

# Scheduling and Control of Flexible Building Loads for Grid Services based on a Virtual Battery Model <sup>\*</sup>

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**Abstract:** This paper presents a framework for modeling, scheduling, and controlling residential thermostatically controlled loads (TCLs) to provide multiple grid services, such as energy shifting, peak load reduction, and ancillary services. A modeling method is proposed to characterize aggregate flexibility from heterogeneous TCLs using a virtual battery model. Based on the flexibility model, a multi-period optimal scheduling formulation is developed to best utilize the flexibility from building loads and maximize total benefits from stacked value streams. An algorithm is proposed to control individual devices to follow the desired power consumption in real-time. The proposed methods are illustrated and validated through simulations.

*Keywords:* Aggregate flexibility, frequency regulation, optimal scheduling, thermostatically controlled load, virtual battery.

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## 1. INTRODUCTION

Flexibility is required to maintain an instantaneous balance between generation and continuously varying demand in electric power systems. Conventionally, flexible resources on the supply side are dispatched and controlled to follow the load. The increasing penetration of renewable generation imposes challenges to this conventional approach due to the natural uncertainty and variability associated with renewable generation (Georgilakis, 2008). Additional flexible resources are required for frequency regulation and load following. Recent developments and advances in energy storage systems (ESS) are making their application a technically feasible solution to grid problems (Wu et al., 2015, 2016; Balducci et al., 2018). In spite of the benefits from these applications, it is still difficult for some ESS projects to be financially viable given their cost at current market rates. Exploiting flexibility from demand-side resources represents an innovative solution for the power grid. Among all loads, thermostatically controlled loads (TCLs) consume about 20% of the electricity in the United States. When properly controlled, TCLs with an inherent ability to store heat in the thermal mass can vary their power consumption to serve power systems with little impact on customers' convenience and comfort. TCLs represent a significant and largely untapped resource for grid services.

Many studies have been dedicated to using TCLs for grid services during the past few years. Abiri-Jahromi and Bouffard (2016) developed an analytical approach to characterize and control reserve capacity from a group of TCLs. A demand-side response model was proposed for

TCLs by Trovato et al. (2018) to enable optimal scheduling of power and energy consumption and to provide multiple ancillary services. When scheduling a large group of TCLs for grid services, it is computationally expensive yet unnecessary to model and consider detailed dynamics and constraints of individual devices. A simplified model that captures aggregate flexibility can help to facilitate the scheduling and coordinating process. Methods were developed to characterize aggregate flexibility from a collection of TCLs using battery-equivalent models or virtual batteries (VBs). A VB is a scalar linear system that resembles simplified battery dynamics parameterized by charging/discharging power limits, energy limits, and self-discharging rate. Hao et al. (2015) proposed analytical methods to approximate aggregate flexibility from homogeneous TCLs using a VB system. Aggregate flexibility from heterogeneous TCLs was characterized as a prototype set that is bounded by two VB systems. The analytical characterization method was validated by Huang and Wu (2019) through simulations using EnergyPlus. To estimate VB parameters for more complex commercial HVAC systems, a simulation-based method using detailed building models is proposed by Hughes et al. (2015), and an optimization-based method using simplified building models is proposed by Hao et al. (2018).

This paper presents a framework for modeling, scheduling, and controlling residential TCLs to provide multiple grid services. An innovative modeling method is proposed to approximate aggregate flexibility from heterogeneous TCLs using a VB system, similarly as for homogeneous TCLs. A VB-based optimal scheduling formulation is then developed to maximize the total benefits from stacked value streams, including energy charge and demand charge reduction, frequency regulation, and critical asset upgrade deferral. In the proposed scheduling formulation, reserve margins are introduced to power and energy constraints to improve signal tracking performance in real-time. Regula-

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<sup>\*</sup> This material is based upon work supported by the U.S. Department of Energy (DOE), Building Technologies Office through the Emerging Technologies program. Pacific Northwest National Laboratory is operated for the U.S. DOE by Battelle Memorial Institute under Contract DE-AC05-76RL01830.

tion up and down capacities are set to be equal to minimize the energy requirement associated with regulation services, and thereby maximize the revenue. Existing priority-stack-based (PSB) algorithms can be used to control TCLs to follow the desired power signal in real-time. A modified PSB is proposed to properly shift flexibility over different time periods and thereby improve signal tracking performance for TCLs with lock time constraints.

The rest of this paper is organized as follows. Section 2 presents VB models and the proposed characterization method to approximate aggregate flexibility from heterogeneous TCLs. A multi-period scheduling formulation is developed in Section 3 to optimally utilize flexibility from TCLs and maximize the total benefits from stacked value streams. In Section 4, a modified PSB algorithm is proposed to shift flexibility between time periods and improve signal tracking performance. The proposed methods are illustrated and validated through case studies in Section 5. Conclusions are offered in Section 6.

## 2. VIRTUAL BATTERY MODELS AND FLEXIBILITY CHARACTERIZATION

This section presents a flexibility characterization method for using a VB system to approximate aggregate flexibility from TCLs, taking air conditioners (ACs) as an example. Considering a collection of ACs, the indoor air temperature dynamics of an AC with index  $i$  can be represented by a first-order differential equation:

$$C_i \frac{d\theta_i(t)}{dt} = \frac{\theta_o(t) - \theta_i(t)}{R_i} - s_i(t)P_i COP_i + w_i(t), \quad (1)$$

where  $\theta_i$  is the indoor air temperature,  $\theta_o$  is the outdoor temperature,  $C_i$  is the thermal capacitance,  $R_i$  is the thermal resistance,  $s_i$  is the AC on/off state that is unity when AC is on and zero when AC is off,  $P_i$  is the rated power,  $COP_i$  is the coefficient of performance, and  $w_i$  is the external disturbance.

Each AC has a temperature setpoint  $\theta_{r,i}$  with a hysteretic on/off local control within a temperature band  $[\theta_{r,i} - \Delta_i, \theta_{r,i} + \Delta_i]$ . The operating state  $s_i$  evolves as a discrete function of air temperature  $\theta_i$ .

$$s_i(t) = \begin{cases} 1, & \theta_i(t) > \theta_{r,i} + \Delta_i, \\ 0, & \theta_i(t) < \theta_{r,i} - \Delta_i, \\ s_i(t - \epsilon), & \text{otherwise,} \end{cases} \quad (2)$$

where  $\epsilon$  is an infinitely small positive number and  $s_i(t) = s_i(t - \epsilon)$  means that the AC maintains the same on/off state as the previous time.

By defining

$$x_i = \frac{C_i(\theta_{r,i} - \theta_i)}{COP_i}, \quad (3)$$

$\alpha_i = \frac{1}{R_i C_i}$ , and  $p_i = s_i(t)P_i - \frac{(\theta_o - \theta_{r,i})}{R_i COP_i}$ , the thermal dynamic model in (1) becomes

$$\dot{x}_i = -\alpha_i x_i + p_i. \quad (4)$$

Herein,  $x_i$  represents the change of energy stored in the thermal mass by deviating from the temperature setpoint. Because  $s_i$  is either 1 or 0,  $p_i$  is either  $P_i - \frac{(\theta_o - \theta_{r,i})}{R_i COP_i}$  or  $-\frac{(\theta_o - \theta_{r,i})}{R_i COP_i}$ . In addition, the indoor temperature  $\theta_i$  is

required to be within  $[\theta_{r,i} - \Delta_i, \theta_{r,i} + \Delta_i]$ . Hence,  $x_i$  must be within  $[-\frac{C_i \Delta_i}{COP_i}, \frac{C_i \Delta_i}{COP_i}]$ .

Hao et al. (2015) found that for a large population of TCLs, their aggregate behavior with the hybrid model can be approximated by the continuous model. The baseline of total power consumption from a group of ACs can be approximated as

$$P_{\text{base}}(t) = \sum_{i=1}^N \frac{\theta_o(t) - \theta_{r,i}}{COP_i R_i}. \quad (5)$$

This is the power consumption to maintain the temperature of those ACs at their setpoints.

A scalar linear system called VB is proposed to characterize aggregate flexibility from a collection of TCLs. The dynamics of the energy state of a VB is expressed as

$$\dot{x}(t) = -\alpha x(t) + p(t), \quad (6)$$

where  $p$  is the charging/discharging power and  $\alpha$  is the self-discharging rate.

For a collection of homogeneous TCLs, i.e., all devices are with the same thermal capacitance and thermal resistance, the aggregate flexibility can be directly approximated by a VB system.

- The energy state corresponds to the average energy state of the TCLs. Hence, the energy upper bound and lower bound of the VB are  $\sum_{i=1}^n \frac{C_i \Delta_i}{COP_i}$  and  $-\sum_{i=1}^n \frac{C_i \Delta_i}{COP_i}$ , respectively.
- The self-discharging rate is simply  $\alpha_i = \frac{1}{R_i C_i}$ .
- The charging/discharging power corresponds to the deviation of total power consumption from the baseline. Hence, the power upper bound and lower bound of the VB are  $\sum_{i=1}^n P_i - P_{\text{base}}$  and  $-P_{\text{base}}$ , respectively.

For a collection of heterogeneous TCLs, the aggregate flexibility of TCLs is defined as the Minkowski sum. The geometry of this set is, in general, unwieldy. Hao et al. (2015) have shown that the aggregate flexibility can be bounded by two VB systems.

For a collection of homogeneous TCLs, the aggregate flexibility can be approximated using a VB system, which can be directly integrated into existing resource scheduling and coordination methods and tools. On the other hand, for a collection of heterogeneous TCLs, characterizing the aggregate flexibility as a set bounded by two VB systems is not convenient for operational planning purposes. To address this issue, a new characterization method is proposed in this paper to approximate aggregate flexibility from heterogeneous TCLs using one VB system.

*Theorem 1.* The aggregate flexibility from a heterogeneous population of  $N$  TCLs can be characterized using a VB system with energy dynamics in (6), where  $\alpha = \frac{\sum_{i=1}^N \frac{R_i \Delta_i}{COP_i}}{\sum_{i=1}^N \frac{C_i \Delta_i}{COP_i}}$ ,  $p \in [-P_{\text{base}}, \sum_{i=1}^N P_i - P_{\text{base}}]$ , and  $x \in [-\sum_{i=1}^N \frac{C_i \Delta_i}{COP_i}, \sum_{i=1}^N \frac{C_i \Delta_i}{COP_i}]$ .

**Proof.** The proof is omitted due to the space limitation.

### 3. OPTIMAL SCHEDULING FOR STACKED VALUE STREAMS

Three most common types of services that can be provided by residential TCLs are considered in this study.

- Energy and demand charge reduction

The energy charge is based on the amount and time when energy is consumed. It reflects the operational cost of electricity generation and delivery. The demand charge is based on the highest power consumption during a day or a month. It is mainly designed to recover the investment in electricity generation and transportation infrastructure. Separate charges for energy consumption and power demand more fairly distribute the power system's operation and investment cost to customers. Flexible residential TCLs can be used for energy shifting through pre-cooling/heating. The economic reward is the rate/price differential between "charging" (increasing load) and "discharging" (decreasing load), minus the cost of additional losses incurred during pre-cooling/heating. In addition, residential TCLs can also be used for demand charge reduction by lowering the peak demand during a day or a month.

- Frequency regulation

The electric power system must maintain a near real-time balance between generation and load. Balancing generation and load instantaneously and continuously is difficult because loads and generators are constantly fluctuating. Regulation up/down services are required to continuously balance generation and load under normal conditions. Regulation is the use of on-line generation, storage, or load that can change output quickly to track the moment-to-moment fluctuations in customer loads and to correct for the unintended fluctuations in generation. Regulation helps to maintain system frequency, manage differences between actual and scheduled power flows between two control areas, and match generation to load within a control area. Regulation service has been identified as one of the best values from energy storage and demand response for increasing grid stability because of the high cost of regulation services.

- Critical asset upgrade deferral

The basic assumption governing the critical asset upgrade deferral analysis is that demand response from building loads could offset part of the investment in generation, substation, or distribution circuit, given the forecast system peaks. To receive the value from a deferral of critical asset investment/upgrade, load reduction from buildings must exceed a certain power level during peak hours.

#### 3.1 Optimal Scheduling Formulation

In day-ahead scheduling, dispatches of a VB for different services need to be optimized to maximize the benefits from stacked value streams. The total benefits highly depend on how these assets are scheduled and operated. Given the limited energy flexibility from TCLs modeled as a VB, operation in different hours is interdependent. For example, decreasing energy consumption of TCLs in one hour helps to reduce the energy cost in that hour,

but results in less flexible energy that can be used in future hours, and therefore may reduce the overall benefits. Hence, an optimal schedule needs to be determined by solving a look-ahead multi-period optimization problem considering energy and demand charge rates, regulation prices, and the power requirements for critical asset upgrade deferral.

The objective function consists of three components including energy cost, peak demand cost, and revenue from regulation services:

$$\sum_{k=1}^K \lambda_k (L_k + P_k) \Delta T + \sum_{j=1}^J \beta_j d_j - \sum_{t=1}^T (\gamma_t^+ h_t^+ + \gamma_t^- h_t^-) \quad (7)$$

where  $\Delta T$  is the time step size,  $\lambda_k$  is the energy charge rate at time step  $k$ ,  $L_k$  is the baseline load,  $P_k$  is the charging/discharging power from a VB,  $\beta_j$  is the demand charge rate for time period  $j$ ,  $d_j$  is the peak demand,  $\gamma_t^+$  and  $\gamma_t^-$  are the regulation up and down price of hour  $t$ , respectively, and  $h_t^+$  and  $h_t^-$  are hourly regulation up and down capacity of hour  $t$ , respectively.

The discrete version of dynamics of a VB energy state is given as

$$X_{k+1} = aX_k + P_k \Delta T, \forall k = 1, \dots, K \quad (8)$$

where  $a = 1 - \alpha \Delta T$ . The initial and required final energy states are given as

$$X_1 = X(1), X_{K+1} = X(K+1) \quad (9)$$

where  $X_k$  represents the VB energy state at time step  $k$ . The power and energy limits need to be enforced:

$$c_k^p P_k \leq P_k \leq c_k^p \bar{P}_k, \forall k = 1, \dots, K \quad (10a)$$

$$c_k^e X_k \leq X_k \leq c_k^e \bar{X}_k, \forall k = 1, \dots, K \quad (10b)$$

Note that factors  $c_k^e$  and  $c_k^p \in [0, 1]$  are reserve margins introduced to improve signal tracking performance in real-time. Without these margins, when the power and energy state are close to their limits, the temperature of an AC or a water heater might hit the temperature lower or upper bounds and the device becomes unavailable to response control signals from the grid, which negatively affects signal tracking performance.

In general, the economic value from critical asset upgrade deferral is much higher than energy shifting and ancillary services, and only requires decreasing building load to certain levels during a few peak hours each year. Therefore, the power requirements are formulated as constraints, with their service values estimated exogenously.

$$P_k \leq -P_k^{\text{req}}, \forall k \in \mathcal{K}^c, \quad (11)$$

where  $\mathcal{K}^c$  is a set that contains all time steps when a VB needs to be discharged for the purpose of critical asset deferral.

There could be multiple peaks within a billing period, operations of these assets need to be deliberately scheduled to coordinate reduction of peaks so that a demand charge can be effectively reduced. For a demand charge  $j$ , the peak demand  $d_j$  can be captured as

$$L_k + P_k \leq d_j, \forall k \in \mathcal{N}_j, \forall j = 1, \dots, J \quad (12)$$

where  $\mathcal{N}_j$  is a set that contains all the time steps of demand charge  $j$ .

Regulation up and down capabilities at time step  $k$  are constrained as

$$0 \leq r_k^+ \leq P_k - c_k^p P_k, \forall k = 1, \dots, K, \quad (13a)$$

$$0 \leq r_k^- \leq c_k^p \bar{P}_k - P_k, \forall k = 1, \dots, K, \quad (13b)$$

where  $r_k^+$  and  $r_k^-$  denote the regulation up and down capability at time step  $k$ , respectively. The hourly regulation up/down capacity is the minimum of regulation up/down capability of all the time steps within that hour, which can be expressed as

$$0 \leq h_t^+ \leq r_k^+, \forall k \in \mathcal{N}_t, \forall t = 1, \dots, T, \quad (14a)$$

$$0 \leq h_t^- \leq r_k^-, \forall k \in \mathcal{N}_t, \forall t = 1, \dots, T, \quad (14b)$$

where  $\mathcal{N}_t$  is a set that contains all the time steps within hour  $t$ . Regulation associated energy is required to follow regulation signals and needs to be included in the energy constraints:

$$c_k^e X_k \leq X_k - \varepsilon h_t^+ \mathcal{T}, \forall k \in \mathcal{N}_t, \forall t = 1, \dots, T, \quad (15a)$$

$$X_k + \varepsilon h_t^- \mathcal{T} \leq c_k^e \bar{X}_k, \forall k \in \mathcal{N}_t, \forall t = 1, \dots, T, \quad (15b)$$

where  $\varepsilon$  represents the energy reserved per unit regulation service. Different amounts of hourly regulation up and down capacity would require a significant amount of energy to follow regulation signals. Given limited energy flexibility from building assets, regulation up and down capacity are set equal to each other in (16) to take advantage of energy-neutral regulation signals and maximize regulation services provided by building assets:

$$h_t^+ = h_t^-, \forall t = 1, \dots, T. \quad (16)$$

In summary, a linear programming problem can be formulated as

$$\min_{P_k, X_k, d_j, r_k^+, r_k^-, h_t^+, h_t^-} \text{objective function in (7)} \quad (17)$$

subject to (7)–(16).

#### 4. A MODIFIED PRIORITY-STACK-BASED CONTROL

PSB control methods were proposed by Lu and Zhang (2013) and Hao et al. (2015) for TCL control. In these methods, TCLs are grouped based on their status at current and previous time steps. Devices in each group are then sorted based on temperatures. Switching operations for individual TCLs are determined based on desired power consumption and priority lists. The PSB control strategy attempts to minimize ON/OFF switching actions for each unit.

In practice, lockout controls are designed to avoid wear and tear resulting from short-cycling of hardware. With lock time constraints, a TCL could be turned off and stay in lock-off mode in addition to for the purpose of following regulation signals. In our previous study (Wang et al., 2019), it was found that a PSB control algorithm may unnecessarily turn off many ACs within a short time period, resulting in poor signal-following performance. There are time periods with insufficient flexibility from TCLs to follow the desired power consumption while there exists redundant flexibility in other time periods. If a control method can appropriately shift power flexibility between different periods, it could provide better signal-following performance. With this idea in mind, a modified PSB algorithm is proposed to better distribute flexibility over time and thereby improve signal tracking performance.

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#### Algorithm 1 Modified PSB Control

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**Input:** Desired power consumption  $P^*$

- 1: Turn on TCLs that are in lock-on mode or with temperatures above the upper bounds. The total rated power of these TCLs is denoted as  $P_{\text{must-on}}$ .
  - 2: Turn off TCLs that are in lock-off mode or with temperatures below the lower bounds.
  - 3: Randomly select a percentage of TCLs  $\rho$  from the rest and sort the selected TCLs by temperature in descending order. Denote the sorted TCLs as  $\text{TCL}_p$ . The total rated power of these TCLs is denoted as  $P_p$ .
  - 4: **if**  $\bar{E}_{\text{th}} \leq E \leq \underline{E}_{\text{th}}$ , where  $E$  is the energy state, **then**
  - 5:      $\rho = 0$ .
  - 6: **end if**
  - 7: Group the remaining TCLs based on their on/off status at previous time step and sort each group by temperature in descending order. Denote the on-group TCLs as  $\text{TCL}_{\text{on}}$  and the total rated power as  $P_{\text{on}}$ . Denote the off-group TCLs as  $\text{TCL}_{\text{off}}$  and the total rated power as  $P_{\text{off}}$ .
  - 8: **switch**  $P^*$  **do**
  - 9:     **case**  $P_{\text{must-on}} \leq P^* < P_{\text{must-on}} + P_{\text{on}}$
  - 10:         Select the first several TCLs in  $\text{TCL}_{\text{on}}$  until the total power consumption is equal  $P^* - P_{\text{must-on}}$ .
  - 11:     **case**  $P_{\text{must-on}} + P_{\text{on}} \leq P^* < P_{\text{must-on}} + P_{\text{on}} + P_p$
  - 12:         Select all TCLs in  $\text{TCL}_{\text{on}}$ ; select the first several TCLs in  $\text{TCL}_p$  until the total power consumption is equal  $P^* - P_{\text{must-on}} - P_{\text{on}}$ .
  - 13:     **case**  $P_{\text{must-on}} + P_{\text{on}} + P_p \leq P^* < P_{\text{must-on}} + P_{\text{on}} + P_p + P_{\text{off}}$
  - 14:         Select all TCLs in  $\text{TCL}_{\text{on}}$  and  $\text{TCL}_p$ ; select the first several TCLs in  $\text{TCL}_{\text{off}}$  until the total power consumption is equal  $P^* - P_{\text{must-on}} - P_{\text{on}} - P_p$ .
  - 15:     **case**  $P_{\text{must-on}} + P_{\text{on}} + P_p + P_{\text{off}} \leq P^*$
  - 16:         Select all TCLs in  $\text{TCL}_{\text{on}}$ ,  $\text{TCL}_p$ , and  $\text{TCL}_{\text{off}}$ .
  - 17: Turn on the selected TCLs and turn off the remaining ones in  $\text{TCL}_{\text{on}}$ ,  $\text{TCL}_p$ , and  $\text{TCL}_{\text{off}}$ .
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In this method, a portion of TCLs are randomly selected and turned on/off only based on temperature, regardless of their previous on/off states. In this way, some TCLs are switched before they reach the temperature bounds such that they fall into lock mode during time periods with redundant flexibility but become available later when there is insufficient flexibility. The remaining TCLs are divided into “on” and “off” stacks and controlled in a similar manner to that of PSB algorithm. The modified PSB algorithm is given in Algorithm 1.

As a portion of TCLs are sorted and switched purely based on their temperatures, ignoring their previous on/off status, some TCLs that were previously in “off” state could be turned on while some other TCLs that were previously in “on” state are turned off. Therefore, TCLs are generally switched more often than with the PSB control in spite of a better signal-following performance. The increased switching frequency negatively affects the lifetime of TCL devices. To mitigate the potential negative effects, one should only select a number of TCLs and switch them on/off purely based on temperature when the flexibility is insufficient. Insufficient flexibility typically occurs when

the VB energy state is close to its energy bounds. Heuristic energy thresholds can be used to determine when to introduce TCLs switching purely based on temperature ignoring previous on/off status. When the energy state is within the thresholds, the proposed algorithm degenerates to a PSB algorithm.

## 5. CASE STUDIES

Residential ACs within a distribution system model developed in our previous work (Reiman et al., 2019) are used in this paper to illustrate and validate the proposed methods. There are 340 residential houses and 152 with electric ACs. The thermal resistance, thermal capacitance, temperature setpoints, and deadbands for all ACs together with the outdoor temperature have been extracted and are used for developing the VB model and simulating individual AC behavior in real-time. An example utility rate structure is used, where the daily demand charge rate is \$0.803/kW, and the time-of-use energy charge rate is \$0.067/kWh from 11 pm to 8 am, \$0.145/kWh from 12 pm to 6 pm, and \$0.092/kWh for the remaining hours on a summer day. Historical regulation prices and signals from online market database in the U.S. are obtained. Based on the regulation signals,  $\varepsilon$  is set to be 0.1 kWh/kW in this study.

### 5.1 Optimal Scheduling Results

For optimal scheduling, three cases are considered with different reserve margins for power and energy.

- Case 1:  $c_k^e = 1$  and  $c_k^p = 1$
- Case 2:  $c_k^e = 0.75$  and  $c_k^p = 0.55$
- Case 3: Time-varying  $c_k^e$  and  $c_k^p$

The reserve margins are constant but at different levels in the first two cases, and are time-varying in the last case. The obtained benefits for different value streams are listed in Table 1. Smaller  $c_k^p$  and  $c_k^e$  lead to more reserves, which tend to reduce the benefits of building assets from different services, but help to improve tracking performance as will be shown later.

- In Case 1, no energy or power margin is reserved. The obtained total benefit is the largest among all three cases.
- In Case 2, the decreasing coefficients lead to tighter constraints and larger reserves in all hours. The benefits from different services are considerably compromised, about 19.91% reduced compared with Case 1.
- In Case 3, time-varying reserves are applied to better balance the trade-off between economic benefits and tracking performance. In particular,  $c_k^p$  is set to be 0.55 and  $c_k^e$  is set to be 0.75 for hours with poor tracking performance in Case 1 and both of them are set to 0.9 for the remaining hours. The total benefits increase by 15.54% compared with Case 2.

Table 1. Benefits with different VB optimal scheduling scenarios

	Energy	Demand	Regulation	Total
Case 1	25.05	96.40	26.45	147.90
Case 2	19.88	84.05	14.53	118.46
Case 3	22.22	91.60	21.86	135.68

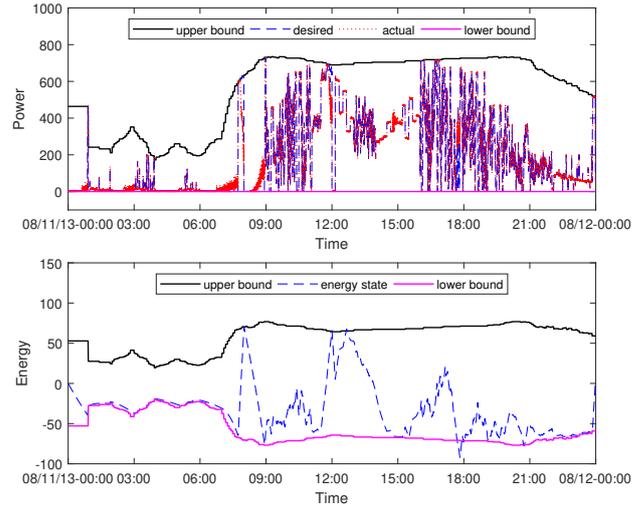


Fig. 1. Signal following results without reserve margins. Top: desired power vs. actual power consumption (kW); Bottom: desired VB energy state (kWh).

### 5.2 Control Results

Based on the optimal schedule and regulation signals, the desired power consumption from ACs is calculated. ACs are then controlled using a PSB algorithm to follow the desired power in each case. The signal tracking results in Case 1 are plotted in Fig. 1. As can be seen, the power consumption cannot well follow the desired value between 6:00 and 9:00, around 12:00 and 15:00. Note that the total power consumption from ACs is discrete, with a step equal to the rated power of a single AC. Therefore, when the tracking error is smaller than the rated power of a single AC, the power signal is considered to be well followed. In this case, the tracking error is larger than 5% of the target power consumption for more than 8% of time.

In Case 2, with sufficient constant reserves for both power and energy, the obtained optimal schedule can be well followed. The tracking results are similar as the last case and therefore are omitted here. The signal tracking results in Case 3 are plotted in Fig. 2. For more than 99.4% of the

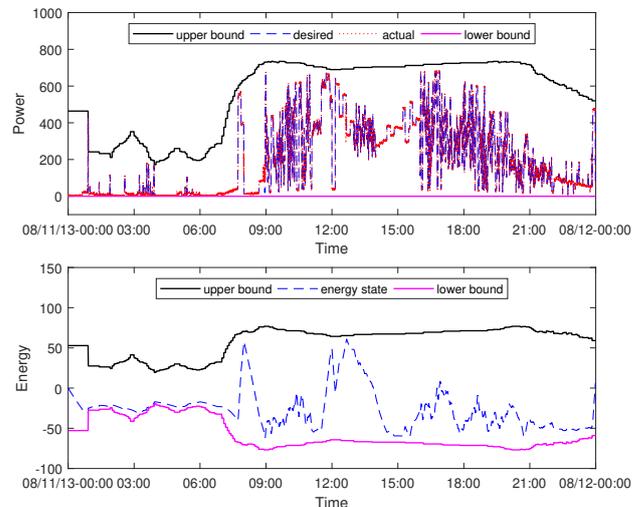


Fig. 2. Signal following results with time-varying reserve margins.

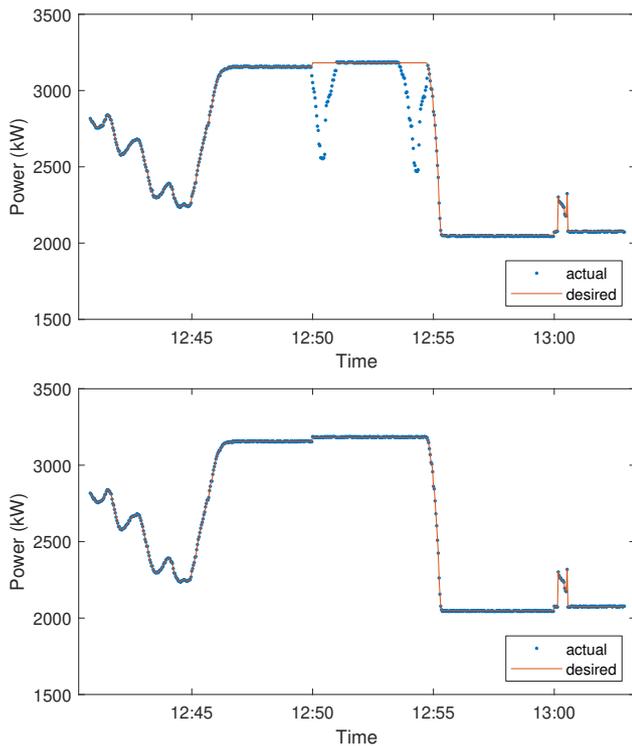


Fig. 3. Signal-following performance of the PSB control (top) and the modified PSB control (bottom).

time, the tracking error is within 5% of the target power consumption.

To test the proposed control algorithm and compare it with the PSB algorithm, simulations were also performed for ACs with a two-minute lock-off time constraint to follow the example power signals within the flexibility limits of the VB. The results for a sampled 25-minute period are shown in Fig. 3. As can be seen, with a two-minute lock-off time constraint, the PSB algorithm fails to control these ACs to follow the desired power consumption in some time periods. It is found that many ACs are in lock-off mode between 12:50 and 12:55 and there are not enough devices to turn on to provide the desired power consumption. On the other hand, using the proposed algorithm with  $\bar{E}_{th}$  set to be 62.5% of the energy upper bound, and  $\underline{E}_{th}$  set to zero (because there is no lock-on time in the example), these ACs can be controlled to follow the regulation signal well all the time, including the time period between 12:50 and 12:55.

## 6. CONCLUSIONS

This paper presents a VB-based flexibility modeling, scheduling, and controlling framework for residential TCLs to provide multiple grid services. With the proposed flexibility characterization, both homogeneous and heterogeneous TCLs can be approximated as a VB system that can be directly integrated into existing resource scheduling and coordination platforms. An optimal scheduling formulation was developed to maximize the total benefits from stacked value streams. It was found that time-varying reserve margins can help to improve signal tracking performance yet with little impacts on total economic benefits. In addition, the PSB algorithm generally provides good

signal tracking performance. The proposed modified PSB algorithm can shift flexibility over time and thereby help to improve the signal tracking performance of ACs with lock time constraints.

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