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Dynamic model for a water distribution network: application to leak diagnosis and quality monitoring ⋆

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Abstract: This paper presents a model based on the rigid water column (RWC) theory to describe the flow and the decay of chlorine in water distribution networks (WDNs), which can be used for developing tools to diagnose leaks and estimate chlorine concentrations. The model includes the continuity equation for each node of the network such that i) the relation of the flow rates entering and leaving the nodes is explicit, and ii) the computation of pressures and flow rates can be simultaneously done. The chlorine decay in each node and in each pipeline section of the WDN is predicted from the computed flow rates by using the third order accurate Warming-Kutler-Lomax (WKL) method. At the end of this paper, it is shown that the chlorine decay rate is well predicted by using the WKL method according to a comparison with simulations results obtained by using the EPANET-MSX software. Furthermore, it is shown that several single leak-diagnosis scenarios can be successfully solved by using an improved sensitivity matrix method together with the proposed model.

Keywords: Fault diagnosis, Water pollution, Networks, Pipelines, Estimation algorithms

1. INTRODUCTION

Drinkable water is supplied to cities and towns by means of distribution systems. First, the water is taken from natural sources (as e.g. lakes, rivers, aquifers, etc.), and it is sent to water-treatment plants to be purified. Then, it is distributed to the population through WDNs. In this process, chlorine is used to keep its quality under prescribed limits up to its consummation.

In the literature some strategies have been proposed to assure the adequate chlorine concentrations through the entire WDNs. For instance, Islam et al. (1997) proposed an inverse method that allows directly calculating the chlorine concentrations that are required at the water sources for having specified concentrations at given locations. Nejjari et al. (2014) presented a methodology to calibrate chlorine decay models via a genetic algorithm for a non-explicit expression of the model. Such a model was successfully applied to the Barcelona drinking water network. Kim et al. (2015) modeled the chlorine decay patterns in the context of the number of transient generation in three different frequencies and by using a pilot-scale pipeline system. In addition, the authors highlighted that in most of studies regarding chlorine decay, steady state models are used but in fact chlorine decay phenomena is clearly affected by transients. Fisher et al. (2017) introduced a new modeling of chlorine-wall reaction for simulating chlorine concentration in drinking water distribution systems. Such a model requires sound mathematical descriptions of decay mechanisms in bulk water and at pipe walls. Flows were constructed within the well-known AQUASIM modeling software.

Together with the assurance of the water quality, the leak diagnosis in WDN is a big concern of the scientific community and practitioners. For this reason, several strategies have been proposed to face it. Many leak-diagnosis approaches are based on principal component analysis (PCA), with leak location supposed to be at the internal nodes, and leaks considered as perturbations in such positions, while no other case is taken into account. In some of these PCA approaches the availability of sensors at internal points is required, which is not always possible in practice (Gertler et al., 2010). Puig and Ocampo-Martinez (2015) proposed an extension to urban uses in which leak identification is considered as a large scale problem due to the number of nodes in the WDN, where pressure head measurements are the core of the leak diagnosis strategy.

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Recently, a leak approach based on a lumped model of a WDN has been proposed in Jiménez-Cabas et al. (2018) on the basis of flow readings. Although such a method seems a good strategy, its implementation could be expensive due to the high cost of flow transducers and the difficulty to install them. Few additional methods can be found on the basis of transient analysis, as in Covas and Ramos (2001), but hardly ever applied in practice.

In the light of the foregoing, this work proposes a model that can be used to address both problems: the prediction of chlorine decay and the leak localization. To do that, firstly a dynamical modeling is derived to compute flows and pressure heads along the network by supposing typical assumptions of the RCW theory. The flow rate computations are used to predict the chlorine decay along the network, whereas the pressure head computations can be used for leak localization purposes. The leak isolability can be ensured somehow since a low number of pressure head sensors installed at some specific inner nodes allow to distinguish each leak via a correlation coefficient.
The paper continues as follows: In Section 2, the dynamic modeling is presented. Section 3 presents the third order accurate Warming-Kutler-Lomax (WKL) method used for chlorine modeling decay purposes. Section 4 introduced the leak isolation strategy by using the pressure head predictions. For the sake of illustration. some simulation results are shown in Section 5. Finally, conclusions and perspectives of future works are given in Section 6.

2. DYNAMIC MODELING

In this contribution, the modeling of a WDN is based on the rigid water column (RWC) theory, which has been previously used by Onizuka (1986); Shimada (1989); Nault and Karney (2016); Kaltenbacher et al. (2017). Therefore, the following assumptions are considered:

(A1) The flow rate is supposed to be one-dimensional.
(A2) The cross-sectional area is constant along each pipeline.
(A3) The conduit walls of each pipeline are rigid and the liquid fluid is incompressible.
(A4) Convective changes in velocity are negligible.
(A5) Energy loss for a given flow velocity during quasi-steady state.

2.1 Component models

A WDN consists of multiple elements (e.g., pipes, valves, leaks, reservoirs) that are characterized by dynamic and algebraic relationships between the flow Q_i through the component i and the pressure drop $\Delta Hi = H_j - H_{j+1}$ across that component, where subscripts j and $j+1$ denote the two ends of component i (De Persis and Kallesoe, 2011). The relationships for the elements considered in this contribution are introduced here below.

Pipe and pipe section: The equation of motion for each pipe (or pipe section) of a WDN is given as

$$\dot{Q}_i = \beta_i(H_j - H_{j+1}) - \mu_i Q_i|Q_i|, \tag{1}$$

where Q_i is the flow rate [m^3/s] trough pipe i, H_j is the piezometric head [mH$_2$O] at the inlet of pipe i, H_{j+1} is the piezometric head [mH$_2$O] at the outlet of pipe i, $\beta_i = gAr_i/L_i$ is the inertial term associated to pipe i, g is the acceleration of gravity, Ar_i is the cross-section area of pipe i, L_i is the length of pipe i, $\mu_i = f_i(Q_i)/2\phi_i Ar_i$, ϕ_i is the inner pipe diameter, and $f(Q_i)$ is the friction coefficient depending on flow rate Q_i according to Swamee and Jain (1976).

Node: The continuity equation for a node can be written as

$$\dot{H}_j = \frac{1}{S_j}(Q_i - Q_{i+1} - Q_k), \tag{2}$$

where Q_k is the flow rate through a branch, leak or demand k connected to the node and S_j is the node area.

2.2 Modeling example

Let us consider the case of a WDN shown in Fig. 1, which has two constant-level reservoirs, 12 pipes, and 7 inner nodes with a base demand of 15 [l/s] with demand pattern shown in Fig. 2.

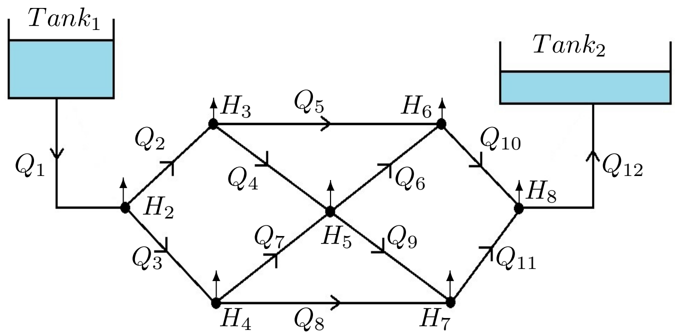

Figure 1. Water distribution network, a synthetic example.

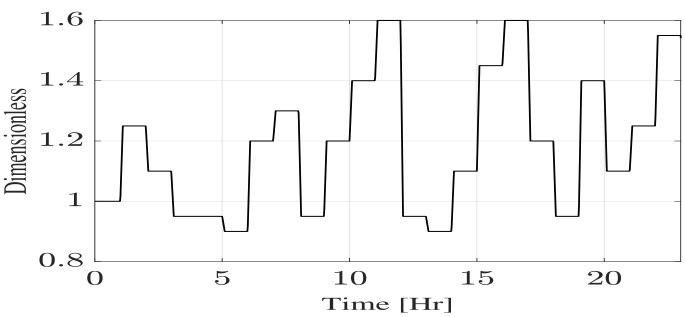

Figure 2. Demand pattern F.

For all pipes: length is 1000 [m], roughness coefficient is 3.81×10^{-4} [m]. Diameter for pipe 1 is 0.4 [m], for pipes 2 and 3 is 0.3 [m], for pipes 4, 7 6 and 9 is 0.015 [m], for pipes 5, 8, 10, 11 and 12 is 0.25 [m], respectively.
A minimal order dynamical representation for the WDN shown in Fig. 1 can be formulated as follows:

$$
\begin{aligned}
\dot{Q}_1 &= \beta_1\left(H^1_{BC} - H_2\right) - \mu_1 Q_1|Q_1| \\
\dot{H}_2 &= \frac{1}{S_2}\left(-Q_2 + Q_1 - Q_3 - Q_d F - Q\ell_2 F\right) \\
\dot{Q}_2 &= \beta_2\left(H_3 - H_2\right) - \mu_2 Q_2|Q_2| \\
\dot{H}_3 &= \frac{1}{S_3}\left(-Q_4 + Q_2 - Q_5 - Q_d F - Q\ell_3 F\right) \\
\dot{Q}_3 &= \beta_3\left(H_2 - H_2\right) - \mu_3 Q_3|Q_3| \\
\dot{H}_4 &= \frac{1}{S_4}\left(-Q_7 + Q_3 - Q_8 - Q_d F - Q\ell_4 F\right) \\
\dot{Q}_4 &= \beta_4\left(H_3 - H_1\right) - \mu_4 Q_4|Q_4| \\
\dot{H}_5 &= \frac{1}{S_5}\left(Q_7 + Q_4 - Q_6 - Q_9 - Q_d F - Q\ell_5 F\right) \\
\dot{Q}_5 &= \beta_5\left(H_4 - H_1\right) - \mu_5 Q_5|Q_5| \\
\dot{H}_6 &= \frac{1}{S_6}\left(Q_5 + Q_6 - Q_{11} - Q_d F - Q\ell_6 F\right) \\
\dot{Q}_6 &= \beta_6\left(H_5 - H_6\right) - \mu_6 Q_6|Q_6| \\
\dot{H}_7 &= \frac{1}{S_7}\left(Q_8 + Q_9 - Q_{10} - Q_d F - Q\ell_7 F\right) \\
\dot{Q}_7 &= \beta_7\left(H_3 - H_2\right) - \mu_7 Q_7|Q_7| \\
\dot{H}_8 &= \frac{1}{S_8}\left(Q_{11} + Q_{10} - Q_{12} - Q_d F - Q\ell_8 F\right) \\
\dot{Q}_8 &= \beta_8\left(H_5 - H_2\right) - \mu_8 Q_8|Q_8| \\
\dot{Q}_9 &= \beta_9\left(H_5 - H_3\right) - \mu_9 Q_9|Q_9| \\
\dot{Q}_{10} &= \beta_{10}\left(H_6 - H_5\right) - \mu_{10} Q_{10}|Q_{10}| \\
\dot{Q}_{11} &= \beta_{11}\left(H_6 - H_4\right) - \mu_{11} Q_{11}|Q_{11}| \\
\dot{Q}_{12} &= \beta_{12}\left(H_7 - H^2_{BC}\right) - \mu_{12} Q_{12}|Q_{12}|
\end{aligned} \tag{3}
$$

where Q_d is the base demand at nodes, $Q\ell_j$ is the base leak-flow that emulates only one leak at time, H^1_{BC} and H^2_{BC} stand for the boundary conditions in the tank 1 and 2, respectively, and F stands for the demand pattern shown in Fig. 2. Such a dynamical modeling will be used for chlorine prediction and for leak localization purposes. In the following section, the third order accurate WKL method is presented to predict the chlorine decay along the WDN.

3. CHLORINE MODELING DECAY

For a first-order decay rate, the following one-dimensional transport equation can describe the movement of a constituent in a pipeline:

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial^2 z} - k_1 C \tag{4}$$

where C is the constituent concentration, V is the advective velocity, D is the dispersion coefficient, z is the spatial coordinate and k_1 is the first-order reaction rate coefficient. A closed-form solution for this partial-differential equation is not available generally speaking and a numerical solution is used instead. The advection and dispersion in the one-dimensional transport process may be solved separately and then combined to obtain the total solution according to Islam and Chaudhry (1998). In order to reduce the numerical diffusion one can solve (4) in two separated steps as follows:

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial z} = 0 \tag{5}$$

$$\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial z^2} + k_1 C = 0 \tag{6}$$

Equation (5) can be solved first for advection transport of the constituent so that the numerical diffusion is reduced. On the other hand, diffusion equation (6) can be solved by means of an explicit finite-difference scheme. To do that it is firstly necessary to consider the mass balance at junctions as follows:

$$C_{nj} = \frac{\sum_{i=1}^{M} Q_i C_i}{\sum_{i=1}^{M} Q_i} \tag{7}$$

where Q_i is the incoming flow in pipe i, C_i is the chlorine concentration of incoming pipe i, C_{nj} is the chlorine concentration at junction j. Note that the chlorine concentration in all flows leaving junction j is the same as C_{nj}. The numerical approximation is based on a third order explicit finite-difference WKL method (Anderson et al., 2016). This method takes the two first steps from MacCormack's method and adds a third step as seen on Rusanov's method (Sod, 1978). If chlorine concentration for the i-th pipe is a function $C^i(z,t)$, then the finite equations for chlorine concentrations can be computed as follows:

- Step 1:

$$\hat{C}^i_{p,k} = C^i_{p,k} - \frac{2}{3}\nu\left(C^i_{p,k+1} - C^i_{p,k}\right)$$

- Step 2:

$$\tilde{C}^i_{p,k} = \frac{1}{2}\left[C^i_{p,k} + \hat{C}^i_{p,k} - \frac{2}{3}\nu\left(\hat{C}^i_{p,k} - \hat{C}^i_{p-1,k}\right)\right]$$

- Step 3:

$$
\begin{aligned}
C^i_{p,k+1} &= C^i_{p,k} - \frac{3}{8}\nu\left(\tilde{C}^i_{p+1,k} - \tilde{C}^i_{p-1,k}\right) \\
&- \frac{1}{24}\nu\left(-2C^i_{p+2,k} + 7C^i_{p+1,k} - 7C^i_{p-1,k} + 2C^i_{p-2,k}\right) \\
&- \frac{\omega}{24}\left(C^i_{p+2,k} - 4C^i_{p+1,k} + 6C^i_{p,k} - 4C^i_{p-1,k} + C^i_{p-2,k}\right)
\end{aligned}
$$

for the p-th point of the grid position along the i-th pipe section, and k being the discrete-time index. ν is the Courant number and $\omega = 4\nu^2 - \nu^4$ for reducing numerical dispersion according to Islam et al. (1997). An explicit first-order finite-difference scheme was adopted to solve the diffusion equation (6) as follows:

$$C^i_{p,k+1} = \lambda C^i_{p-1,k} + \left(1 - 2\lambda - k_1\Delta t\right) C^i_{p,k} + \lambda C^i_{p+1,k}$$

where $\lambda = D\Delta t/\left(\Delta z\right)^2$, the dispersion coefficient in (4) is estimated from the following expression:

$$D = 10.1 r\sqrt{\frac{\tau_0}{\rho}} \tag{8}$$

where r is the pipe radius, τ_0 is the shear stress at the wall, and ρ is the density of the fluid.

Since the advection scheme is first order, the points at the upstream and downstream of each pipeline section are adjusted. This is done in order to determine all points in the $t-z$ grid. In this case, the space z along the $i-th$ pipe is divided into $P = 100$ points uniformly distributed and the t-grid is considered to be 86400 [s] (1 day). Thus, the chlorine concentration must be fixed at upstream of pipe 1, that is, $C^1_{1,k}$ is a supplied chlorine rate at upstream [mg/L]. The chlorine concentration can be then predicted along the WDN (shown in Fig. 1) according to the following expressions coming from (7):

$$
\begin{aligned}
C_{1,k}^{2} &= C_{1,k}^{3} = C_{P,k}^{1} \\
C_{1,k}^{4} &= C_{1,k}^{5} = C_{P,k}^{2} \\
C_{1,k}^{6} &= C_{1,k}^{7} = C_{1,k}^{9} = C_{P,k}^{1} \\
C_{1,k}^{8} &= \frac{Q_3 C_{P,k}^{3} + Q_7 C_{P,k}^{7}}{Q_3 + Q_7 + Q_{d_4}} \\
C_{1,k}^{10} &= \frac{Q_8 C_{P,k}^{8} + Q_9 C_{P,k}^{9}}{Q_8 + Q_9 + Q_{d_7}} \\
C_{1,k}^{11} &= \frac{Q_5 C_{P,k}^{5} + Q_6 C_{P,k}^{6}}{Q_5 + Q_6 + Q_{d_4}} \\
C_{1,k}^{12} &= \frac{Q_{10} C_{P,k}^{10} + Q_{11} C_{P,k}^{11}}{Q_{10} + Q_{11} + Q_{d_8}}
\end{aligned}
$$

Remark. *Note that to predict the chlorine decay along the WDN, only flow rate computations are needed. To take advantage from pressure head computations obtained from the model* (3)*, in the following section a leak localization strategy is described which only requires such pressure head computations.*

4. LEAK LOCALIZATION STRATEGY

Let us consider that one leak can appear at a node in the WDN (see Fig. 1). Here, the idea considered to localize it by using a fault sensitivity matrix approach on the basis of pressure head measurements as in Perez et al. (2014), but improving the leak modeling at nodes. The leak identification strategy basically consists in comparing the monitored pressure disturbances caused by leaks at *certain* inner nodes of the WDN with the *theoretical* pressure disturbances caused by all potential leaks of the WDN and stored in a fault sensitivity matrix $FSM \in \mathbf{R}^{n_s \times n_p}$

$$
FSM = \begin{bmatrix} \mathbf{\Delta H_{1,1}} & \dots & \mathbf{\Delta H_{1,n_p}} \\ \vdots & \ddots & \vdots \\ \mathbf{\Delta H_{n_s,1}} & \dots & \mathbf{\Delta H_{n_s,n_p}} \end{bmatrix} \tag{9}
$$

where $\mathbf{\Delta H_{l,j}} = (H_l|_{Q\ell_j} - H_l|_0)/Q\ell_j$, with $l = 1, 2, ..., n_s$ and $j = 1, 2, ..., n_p$. n_s is the number of sensors and n_p is the number of total nodes. $H|_{Q\ell}$ stands for the pressure head H under effect of leak $Q\ell$. $Q\ell_j = \alpha_j \sqrt{H_j}$, α_j is associated to a discharge coefficient and to the cross-section area of the leak at the $j-th$ node (an average diameter of the pipes delivering water to final consumers could be used to approximate such a parameter). H_j is the pressure head at the $j-th$ inner node. The residual vector $r_e \in \mathbf{R}^{n_s}$ is determined by the difference between the measured pressure at inner nodes $H \in \mathbf{R}^{n_s}$, and the estimated pressure at those nodes computed by a nominal model, $\hat{H}_0 \in \mathbf{R}^{n_s}$, it is given at the time instant k as follows:

$$
r_e(k) = H(k) - \hat{H}_0(k) \tag{10}
$$

Clearly the size of vector r_e depends on the number of pressure head sensors. Finally, the leak location method is based on comparing residual vector r_e with the theoretical fault signatures which are the columns of the FSM by means the following correlation coefficient

$$
Cc_{i_{FSM}, r_e} = \frac{cov(c_{i_{FSM}, r_e})}{\sqrt{cov(c_{i_{FSM}}, c_{i_{FSM}}) cov(r_e, r_e)'}} \tag{11}
$$

where Cc_{i_{FSM}, r_e} is the correlation coefficient, $c_{i_{FSM}}$ is the $i-th$ column of FSM matrix, $cov(\gamma, \delta)$ is the covariance function between γ, δ. The biggest correlation value is related with the most probable candidate node to have the leak. Of course the reliability of our approach depends, among other things, on the number of sensors. In order to determine the number of sensors such that all possible leaks be detected the following procedure has been adopted:

Let us assume that $n_s = n_p$ sensors are installed (ideal case) and by checking equation (11) each leak can be clearly identified. Then, a first pressure head sensor is randomly removed, that is $(n_s = n_p - 1)$. One can verify that even in this case all leaks are distinguishable by checking equation (11). Then, a second pressure head sensor is also removed randomly, that is $n_s = n_p - 2$, and once again all leaks are well identified via (11). Similar procedure can be repeated up to $n_s = n_p - k$ keeping all leaks distinguishable via (11). In this way, it is possible to determine the lower number of required sensors such that all possible leaks could be properly identified. Following this procedure, a couple of sensors at nodes 2 and 7 are enough to identify all leaks in our example. Of course, this method is quite simple and general, and it could be reformulated and improved to face large-scale problems.

5. SIMULATION RESULTS

The network model (3) is simulated in Matlab environment with the solver *ode*4: *Runge-Kutta*. Flow rate computations are compared with those obtained from the well-known $EPANET - MSX$ software for validation purposes, see Figs. 3 and 4. In addition, pressure head computations are also compared, see Figs. 5 and 6, for instance, one can see an excellent agreement that evidences the effectiveness of our modelling.

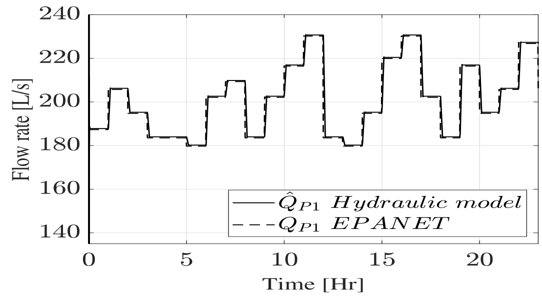

Figure 3. Flow rate in pipe section 1.

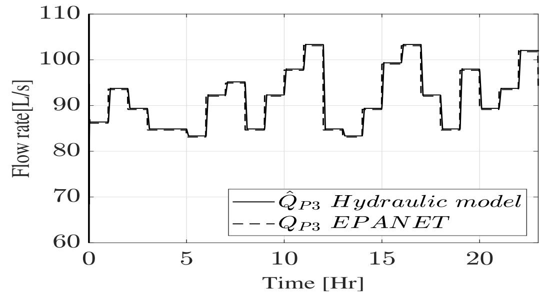

Figure 4. Flow rate in pipe section 3.

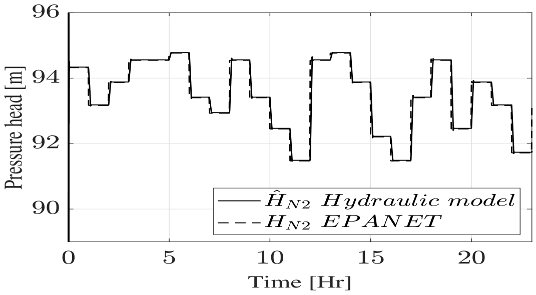

Figure 5. Pressure head at node 2.

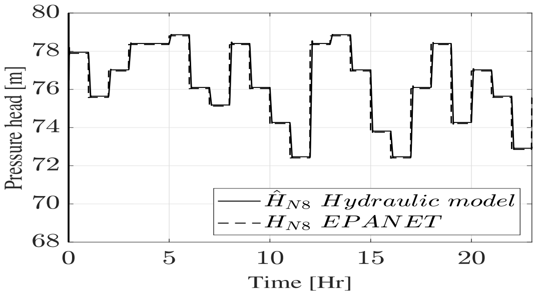

Figure 6. Pressure head at node 8.

5.1 Chlorine prediction

Flow rate computations are taken from the simulation of the model (3) and used to predict the chlorine decay along the WDN. A Chlorine concentration is fixed at upstream, i.e., $C^1_{1,k} = 8$ [mg/L]. Then, a simulation is performed in parallel. The first one by using our dynamic model and the WKL method to predict the chlorine decay rate. On the other hand, *EPANET-MSX* is used to computate the chlorine concentration. In Figs. 7, 8 9 and 10, some examples of chlorine concentration at nodes and at pipes are depicted, once again they show an excellent agreement.

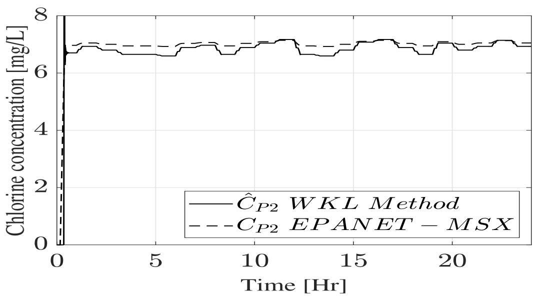

Figure 7. Chlorine concentration at pipe section 2

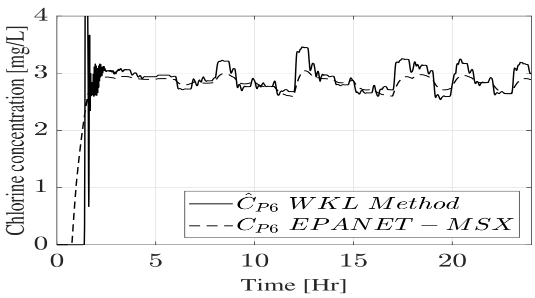

Figure 8. Chlorine concentration at pipe 6

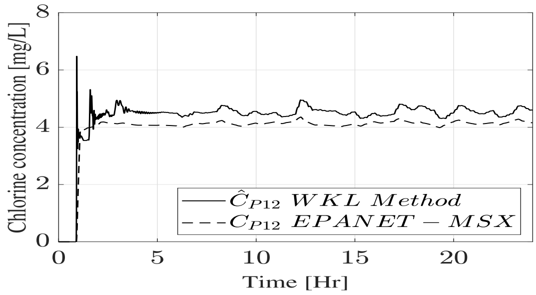

Figure 9. Chlorine concentration at pipe 12

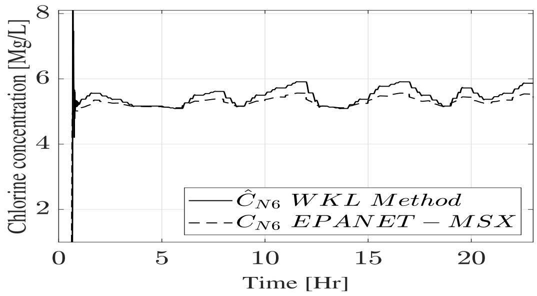

Figure 10. Chlorine concentration in node 6

5.2 Leak isolation

Two different leak scenarios are here shown, namely, cases of a leak at node 3 and at node 6 are presented. The Figs. 11 and 13 show the correlation coefficient which indicates the most probable node to present the leak. Figs. 12 and 14 show the 2-D representation. It should be noted that in this free-noise scenario all cases are solved with success.

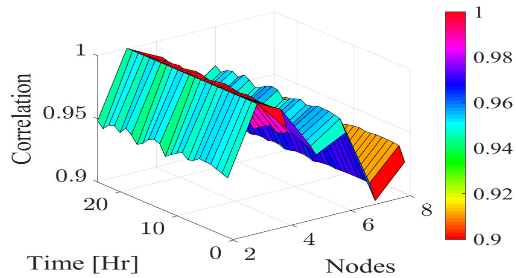

Figure 11. Leak identification in node 3.

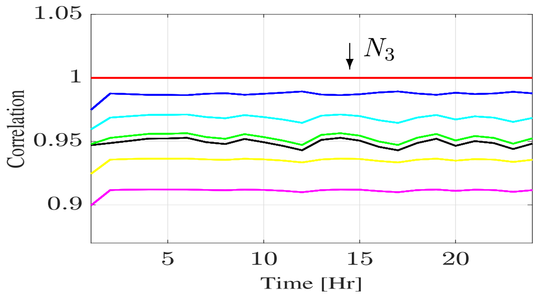

Figure 12. 2-D leak identification in node 3.

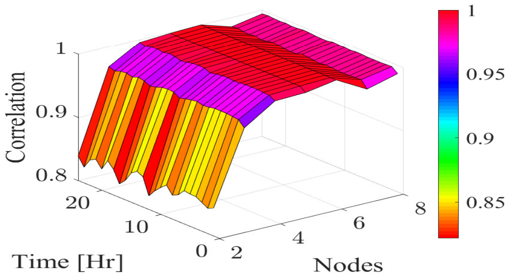

Figure 13. Leak identification in node 6.

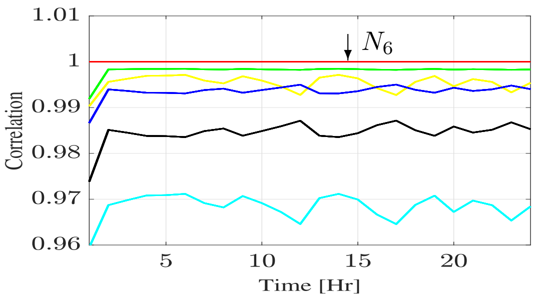

Figure 14. 2-D leak identification in node 6.

6. CONCLUSIONS

The dynamical modeling of the WDN allows to address two different and realistic issues. On the one hand, flow rates are used to predict the chlorine decay along the WDN, the results match those coming from EPANET-MSX software. On the other hand, pressure computations are used to identify a leak on the basis of the sensitivity matrix approach which has been used in real leak problems. As future work, the sensor placement procedure will be formulated to face large-scale problems and applications of this methodology will be part of future developments.

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