Comfort-aware Cooperative Cruise Control of Multiple High-speed Trains: An Artificial Potential Field Approach *

Pingping Wang^{*}, Zhiwu Huang^{*}, Feng Zhou^{**}, Bin Chen^{*}, Yue Wu^{*}, Yongjie Liu^{*}, Fei Li^{*}, Jun Peng^{***}

* School of Automation, Central South University, Changsha, China 410083 (e-mail: wangpp@csu.edu.cn)
** School of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, China 410114 (Corresponding author e-mail: zhoufengcsu@gmail.com)
*** School of Computer Science and Engineering, Central South University, Changsha, China 410083

Abstract: In this work, the cruise control problem of multiple high-speed trains movements is investigated. Different from the classical PID control method applied in the practical highspeed train operation system, in this paper, a cooperative cruise control strategy considering both safety and passenger comfort based on layered potential function is proposed. First, the cyber-physical modeling of the high-speed trains system is presented, where the physical layer models the dynamic characteristic, and the cyber layer describes the communication topology. Second, a cooperative control algorithm based on layered potential function is designed. The underlying artificial potential function is introduced to keep a safe distance between adjacent high-speed trains, and the speed consensus rule is realized through the consensus algorithm. The hyperbolic tangent function is selected as the upper artificial potential function to ensure the acceleration of the high-speed train within a comfortable range. Finally, the stability of the control system is proved by the Lyapunov stability theorem, and simulations verify the effectiveness of the control strategy.

Keywords: High-speed trains, Ride comfort, Cruise operation, Cooperative control, Artificial potential field.

1. INTRODUCTION

High-speed trains are playing an increasingly important role in urban transportation, because of the advantages such as high speed, large volume, and safe conditions. The cruise control problem, that is, automatically controls the speed of high-speed train to track the desired trajectory, is challenging due to the complex operating environment. To ensure the operation safety, recent years have seen a growing interest in the research on cruise control of the high-speed trains.

Many researches have been carried out to improve the cruise operation performance of high-speed trains. Faieghi et al. (2014) provides an adaptive control strategy based on Lyapunov method to achieve the speed tracking error asymptotically stable. In order to improve the operation efficiency, Wang et al. (2019) presents a periodically intermittent cruise controller on the basis of practical driving experience. A finite-State Markov modeling of communication-based train control systems is proposed by Wang et al. (2013), in which the moving authority of a train is received from the zone controller. However,

the indeterminacy of environment tends to degrade the performance of cruise control, since the high-speed trains receive instructions from the control center without realtime operation information of other high-speed trains.

As a novel control method, cooperative control of highspeed trains based on train-train communication, instead of passively receiving instructions from the control center, can improve the real-time performance and comfortableness. Utilizing the self-triggered model predictive control method, a cooperative headway regulation control algorithm is proposed by Xun et al. (2019), which can effectively reduce the impact caused by several practical constraints. Bai et al. (2019) models high-speed train as an intelligent agent that can communicate with its neighbors, and exploits a cooperative cruise control law to achieve the consistency goal. According to Wu et al. (2020), the potential field method handles real-time constraints more flexibly. By designing an artificial potential function, Li et al. (2015) presents a new cooperative cruise control strategy to keep a safe distance between adjacent highspeed trains. However, the artificial potential function in Li et al. (2015) may result in a comparatively large acceleration when the distance between two high-speed trains is at the boundary of the function. Large acceleration tends

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to put more stress on the internal organs of passenger, which reduces the passenger comfort.

At present, many researchers investigate the physical factors affecting ride comfort, such as vibration by Zhai et al. (2015), noise by Tokunaga et al. (2016), and so on. Besides, some scholars consider that kinematic parameters, especially accelerations, dramatically affect passenger comfort. For example, to guarantee passenger comfort, Yu et al. (2017) restricts the change rate of the acceleration, Xun et al. (2017) sets an appropriate acceleration threshold. In this paper, passenger comfort is a major consideration in the operation of high-speed trains, and a potential function is used to adjust the acceleration to ensure that passengers are within a comfortable rating range.

In this paper, we present a cooperative control strategy to address the safety issue and comfort issue of highspeed trains based on a hierarchical artificial potential field structure. In the bottom control layer, integration of the consensus algorithm and artificial potential field is to obtain the traction of each high-speed train. The consistency of high-speed trains is implemented by the consensus algorithm based on an adjacent communication topology, and the artificial potential function is constructed to maintain a safe distance of high-speed trains. In the top control layer, the tractions acquired from the bottom controller is input to the hyperbolic tangent potential function. The acceleration of high-speed trains can be maintained within a comfortable rating range. The stability of the control system is proved by the Lyapunov stability theorem. The simulation results illustrate the effectiveness of the proposed control strategy.

In general, the main contributions of this paper are presented as follows:

1. Taking advantage of the speed information of adjacent trains, the consensus algorithm is adopted to realize the cooperative cruise operation of multiple high-speed trains.

2. The artificial potential field is constructed to depict the distance between neighboring high-speed trains. The safe operation is guaranteed by introducing the negative gradient direction vector of the artificial potential function.

3. To achieve passenger comfort, the acceleration of the high-speed train is restricted in a range by introducing the hyperbolic tangent function.

The rest of this paper is organized as follows. In Section 2, the cyber-physical modeling of the control system is presented. In Section 3, the cooperative cruise control strategy considering safety and comfort is designed. In Section 4, simulation results are given to illustrate the effectiveness of the proposed strategy. We conclude this paper in Section 5.

2. SYSTEM MODELING

In this section, the system modeling of multiple high-speed trains and some preliminaries are given. A cyber-physical modeling of high-speed trains control system is presented, in which the physical layer models the motion of highspeed trains, and the cyber layer describes the communication topology. The evaluation of comfort magnitude needed in control strategy designing is introduced.



Fig. 1. The cyber-physical model of cooperative cruise control system, in which the physical layer models the dynamic characteristics, the cyber layer describes the communication topology of high-speed trains, and the control layer presents the structure of controller.

2.1 Dynamic model of multiple high-speed trains

Assume that a set of n high-speed trains are running on a straight and flat railway line, as is depicted in the physical layer of Fig. 1. Ignore additional resistance caused by slopes, curves, and tunnels. The high-speed train is subject to friction resistance and air resistance, which can be expressed as the Davis equation according to the research of Davis (1926). The resistance of unit train mass is given by

$$R_i = c_{i0} + c_{i1}v_i(t) + c_{i2}v_i^2(t), \tag{1}$$

where the first two terms are friction resistance, the third term is air resistance, $v_i(t)$ represents the speed of *i*th high-speed train at time t, c_{i0} , c_{i1} , c_{i2} are the resistance coefficients obtained via wind tunnel experiment.

The widely used single-particle model of high-speed train is presented as follows, in which the train is formalized as a rigid particle neglecting the coupling between adjacent carriages.

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ m_i \dot{v}_i(t) = u_i(t) - m_i R_i, \\ i = 1, 2...n, \end{cases}$$
(2)

where $x_i(t)$ represents the position of the *i*th high-speed train at time *t*, and m_i is the mass of the *i*th high-speed train, and $u_i(t)$ is the traction or braking force to be designed.

2.2 Communication topology

Cooperative control of multiple high-speed trains requires real-time and reliable communication, which directly affects the control feasibility. As is shown in Fig. 1, each high-speed train has a corresponding node storing the realtime speed and location information in the cyber layer. To prevent data loss and data error, each high-speed train communicates with its neighbor trains.

Algebraic graph theory is recommended to describe the communication relationship among high-speed trains. The communication topology can be modeled as a graph $G = (v, \varepsilon)$, where v represents nodes 1, ..., n, and ε denotes a set of edges. The weighted adjacency matrix of the graph is defined as $A = [a_{ij}]_{n \times n}$, which is used to characterize the communication relationship. If node i can receive information from node j, then j is called a neighbor of node i, and $a_{ij} = 1$, otherwise $a_{ij} = 0$. If for any $a_{ij} = 1$, $a_{ji} = 1$, the graph is called undirected. As mentioned above, every train communicates with its adjacent trains. A can be expressed as:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 1 & \cdots & 0 \\ \cdots & & & & \\ 0 & \cdots & 1 & 0 & 1 \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}.$$

Laplacian matrix L = D - A characterizes the convergence of the control system, where in-degree matrix D is known as $D = diag(d_1, d_2, ...d_n), d_i$ represents the number of neighbors of node *i*. According to A above, the corresponding L is obtained as:

$$L = D - A = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ \cdots & & & & \\ 0 & \cdots & -1 & 2 & -1 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}.$$

The communication topology of high-speed trains remains fixed in the operation, so matrix A and L remain the same.

2.3 Comfort evaluation

Comfort is the objective requirement of passengers in the high-speed train operation. The definition of comfort in sitting is under debate. De Looze et al. (2003) points out that comfort is under the influence of various properties, including physical, physiological, and psychological factors. Yang et al. (2013) presents a model considering 6 factors to evaluate subjective comfort, including vibration, noise, air pressure, temperature, humidity, and intensity of illumination. Eboli et al. (2016) evaluates comfort levels by considering the passenger subjective opinion together with acceleration values, which indicates that the comfort level is bad when acceleration exceeds the threshold. ISO2631 provides the test results of comfort based on acceleration, as is shown in table 1.

Table 1. Comfort evaluation standard

Comfort level	$\operatorname{acceleration}(m/s^2)$	uncomfort magnitude
level1	< 0.315	not uncomfortable
level2	0.315 - 0.63	a little uncomfortable
level3	0.5-1	fairly uncomfortable
level4	0.8-1.6	uncomfortable
level5	1.25 - 2.5	very uncomfortable
level6	>2	extremely uncomfortable

According to table 1, passengers will feel uncomfortable if the acceleration is greater than $1m/s^2$. So the range of acceleration should be limited in $(1, 1)m/s^2$.

3. HIERARCHICAL COOPERATIVE CONTROL STRATEGY

In this section, a coordinated cruise control strategy based on a hierarchical control structure is designed to enable each train to track the desired speed, keep the distance between adjacent trains within a safe range, and ensure the comfort of passengers during operation. In the bottom layer, the consensus of high-speed trains is implemented by integrating the consensus algorithm and artificial potential field. While in the top layer, the comfortable operation is ensured by using the hyperbolic tangent function.

3.1 The bottom control layer

For the model presented in section2, a control force component is required to overcome the running resistance, which is expressed as equation 3:

$$u_{i0}(t) = c_{i0}m_i + c_{i1}m_iv_i(t) + c_{i2}m_iv_i^2(t).$$
 (3)

The speed of each high-speed train should track the expected speed v_r , that is to say, the speed deviation between actual speed and desired speed converges to zero. u_{i1} is designed as equation 4:

$$u_{i1}(t) = m_i(v_r - v_i(t)).$$
(4)

The first-order consensus algorithm is employed to align the speed of adjacent high-speed trains. Design the corresponding control components u_{i2} as equation5:

$$u_{i2}(t) = m_i((v_{i-1}(t) - v_i(t)) + (v_{i+1}(t) - v_i(t))).$$
(5)

The distances between neighboring high-speed trains should be kept in a safe range, which can be realized by introducing an artificial potential function. The introduced potential function needs to meet the following requirements: when the distance is within the safe range, the potential energy is small, but once the distance is closed to the safe distance, the energy increases quickly. The potential function is defined as equation 6, where d_1 and d_2 represent the minimum and maximum safe distance respectively, x_{ij} is the distance between *i*th high-speed train and *j*th high-speed train. Let $d_1 = 2, d_2 = 4$, Fig. 2 shows the graph of artificial potential function.

$$U_{ij}(x_{ij}) = \frac{1}{(x_{ij}^2 - d_1^2)} + \frac{1}{(d_2^2 - x_{ij}^2)}.$$
 (6)

Once the distance between the trains is closed to the maximum or minimum safe distance, it should be restored to the safe range as soon as possible. The vector on the negative gradient direction of the artificial potential function, in which the function value decreases fastest, is adopted in the control strategy. The corresponding control component is expressed as equation 7:

$$u_{i3}(t) = -m_i (\nabla_{xi} U_{i(i-1)} + \nabla_{xi} U_{i(i+1)}).$$
(7)

If i = 1, $x_{i-1} = x_i$, if i = n, $x_{n+1} = x_n$.



Fig. 2. The artificial potential function of the cooperative cruise control system depicts the distance between neighboring high-speed trains.

3.2 The top control layer

According to table 1, to guarantee the comfort magnitude of passengers, meanwhile the high-speed train can accelerate as soon as possible, the range of acceleration should be restricted in $(-1, 1)m/s^2$ using a comfort function. On account of the complexity of theoretical analysis, the piecewise function is not considered. The comfort function should satisfy the following three requirements: (1) The output range is (-1, 1), (2) It should have a linear interval near zero, (3) The property of the piecewise function is required on the interval far away from zero. The hyperbolic tangent function well meets these requirements, hence it is employed to restrict the acceleration of high-speed train. The expression of the hyperbolic tangent function is given by equation 8. Fig. 3 shows its graph.

$$f(z) = tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}.$$
(8)

According to table 1, passengers will feel extremely uncomfortable if the acceleration is greater than $2.5m/s^2$. Fig. 3 shows that f(z) is nearly equal to 1 when input z = 2.5, which implies that the comfort function converts extremely uncomfortable state to fairly uncomfortable state.

Based on the above discussions, the hierarchical cooperative control strategy of *i*th high-speed train is established as:

$$u_i(t) = u_{i0} + F_0 \tanh \frac{\alpha u_{i1}(t) + \beta u_{i2}(t) + \gamma u_{i3}(t)}{F_0}, \quad (9)$$

where $F_0 = m \cdot a_{max}$, the weight coefficient $\alpha > 0$, $\beta > 0$, $\gamma > 0$. The architecture of the cooperative cruise control system is presented in the control layer of Fig. 1. The control components in the dotted box are the bottom control strategy, where $u_{i0}(t)$ overcomes the running resistance, $u_{i1}(t)$ is used to track expected speed, $u_{i2}(t)$ realizes speed consensus, $u_{i3}(t)$ keeps the distances between adjacent trains within a safe range. The comfort function is corresponding to the top control layer, which restricts the acceleration range to guarantee the ride comfort.



- Fig. 3. The comfort function restricts acceleration within $(-1,1)m/s^2$, which converts extremely uncomfortable state to fairly uncomfortable state.
- 3.3 Stability of the system

Apply the cooperative control strategy to multiple highspeed trains. The stability of the system is proved based on the Lyapunov stability theorem. Choose the following Lyapunov candidate as:

$$V(t) = \hat{v}^T \hat{v} + a\gamma \sum_{i=1}^n \sum_{j=1}^n (a_{ij} \hat{U}_{ij}), \qquad (10)$$

where $\hat{v} = (\hat{v}_1(t), \hat{v}_2(t), ... \hat{v}_n(t)), \ \hat{v}_i(t) = v_r - v_i$ represents speed deviation of *i*th high-speed train, $a > 0, \ \hat{U}_{ij} = U(\hat{x}_{ij}), \ \hat{x}_{ij}$ denotes position deviation between *i*th and *j*th high-speed train, it holds $V(t) \ge 0$.

The derivative of V(t) is calculated along the trajectory of equation 2 yields.

$$\frac{dV(t)}{dt} = 2\left(1 - \tanh^2 u\right) \sum_{i=1}^n \hat{v}_i(t) \left[-\alpha \hat{v}_i(t) + \beta \left(\hat{v}_{i-1}(t) - \hat{v}_i(t) + \hat{v}_{i+1}(t) - \hat{v}_i(t) \right) + \gamma \sum_{j=1}^n a_{ij} \nabla_{\hat{x}_i} \hat{U}_{ij} \right]
+ a\gamma \sum_{i=1}^n \sum_{j=1}^n a_{ij} \dot{\hat{U}}_{ij}
= 2\left(1 - \tanh^2 u\right) \left[-\gamma \sum_{i=1}^n \sum_{j=1}^n a_{ij} \hat{v} \nabla_{\hat{x}_i} \hat{U}_{ij} - \alpha \hat{v}^T \hat{v} - \beta \hat{v}^T L \hat{v} \right] + a\gamma \sum_{i=1}^n \sum_{j=1}^n a_{ij} \dot{\hat{U}}_{ij}.$$
(11)

Note that

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \dot{\hat{U}}_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} \dot{\hat{x}}_{ij} \nabla_{\hat{x}_i} \hat{U}_{ij})$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} (\hat{v}_i \nabla_{\hat{x}_i} \hat{U}_{ij} + \hat{v}_j \nabla_{\hat{x}_i} \hat{U}_{ij}) \quad (12)$$

$$= 2 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \hat{v}_i \nabla_{\hat{x}_i} \hat{U}_{ij}.$$



Fig. 4. The simulation results of applying only the bottom layer control strategy. (a)The speed curves, (b)The acceleration curves, (c)The distance between adjacent trains.

Let $a = 1 - \tanh^2 u$, combined with the equation 11 and equation 12, we can obtain that

$$\frac{dV(t)}{dt} = 2\left(1 - \tanh^2 u\right) \left(-\alpha \hat{v}^T \hat{v} - \beta \hat{v}^T L \hat{v}\right).$$
(13)

Notice that $0 < \tanh^2 u < 1$, $\alpha > 0$, $\beta > 0$, $L \ge 0$, we have $dV(t)/dt \le 0$. Hence, the control system in equation 2 is stable.

When dV(t)/dt = 0, i.e., $\hat{v} = 0$, which implies that each high-speed train can track the desired speed. Integrating both sides of $dV(t)/dt \leq 0$, we can get that $V(t) - V(0) \leq 0$. V(t) is bounded for the boundedness of V(0). Note that if $x_{ij} \longrightarrow d_1$, or $x_{ij} \longrightarrow d_2$, $U_{ij} \longrightarrow \infty$. According to continuity and boundedness of U_{ij} , it follows that $d_1 < x_{ij} < d_2$, that is, the distances between adjacent trains are kept in the safe range.

4. SIMULATION RESULTS

In this section, a numerical example is provided to evaluate the effectiveness of the proposed cooperative cruise control

Table 2. Resistance coefficient of high-speed train.

Symbol	Value	Unit
c_{i0}	0.01176	N/kg
c_{i1}	0.00077616	$N \cdot s/(m \cdot kg)$
c_{i2}	0.00016	$N\cdot s^2/(m^2\cdot kg)$



Fig. 5. The simulation results of applying proposed hierarchical control strategy. (a)The speed curves, (b)The acceleration curves, (c)The distance between adjacent trains.

strategy. The values of positive coefficient c_{i0} , c_{i1} and c_{i2} obtained by Chen and Zhang (1998) are listed in table 2.

Consider 4 homogeneous high-speed trains running on the railway. The mass of high-speed trains is 400t. Choose the weight coefficient of each control component as $\alpha = 0.01$, $\beta = 0.01$, $\gamma = 0.1$. The expected speed is $v_r = 85m/s$. The employed safe distance range is [15, 20]km. Simulation time horizon considers [0, 350]s. The initial position and speed of each high-speed train are presented in table 3. The initial distances between any two neighboring trains are in the safe range [15, 20]km.

Under the bottom control strategy, the speed curves of all trains are shown in Fig. 4(a). In the beginning, highspeed trains approach to speed consensus state, which is the result of cooperative term u_{i2} . All the trains achieve expected speed 85m/s at about 100s and run with the expected speed in [100, 350]s. The acceleration curves are plotted in Fig. 4(b). Variable initial speeds of highspeed trains lead to different acceleration ranges. All the accelerations converge to 0 at about 100s. The plot of distances between neighboring high-speed trains are shown in Fig. 4(c), in which the distances between adjacent

Table 3. The initial position and speed of 4 high-speed trains.

i	1	2	3	4
$x_{i0}(km)$	51	34	18	1
$v_{i0}(m/s)$	83	61	65	5

trains are stabilized within the safety range. The distance adjustment between 3th and 4th high-speed train is larger than others because their initial speed deviation is larger than the other. All the distances no longer change in about [80, 350]s, which indicates that the speed consensus of high-speed trains is achieved before reaching the expected speed. The simulation results reveal that by applying the bottom control strategy, all high-speed trains can track expected speed, the distances between adjacent trains are limited in the safe range.

Due to the small initial speed of 4th train, its initial acceleration, about $4.8m/s^2$, is obviously larger than others. According to table 1, passengers will feel extremely uncomfortable. Apply the proposed hierarchical control strategy to the multiple high-speed trains. The initial speeds, positions, and parameters are set the same as the above example. The plots of speeds, accelerations, and distances are presented in Fig. 5 respectively. We can see that the accelerations are restricted within $(-1,1)m/s^2$. The 4th high-speed train speeds up with a maximum allowable acceleration of $1m/s^2$ in [0, 30]s, i.e., high-speed trains accelerate as fast as possible on the premise of ensuring passenger comfort. The distances between adjacent trains are still kept in the safe range. In comparison with the above example, it takes longer to reach a steady-state, but the safety and ride comfort are guaranteed at the same time. The first train speed decreases due to disturbance such as gust in [210, 240]s, meanwhile, acceleration becomes positive to track the desired speed. The other trains slow down to ensure operation safety under the action of cooperative term. When the disturbance disappears, the high-speed trains gradually reach the steady-state, and the distances stabilize at another value within the safe range. The simulation results illustrate that the proposed cooperative cruise control strategy is effective, also has the ability to resist disturbance.

5. CONCLUSION

In this paper, the cooperative cruise control problem of multiple high-speed trains considering the safety and comfort has been studied. To address the problem, we proposed a hierarchical artificial potential field architecture based on the cyber-physical modeling of high-speed trains system. In the bottom layer, the consensus algorithm is employed to ensure speed alignment, and artificial potential function is to keep a safe distance between adjacent high-speed trains. The hyperbolic tangent function in the top control layer is adopted to restrict the acceleration within a comfortable range. The simulation results demonstrate the effectiveness and anti disturbance of the proposed control strategy.

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