**Preprints of the 21st IFAC World Congress (Virtual)**
**Berlin, Germany, July 12-17, 2020**

# Passivity-Based Nonlinear Active Suspension Control Utilizing Relative Information

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**Abstract:** In this paper we present the design of active suspension system by using a kind of passivity-based control method, where the proposed suspension system provides the good ride comfort and the good road holding simultaneously and only uses relative displacement and velocity. We show that the proposed method can be extended to nonlinear case easily. The robustness of proposed method is also analyzed.

*Keywords:* vibration, ride comfort, road holding, suspension system, skyhook

## 1. INTRODUCTION

Vibration suppression is a fundamental problem in the design of mechanical systems. Vehicle suspension system, which is a classical example of the problem, should be designed adequately for the sake of suppressing the vibration of vehicles' body. The performances of a suspension system include the ride comfort, the road holding ability, the size of rattle space, and the dynamic tire force as reported in Hrovat (1997). Among these requirements, the performances which are focused in most of studies are the ride comfort and the road-holding ability of vehicle.

As a representative ride comfort control method, the skyhook control which can reduce the resonant peak of the sprung mass quite significantly is extensively studied (see, e.g., Karnrop et al. (1974), Alanoly and Sanker (1988), Sammier et al. (2003), Emura et al. (1994), Nagarajaiah et al. (1993), and Priyandoko et al. (2009).) In order to extend the vibration suppression effect to 5 Hz (4–8 Hz), which is known to be asensitive frequency range to human body according to ISO 2631, and also for improving the vibration suppression effect, the preview suspenstion systems that utilize the information of unsprung mass and road are studied. However, those proposed methods are not only unavoidable to complicate control laws but also require the addition of sensors for the sake of detailed suspension-state observation.

On the other hand, it is known that the skyhook control method does not focus on the vibration of unsprung mass. The direct utilization of skyhook control method often causes a deterioration in the road-holding ability of the vehicle. To solve this problem, some modified skyhook methods and the methods using active force control (AFC) have been proposed by Ahmadian et al. (2004), Besinger et al. (1995), Novak and Valasek (1996), Hewit and Burdess (1981), and Hewit and Marouf (1996).

Although the utilization of aforementioned methods can bring nice performance in a limited frequence range, they are based on an assumption that all the state are measurable, while some of the vehicle states are hard to measured in actual systems. In particular, most methods require the utilization of absolute information of sprung and unsprung masses, but inexpensive sensors can only measure the information of relative positions and velocities. From this point of view, the methods based on linear-quadratic-Gaussian (LQG) methodology (e.g. Ulsoy et al. (1994)), the $H_\infty$ control technique (e.g. Li et al. (2014), Moran and Nagai (1992)), and the saturated adaptive robust control (ARC) strategy (Sun et al. (2013)) have been proposed.

The limitation of the aforementioned studies is that the application of these methods to nonlinear cases are tricky, while most components, such as springs and dampers, containing nonlinearities. The dissipative properties of Euler–Lagrange (or Hamiltonian) systems guarantee the asymptotic stabilities of the nonlinear controlled systems. However, under linear state feedbacks, which are designed based on linear approximations, the global asymptotical stability of the closed-loop systems is no longer guaranteed, because the feedbacks destroy the structures of Hamiltonian systems. Namely, methods preserving the structure of Hamiltonian systems are required for the control of nonlinear systems. Otherwise, some Hamilton-Jacobi partial differential equations should be solved for the global asymptotic stability.

Consequently, a new suspension system which can obatin the good ride comfort and good road holding performance simultaneously by only using relative information is expected. Furthermore, the application to the nonlinear cases should be easy. In this paper, a powerful controller design technique that is widely applied in equilibrium stabilization problem so called the interconnection and damping assignment passivity-based control (IDA-PBC) methods Ortega et al. (2002) is adopted, because this method can be applied to nonlinear systems and preserve the structure of generalized Hamiltonian systems. As a kind of energy shaping method, this method is suitable for applying the main idea of skyhook control. The aim of this paper is to present the design of active suspension system by using IDA-PBC, where the proposed suspension system

⋆ This work was supported by JSPS KAKENHI Grant Number JP19H02157.

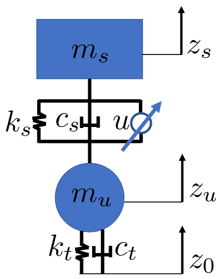


Fig. 1. Quarter-Car Model.

provides the good ride comfort and the good road holding simultaneously and only uses relative displacement and velocity. Moreover, besides the damping term, we utilize the characteristic of energy shaping method to change the mass of sprung mass and unsprung mass, so that the vibration suppression effect can be strengthened.

The rest of this paper is organized as follows: In Section 2, the port-Hamiltonian system of a 2-DOF quarter-car model is derived. In Section 3, we briefly introduce the standard formulation of IDA-PBC method. In Section 4, we apply the IDA-PBC to derive the control law that is only using relative displacement and velocity. In Section 5, we propose the guideline for parameter selction of control law. In Section 6, the performance of the proposed suspension system is evaluated by numerical calculation. In Section 7, we show that the proposed method can be extended to nonlinear case easily. Conclusions are given in Section 8.

## 2. PROBLEM FORMULATION

In this paper, we mainly deal with linear systems since the analysis of linearly approximated systems is sufficient for the performance evaluation. Our method can be extended to nonlinear cases by using our previous result (Hao et al. (2018)), which will be mentioned in Section 7.

A 2-DOF linear quarter-car model is shown in Fig. 1. This is the one which is widely used for suspension analyses. The dynamic model of a quarter-car can be described by the following equations:

$$m_s\ddot{z}_s = c_s(\dot{z}_u - \dot{z}_s) + k_s(z_u - z_s) + u, \tag{1}$$

$$m_u\ddot{z}_u = c_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) \tag{2}$$

$$+ c_t(\dot{z}_0 - \dot{z}_u) + k_t(z_0 - z_u) - u, \tag{3}$$

where $m_s$ stands for a quater of the suspension mass; $m_u$ is the unsprung mass; $z_s$ and $z_u$ are the vertical displacements of sprung mass and unsprung mass, respectively; $z_0$ represents the road profile; $k_t$ is the tire stiffness, whereas $k_s$ is the stiffness of the spring between the tire and the chassis; and $c_s$ is the damping of a passive damper that provides a damping force proportional to the velocity $\dot{z}_s - \dot{z}_u$. The Hamiltonian can be written as

$$H = \frac{1}{2}(m_s\dot{z}_s^2 + m_u\dot{z}_u^2 + k_s(z_s - z_u)^2 + k_t(z_u - z_0)^2). \tag{4}$$

Since one of our purposes is only using relative information, it will become convenient to design the control law if the state of the system is described as relative information. By rewriting the Hamiltonian with new state

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} z_s - z_u \\ z_u - z_0 \end{bmatrix}, \tag{5}$$

the port-Hamilton system can be described as

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = (J - R)\frac{\partial H}{\partial x}^\top + Dw + Bu, \tag{6}$$

where

$$H(q,p) = \frac{1}{2}p^\top M^{-1}p + V$$
$$M = \begin{bmatrix} m_s & m_s \\ m_s & m_s + m_u \end{bmatrix},\ p = M\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 + w \end{bmatrix},\ w = \dot{z}_0,$$
$$V = \frac{1}{2}(k_s q_1^2 + k_t q_2^2),$$
$$J = \begin{bmatrix} O & I \\ -I & O \end{bmatrix},\ R = \begin{bmatrix} O & O \\ O & C \end{bmatrix},\ C = \begin{bmatrix} c_s & 0 \\ 0 & c_t \end{bmatrix},$$
$$B = (0\ 0\ 1\ 0)^\top$$
$$D = (a^\top\ \ -Ca^\top)^\top,\ a = (0\ \ -1)^\top.$$

The ride comfort performance can be evaluated by the sprung mass acceleration $\ddot{z}_s$, and the the road holding performance can be evaluated by the tire deflection $q_2$. Hence, the purpose of this paper is to decrease the value of sprung mass acceleration $\ddot{z}_s$ and tire deflection $q_2$ simultaneously with feedback law that only utilizes relative information.

## 3. STANDARD IDA-PBC FORMULATION

The IDA-PBC method is a powerful controller design technique to solve the stabilization problem and the discussed dynamics are often written as

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & I_n \\ -I_n & 0_{n\times n} \end{bmatrix}\frac{\partial H}{\partial x}^\top + \begin{bmatrix} 0_{n\times m} \\ G(q) \end{bmatrix}u, \tag{7}$$

where $q, p \in \mathbb{R}^n$ are the generalized position and momentum, respectively, $u \in \mathbb{R}^m$ is the control input, $G(q) \in \mathbb{R}^{n\times m}$, with rank$(G)= m$. The controlled system is underactuated when $m < n$. The Hamiltonian function $H$ is defined as,

$$H(q,p) = \frac{1}{2}p^\top M^{-1}(q)p + V(q) \tag{8}$$

where $M \in \mathbb{R}^{n\times n}$ is the positive definite inertia matrix and $V \in \mathbb{R}$ is the potential energy.

The control objective is to design a static, state feedback that assigns to the closed loop a desired stable equilibrium $(q,p) = (q^\star, 0)$, $q^\star \in \mathbb{R}^n$. This is achieved in IDA-PBC by matching the port-Hamiltonian (pH) target dynamics

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & M^{-1}(q)M_d(q) \\ -M_d(q)M^{-1}(q) & J_2(q,p) - R_d(q) \end{bmatrix}\frac{\partial H}{\partial x}^\top, \tag{9}$$

with the new Hamiltonian function

$$H_d(q,p) = \frac{1}{2}p^\top M_d^{-1}(q)p + V_d(q), \tag{10}$$

where the desired mass matrix $M_d \in \mathbb{R}^{n\times n}$ is positive definite, the desired potential energy $V_d \in \mathbb{R}$ verifies

$$q^\star = \text{argmin}V_d(q), \tag{11}$$

and the desired damping matrix is defined by

$$R_d(q) = G(q)K_pG^\top(q) \geq 0,$$

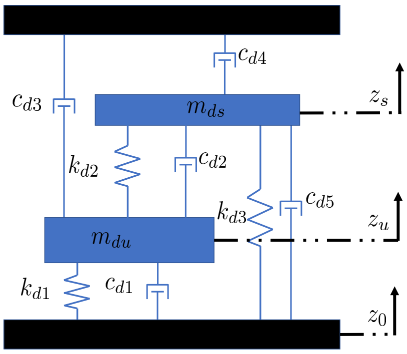


Fig. 2. Desired system.

with $K_p \in \mathbb{R}^{m\times m}$ a free positive definite matrix. The matrix $J_2 \in \mathbb{R}^{n\times n}$ is free to the designer and fulfills the skew-symmetry condition

$$J_2(q,p) = -J_2^\top(q,p). \tag{12}$$

The closed-loop system (9) has a stable equilibrium point at $(q^\star, 0)$ with Lyapunov function $H_d$, which verifies

$$\dot{H}_d = -(G^\top M_d^{-1}p)^\top K_p (G^\top M_d^{-1}p) \leq 0. \tag{13}$$

By matching the right-hand sides of (7) and (9), we can derive the expression of the static state feedback law $u(q,p)$.

## 4. APPLICATION OF IDA-PBC

### 4.1 Overview

In this paper, we match the desired system whose structure is like Fig 2 with the controlled system.

### 4.2 Desired system

We construct the desired system with an artificial structure matrix as follows:

$$\dot{x} = \begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} = (J_d(q_1) - R_d(q_1))\frac{\partial H_d}{\partial x}^\top + D_d\omega, \tag{14}$$

where

$$H_d(q,p) = \frac{1}{2}(p^\top M_d^{-1}p + V_d(q_1,q_2)), \tag{15}$$

denotes the Hamiltonian of desired system, and

$$M_d(q_1) = \begin{bmatrix} m_{ds} & m_{ds} \\ m_{ds} & m_{ds}+m_{du}\end{bmatrix},$$

$$J_d(q_1) = \begin{bmatrix} O & M^{-1}M_d \\ -M_dM^{-1} & J_2\end{bmatrix},$$

$$J_2(q_1) = \begin{bmatrix} 0 & j_e \\ -j_e & 0\end{bmatrix},$$

$$V_d(q_1,q_2) = \frac{1}{2}q^\top K_d q,\ K_d = \begin{bmatrix} k_{d2}+k_{d3} & k_{d3} \\ k_{d3} & k_{d1}+k_{d3}\end{bmatrix},$$

$$R_d = \begin{bmatrix} O & O \\ O & C_d\end{bmatrix},$$

$$C_d = \begin{bmatrix} c_{d2}+c_{d4}+c_{d5} & c_{d4}+c_{d5} \\ c_{d4}+c_{d5} & c_{d1}+c_{d3}+c_{d4}+c_{d5}\end{bmatrix},$$

$$D_d = (0\ \ -1\ c_{d5}\ c_t)^\top.$$

Here, $J_d(q_1)$, $R_d(q_1)$, $V_d(q)$, and $M_d(q_1)$ denote an artificial skew-symmetric structure matrix, a positive semidefinite damping matrix, a potential energy, and the inertia matrix in the desired Hamiltonian, respectively.

### 4.3 Matching Dynamics

The expression for the feedback law with equality and inequality constraints of the parameters of the desired system can be derived by matching the dynamics of the desired system with that of the controlled system as follows:

$$(J_d - R_d)\frac{\partial H_d}{\partial x}^\top = (J-R)\frac{\partial H}{\partial x}^\top + B_2u + (D - D_d)w. \tag{16}$$

We define mass ratios

$$r_1 = \frac{m_{ds}}{m_s},\quad r_2 = \frac{m_{du}}{m_u}. \tag{17}$$

The following equality constraints can be derived from the matching equation.

$$c_{d4} + c_{d5} = -j_e, \tag{18}$$
$$c_{d1} = c_t - c_{d5}, \tag{19}$$
$$c_{d3} = c_t r_2 - c_t - c_{d4}, \tag{20}$$
$$r_2 k_{d1} = k_t - r_1 k_{d3},\ r_1 k_{d3} = (r_2 - r_1)k_{d2}. \tag{21}$$

From the third equation of (16), a feedback law $u = \alpha_{\text{raw}}(q,p,\omega)$ is obtained. Because the feedback should be a function of $q$ and $\dot{q}$ only, we disassemble the feedback law as

$$\alpha_{\text{raw}}(q, M(\dot{q} - a\omega), \omega) = \alpha(q,\dot{q}) + \alpha_{\text{rest}}(q,\dot{q})\omega.$$

The coefficient $\alpha_{\text{rest}}(\cdot)$ should be zero identically, and hence we decompose it again as

$$(c_{d2}+c_{d4}+c_{d5})(r_1 - r_2) = r_1(2(c_{d4}+c_{d5}) - c_{d5}r_2) \tag{22}$$

With these equality constraints, we obtain a feedback law

$$\begin{aligned} u =& c_s\dot{q}_1 + \frac{(2c_{d2}+c_{d5}r_2)}{(r_1+r_2)}\dot{q}_1 + c_{d5}\dot{q}_2 + k_s q_1 \\ &- r_2k_{d2}q_1 - (r_2 - r_1)\,k_{d2}q_2. \end{aligned} \tag{23}$$

Because of the feature of IDA-PBC, the closed-loop system is identical to the desired system.

According to the definition, some parameters should be positive definite to ensure the asymptotic stability. To make the new Hamiltonian positive definite, $M_d$ and $V_d$ should be positive definite, i.e.,

$$r_1 > 0,\ r_2 > 0 \tag{24}$$
$$K_d > 0. \tag{25}$$

The (25) is equivalante to

$$k_{d2} + k_{d3} > 0 \tag{26}$$
$$\text{det}K_d > 0. \tag{27}$$

Both of these inequality constraints can be satisfied by

$$k_{d2} > 0. \tag{28}$$

Moreover, for the asymptotical stability of the desired system, $C_d$ should be positive definite as

$$c_{d2} + c_{d4} + c_{d5} > 0, \tag{29}$$
$$\det C_d > 0. \tag{30}$$

Since (30) can be ensured by setting

$$c_{d4} + c_{d5} = \frac{c_t r_1 r_2}{r_1 - r_2}, \tag{31}$$

we transform (29) to

$$\frac{r_1r_2}{r_2^2 - r_1^2}\left[c_t r_1\,(r_2 - 2) + c_{d4}\,(r_2 - r_1)\right] > 0, \tag{32}$$

by combining (22) and (31). While designing the parameters for desired system, we must take into consideration that all of the parameters should satisfy the equality constraints (18), (19), (20), (21), (22), and (31), and the inequality constraints (24), (28), and (32). Therefore, we choose $r_1, r_2, k_{d2}$, and $c_{d4}$ as free parameters. The other parameters are determined from the equality constraints.

## 5. GUIDELINE FOR PARAMETER SELECTION

As we mentioned before, our purpose is to decrease the value of sprung mass acceleration $\ddot{z}_s$ and tire deflection $q_2$ simultaneously. Considering the empirical knowledge for skyhook system Karnrop et al. (1974), we expect that large $c_{d4}$ and $m_{ds}$ enhance the vibration suppressoin/isolation effects with respect to the sprung mass $m_{ds}$. For the tire deflection $q_2$, large $c_{d1}$ should be selected. We rewrite (19) as

$$c_{d1} = \left(1 - \frac{r_1 r_2}{r_1 - r_2}\right) c_t + c_{d4}, \tag{33}$$

and we can show that selecting $0 < r_2 < 1$ and large $r_1$, $c_{d4}$ will lead to a large $c_{d1}$.

On the other way, a sufficient condition of (32) can be written as

$$r_1 > 2 > r_2 > 0, \tag{34}$$

which is satisfied by aforementioned setting.

Consequently, the guideline for parameter selection of desired system is setting $0 < r_2 < 1$ and large $r_1$, $c_{d4}$. The other parameters are determined from the equality constraints, and the inequality constraints which guarantee the asymptotical stability of the desired system are satisfied naturally by aforementioned parameter setting.

*Remark 1.* From the point of view of the desired system, setting large $r_1$ and small $r_2$ makes the virtual vehicle body and virtual unsprung mass (usually the tire structure) heavy and light, respectively. It is well known that heavy body is effective to suppress the vibration, and the light unsprung mass is effective to follow the undulation of road. In general, it requires a large lateral force to control a heavy body, and the force may exceed the tire capacity. However, our feedback law only consider the vertical direction, which means the horizontal performance of desired system will be the same as the original one.

## 6. SIMULATION RESULT

In this section, we verify the suspension effect of the feedback law with an example. The parameters of controlled object are set as table 1. In this paper, we compare the

Table 1. Parameters for calculation

| Parameters | Symbol | Unit | Value |
|---|---|---|---|
| Sprung mass | $m_s$ | Kg | 500 |
| Unsprung mass | $m_u$ | Kg | 50 |
| Spring stiffness | $k_s$ | N/m | 30,000 |
| Damping coefficient 1 | $c_s$ | N/(m/s) | 2,000 |
| Damping coefficient 2 | $c_t$ | N/(m/s) | 200 |
| Tire stiffness | $k_t$ | N/m | 300,000 |

vibration suppression effect of proposed method with the performance that is under skyhook damper controller. The control law can be descibed as

$$u = -c_{sh} \dot{z}_u.$$

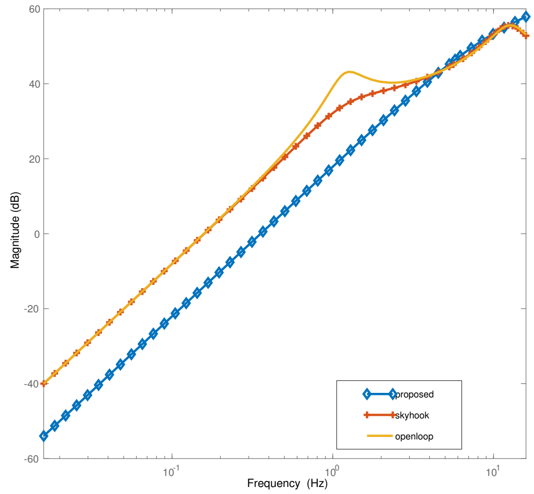


Fig. 3. Comparison of the body accelerations

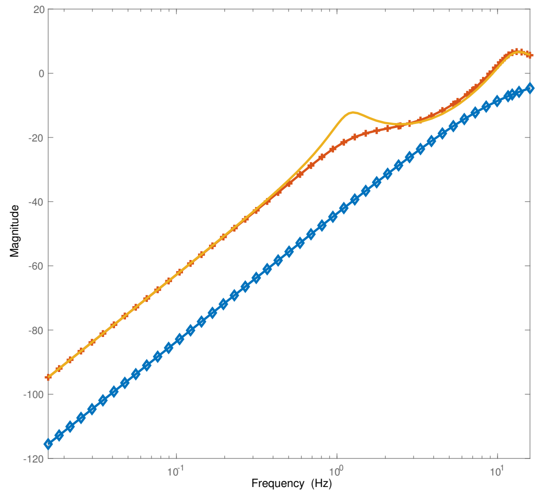


Fig. 4. Comparison of the tire deflections

For comparison, we choose skyhook damper coefficients of conventional controller and proposed controller as the same value. We select $c_{d4} = c_{sh} = 3000$, and $r_1 = 1000$, $r_2 = 0.1$, $k_{d2} = 0.1$. The other parameters of controller will be derived from matching equations.

The given results consists of sprung mass acceleration, sprung mass displacement, suspension deflection, and tire deflection as shown in Fig. 3 and Fig . 4. The main purpose of our study, sprung mass acceleration and tire deflectgion, are notably improved as compared to openloop system and skyhook damper system.

## 7. APPLICATION TO A NONLINEAR ACTIVE SUSPENSION SYSTEM

Although the previous sections have showed the effectiveness of proposed method sufficiently, the necessity of utilizing the IDA-PBC method is not explained clearly. If we only take the linear suspension system into the consideration, some other methods can obtain the similar result. However, one of our main purposes of using the IDA-PBC method in this study are that the IDA-PBC method can be applied to nonlinear system easily and the stability of the closed-loop system can be guaranteed theoreitcally. Moreover, most of the aforementioned methods require the solution of Hamilton - Jacobi partial differential equations. In this section, we apply our proposed method to a 2-DOF system with nonlinear inertia matrix. We assume that the Hamiltonian of considered system can be written as

$$\begin{aligned} H(q_1, p) &= \frac{1}{2} p^\top M(q_1)^{-1} p + V_1(q_1) + V_2(q_2) \\ M(q_1) &= \begin{bmatrix} m_1(q_1) & m_2(q_1) \\ m_2(q_1) & m_3(q_1) \end{bmatrix} \end{aligned} \tag{35}$$

The states $q, p$ are set as the same as linear case. It is assumed that the additional potential $V_2(q_2)$ is positive definite with respect to $q_2$ and satisfies $\partial V_2/\partial q_2 \neq 0$ ($q_2 \neq 0$). The port-Hamiltonian system is described as

$$\dot{x} = (J - R)\frac{\partial H}{\partial x}^\top + D\omega + Bu, \tag{36}$$

The desired system is expressed as

$$\begin{aligned} \dot{x} = (J_d(q_1) - R_d(q_1))\frac{\partial H_d}{\partial x}^\top + D_d(q_1, \dot{q})\omega \\ + D_{d\omega}(q_1, \dot{q})\omega^2, \end{aligned} \tag{37}$$

where

$$H_d(x) = \frac{1}{2} p^\top M_d(q_1)^{-1} p + V_d(q_1, q_2) \tag{38}$$

$$M_d(q_1) = \begin{bmatrix} m_{d1}(q_1) & m_{d2}(q_1) \\ m_{d2}(q_1) & m_{d3}(q_1) \end{bmatrix} \tag{39}$$

is the Hamiltonian of desired system, and

$$\begin{aligned} J_d(q_1) &= \begin{bmatrix} O & M(q_1)^{-1} M_d(q_1) \\ -M_d(q_1) M(q_1)^{-1} & J_2(q_1) \end{bmatrix} \\ J_2(q_1) &= \begin{bmatrix} 0 & j_e(q_1) \\ -j_e(q_1) & 0 \end{bmatrix}, \ R_d(q_1) = \begin{bmatrix} O & O \\ O & C_d(q_1) \end{bmatrix} \\ C_d(q_1) &= \begin{bmatrix} c_{d1}(q_1) & c_{d2}(q_1) \\ c_{d2}(q_1) & c_{d3}(q_1) \end{bmatrix} \\ D_d(q_1, \dot{q}) &= (0 \ -1 \ d_1(q_1, \dot{q}) \ d_2(q_1))^\top \\ D_{d\omega}(q_1) &= (0 \ 0 \ d_3(q_1) \ d_4(q_1))^\top. \end{aligned}$$

Let $J_d(q_1)$, $R_d(q_1)$, $V_d(q)$ and $M_d(q_1)$ denote an artificial skew-symmetric structure matrix, a positive semidefinite damping matrix, the potential energy, and the inertia matrix in the desired Hamiltonian, respectively.

The feedback law and the constraints of parameters are derived from the matching equation

$$\begin{aligned} (J_d - R_d)\frac{\partial H_d}{\partial x} = (J - R)\frac{\partial H}{\partial x} + Bu \\ + (D - D_d(q_1, \dot{q}))\omega - D_{d\omega}(q_1)\omega^2. \end{aligned} \tag{40}$$

For simplicity of calculations, we define

$$\begin{aligned} S(q_1) &= M^{-1}(q_1) = \begin{bmatrix} s_1(q_1) & s_2(q_1) \\ s_2(q_1) & s_3(q_1) \end{bmatrix} \\ S_d(q_1) &= M_d^{-1}(q_1) = \begin{bmatrix} s_{d1}(q_1) & s_{d2}(q_1) \\ s_{d2}(q_1) & s_{d3}(q_1) \end{bmatrix}. \end{aligned} \tag{41}$$

Each side of (40) is four dimensional vector. The first two components of (40) are already satisfied for all $x$ and $\omega$. By focusing on the coefficients of $p_1^2$, $p_1 p_2$, and $p_2^2$ in the third component of (40), we obtain

$$\begin{aligned} {s_{d1}}' &= \frac{|S_d| {s_1}'}{s_1 s_{d3} - s_2 s_{d2}}, \\ {s_{d2}}' &= \frac{|S_d| {s_2}'}{s_1 s_{d3} - s_2 s_{d2}}, \\ {s_{d3}}' &= \frac{|S_d| {s_3}'}{s_1 s_{d3} - s_2 s_{d2}}, \end{aligned} \tag{42}$$

where $*'$ means the derivative with respect to $q_1$.

The coefficients of $p_1$ and $p_2$ in the third component of (40) derive the following relations:

$$c_{d1}(q_1) = \frac{\mu}{|S_d|}(s_1 s_{d3} - s_2 s_{d2}), \tag{43}$$

$$j_e(q_1) = c_{d2}(q_1) + \frac{\mu}{|S_d|}(s_1 s_{d2} - s_2 s_{d1}). \tag{44}$$

The rest of the third component of (40) leads an equation for the potential energy

$$\frac{s_2 s_{d2} - s_1 s_{d3}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_1} + \frac{s_3 s_{d2} - s_2 s_{d3}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_2} + V_1' = 0.$$

The general solution of the above equation is

$$\begin{aligned} V_d(q) = P\left[ q_2 + \int_0^{q_1} \left. \frac{s_3 s_{d2} - s_2 s_{d3}}{s_1 s_{d3} - s_2 s_{d2}} \right|_{q_1 = \tau} d\tau \right] \\ + \int_0^{q_1} \left. \frac{V_1' |S_d|}{s_1 s_{d3} - s_2 s_{d2}} \right|_{q_1 = \tau} d\tau, \end{aligned} \tag{45}$$

where $P$ will be an arbitrary positive-definite function.

By solving the matching equation with respect to $u$, we can obtain a feedback law $u = \alpha_{\rm raw}(q, p, \omega)$. Notice that the feedback should be a function of $q$ and $\dot{q}$ only. Hence, we decompose $\alpha_{\rm raw}$ as

$$\alpha_{\rm raw}(q, M(q_1)(\dot{q} - a\omega), \omega) = \alpha(q, \dot{q}) + \alpha_{\rm rest}(q, \dot{q}, \omega)\omega.$$

The coefficient $\alpha_{\rm rest}(\cdot)$ should be identically zero, and thus we decompose it again as

$$\begin{aligned} &\alpha_{\rm rest}(q, S(q_1)p + a\omega, \omega) = \\ &\quad \alpha_1(q_1) + \alpha_2(q_1)p_1 + \alpha_3(q_1)p_2 + \alpha_4(q_1)\omega. \end{aligned}$$

By solving $\alpha_i(q_1) = 0$ ($i = 1, \ldots, 4$)with respect to $d_1(q_1), \ldots, d_4(q_1)$ and applying (42), we obtain additional equality constraints

$$\begin{aligned} d_1(q_1) = \frac{1}{|S|}\{(s_1 s_{d3} - s_2 s_{d2}) c_{d3}(q_1) \\ + (s_1 s_{d2} - s_2 s_{d1})(j_e(q_1) + c_{d2}(q_1)\}, \end{aligned} \tag{46}$$

$$(d_2(q_1) \ d_3(q_1)) = g(q_1) \cdot (0 \ 1) M' S, \tag{47}$$

$$d_4(q_1) = \frac{g(q_1)}{2} \cdot (0 \ 1) M' (0 \ 1)^\top, \tag{48}$$

where $M' = \partial M / \partial q_1$ and

$$g(q_1) = \frac{s_2 s_{d1} - s_1 s_{d2}}{s_1 s_{d3} - s_2 s_{d2}}.$$

The control input can be rewritten as

$$\begin{aligned} u &= \alpha(q, \dot{q}) \\ &= \frac{(s_2 s_{d3} - s_3 s_{d2}) c_{d3} - (s_3 s_{d1} - s_2 s_{d2})(c_{d2} + j_e)}{|S|} \dot{q}_1 \\ &+ (c - d_1(q_1)) \dot{q}_2 + \frac{g(q_1)}{2} \cdot \dot{q}^\top M' \dot{q} + \frac{\partial V_2(q_2)}{\partial q_2} \\ &+ \frac{s_1 s_{d2} - s_2 s_{d1}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_1} - \frac{s_3 s_{d1} - s_2 s_{d2}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_2}. \end{aligned} \tag{49}$$

Because of the feature of IDA-PBC, the closed-loop system is identical to the desired system. Therefore, the asymptotic stability of zero-disturbance case can be guaranteed by the nature of pH system. Thus we need to ensure the positive definiteness of $M_d, V_d$ and $C_d$, and the following inequality constraints can be derived:

$$s_{d3}(q_1) > 0, \quad |S_d(q_1)| > 0, \tag{50}$$

$$s_1 s_{d3} - s_2 s_{d2} > 0, \quad {}^\forall q_1, \tag{51}$$

$$|C_d(q_1)| > 0, \tag{52}$$

$$P[\sigma] > 0, \quad \sigma \neq 0. \tag{53}$$

Inequalities (50) show the positive definiteness of the inertia matrix of the desired system. We can show $c_{d1}(q_1) > 0$ from (51) and (43), and therefore (51) and (52) means that the damping matrix of the desired system is positive definite. Because of (51), the positivity of the second term of (45) will be automatically satisfied if $q_1 V_1' \geq 0$. Hence, under the constraint (53), the potential energy function $V_d(q)$ is positive definite.

We can gain $s_{di}(q_1)$ by solving (42), while the initial value $S_d(0) = S_{d0}$ is a degree of freedom. The inequality constraints of parameters are (50), (51), (52), and (53). The equality constraints of parameters are (43), (44), (45), (46), (47), (48), and (49).

From the above derivation, we can see that our proposed method can be applied to nonlinear case easily without solving any Hamilton-Jacobi partial differential equations.

## 8. CONCLUSION

In this paper, we propose a suspension system which can have a good ride comfort and road holding ability only utilizing relative information. The numerical simulation comparing with the result of skyhook damper method and openloop system is verified. Our future work is to apply the proposed method to semi-active suspension systems.

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