while the measured T_u was 58.5 µs. Table 2 provides the PID controller parameters for the MRFT auto-tuned controller and the optimal PID controller described in the previous section.

The transient response of V_o for a 2 V to 2.2 V step in $V_{o\text{-}ref}$ is compared for the MRFT and optimal controllers in Fig. 6. The MRFT auto-tuned controller shows only slightly higher overshoot and slightly longer settling time compared to the optimal controller. As earlier mentioned, the MRFT auto-tuned controller is not specifically designed to provide optimal transient performance for the system under experiment. However, it does guarantee stability with a gain margin of 3 for any system and through that provide decent performance for any converter.

Table 2. Parameters of MRFT & Optimal Controllers

MRFT	$K_p = 7.35$	$T_i = 185.5$ µs	$T_d = 3.4$ µs
Optimal	$K_p = 8.19$	$T_i = 185.2$ µs	$T_d = 12.1$ µs

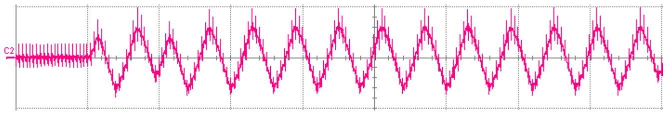


Fig. 4. MRFT oscillations in V_o measured with oscilloscope (scale: 100 mV/div, 100 us/div)

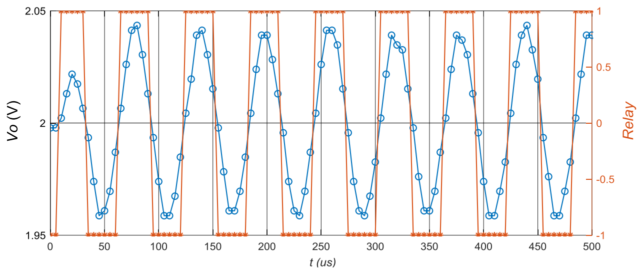
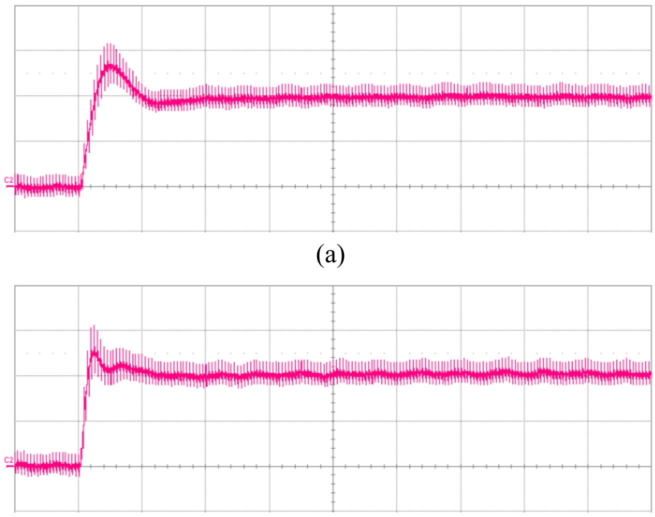


Fig. 5. MRFT periodic oscillations (blue) and relay status (red) as recorded in the microcontroller



(a)

(b)

Fig. 6. Transient response of V_o for 2 V → 2.2 V step in $V_{o\text{-}ref}$ (a) MRFT auto-tuned controller, and (b) Optimal controller (scale: 100 mV/div, 100 µs/div)

In the following, some points related to the practical application of RFT/MRFT to switching converters are given.

5.1. Effect of Sampling on MRFT

Often in switching converters, the sampling frequency (f_s) is set very high, at least as fast as the switching frequency (f_{sw}). For successfully applying RFT/MRFT, it is recommended that the RFT/MRFT oscillation frequency, Ω_0, be about 30 to 40 times lower than f_s. This provides a large enough number of sample in the digitally recorded error signal oscillations, thus allowing for a more accurate detection of Ω_0 and a_o. Also, specific to MRFT, a very low f_s/Ω_0 ratio would result in a delay in applying the relay. For example, consider the experimentally measured oscillations in Fig. 5. There are about 12 samples per cycle of oscillation. Considering only the zero positive crossings, no crossing has a sample that is exactly at $e(t) = 0$. This means that if a relay with zero hysteresis is to be applied, the effective hysteresis would be some positive value and not zero. This problem becomes more severe as the f_s/Ω_0 ratio decreases.

Although it may not be possible to completely mitigate such issue, certain measures may be taken to improve it. First, a higher f_{sw} and hence higher f_s may be used. Also, whenever a test is preformed, a quick calculation of the average deviation of the actual hysteresis versus the intended hysteresis may be made. If the deviation is above a certain threshold, the test may be repeated. This is affordable in practice since each test lasts for a very short time and results in minimal disturbance to the system.

5.2. Value of Relay Magnitude

For a given system the choice of the relay magnitude, h, must be thoroughly tested to ensure that the test does not fail when the system is put in operation. For positive or zero values of β (delayed switching of the relay), usually a smaller h would suffice. But for negative β (advance switching of the relay), the gain of the plant is usually much lower, and so a higher h is required to obtain oscillations of an amplitude convenient for measurement. This is important since measurement noise may prevent proper oscillations from developing in case h is not properly sized (i.e. if h is not high enough).

6. CONCLUSIONS

This paper presents a practical implementation of an auto-tuning PID controller for a buck converter, based on the modified relay feedback test (MRFT). The tuning approach involves coordinated application of the test and tuning rules. Tuning rules guaranteeing the gain margin of 3, when used together with the MRFT having the parameter $\beta = -0.3$, are proposed. Proof that the proposed tuning rules indeed result in the specified gain margin is provided. Experimental results for the application of this auto-tuning method to a DC-DC power electronic buck converter are provided. They show acceptable performance of the developed auto-tuning method, with the performance being not very far from optimal. It is important to note that the tuning rules were not designed for the given experimental prototype, and that the used experimental prototype was a rather random object for an application of the developed method. The method along with the proposed tuning rules is applicable to the whole class of DC-DC buck converters, though it is expected that performance will be better for some buck converter designs rather than others. Important practical considerations related to the application of the MRFT to PWM switching converters are also discussed. Overall, the results of applying the MRFT-based auto-tuned PID controller using coordinated test and tuning rules on a buck converter are promising as providing better than satisfactory performance and are easy to implement. Future work will focus on optimizing the tuning rules for certain transient performance goals.

REFERENCES

D. P. Atherton (2006). Relay Autotuning: An Overview and Alternative Approach. *Ind. & Eng. Chem. Res.*, vol. 45, no. 12, pp. 4075-4080.

I. Boiko (2012). Loop tuning with specification on gain and phase margins via modified second-order sliding mode control algorithm. *Internat. J. System Science*, vol. 43, no. 1, pp. 97-104.

I. Boiko (2013). *Non-Parametric Tuning of PID Controllers: A Modified Relay-Feedback-Test Approach*. Springer, London, UK.

L. Corradini, P. Mattavelli, and D. Maksimovic (2007). Robust Relay-Feedback Based Autotuning for DC-DC Converters. *2007 IEEE Power Electron. Specialists Conf. - PESC*, Orlando, FL, pp. 2196-2202.

M. Faraji-Niri and A. Shaheydari (2016). Autotuing a PID Controller for DC-DC Buck Converter by Improved Relay Feedback Test. *2016 IEEE 1st Int. Conf. on New Res. Achievements in Elect. and Comput. Eng. – ICNRAECE*.

T. Liu, Q. G. Wang, and H. P. Huang (2013). A tutorial review on process identification from step or relay feedback test. *J. Process Control*, vol. 23, no. 10, pp. 1597–1623.

D. Maksimovic, R. Zanem and R. Erickson (2004). Impact of digital control in power electronics. *Proc. 16th Int. Symp. Power Semiconductor Devices and ICs*, Kitakyushu, Japan, pp. 13-22.

B. Miao, R. Zane, and D. Maksimovic (2005). Automated Digital Controller Design for Switching Converters. *36th Power Electron. Specialists Conf. – PESC*, Recife, Brazil, pp. 2729-2735.

V. Serrano and K. Tsakalis (2016). A study on the on-line system identification and PID tuning of a buck converter. *13th Int. Conf. on Networking, Sensing, and Control – ICNSC*, Mexico City, Mexico, pp. 1-5.

M. Shirazi, R. Zane, D. Maksimovic, L. Corradini, and P. Mattavelli (2007). Autotuning techniques for digitally-controlled point-of-load converters with wide range of capacitive loads. *Proc. 22nd Annu. IEEE Applied Power Electron. Conf. and Expo. – APEC*, Anaheim, CA, pp. 14-20.

M. Shirazi, R. Zane, and D. Maksimovic (2009). An Autotuning Digital Controller for DC–DC Power Converters Based on Online Frequency-Response Measurement. *IEEE Trans. Power Elec.*, vol. 24, no. 11, pp. 2578-2588.

A. Shehada, Y. Yan, A. R. Beig, and I. Boiko (2019). Comparison of Relay Feedback Tuning and Other Tuning Methods for a Digitally Controlled Buck Converter. *IECON 2019 - 45th Annual Conference of the IEEE Industrial Electronics Society*, Lisbon, Portugal, pp. 1647-1652.

W. Stefanutti, P. Mattavelli, S. Saggini, and M. Ghioni, (2007). Autotuning of digitally controlled DC–DC converters based on relay feedback," *IEEE Trans. Power Electron.*, vol. 22, no. 1, pp. 199-207.

Z. Zhao and A. Prodic (2007). Limit-Cycle Oscillations Based Auto-Tuning System for Digitally Controlled DC–DC Power Supplies. *IEEE Trans. Power Elec.*, vol. 22, no. 6, pp. 2211-2222.

J. G. Ziegler and N. B. Nichols (1942). Optimum Settings for Automatic Controllers. *Trans. ASME*, vol. 64, 1942, pp. 759-768.