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A Multi-stage Economic NMPC for the Tennessee Eastman Challenge Process

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Abstract: This paper addresses the design and implementation of a robust nonlinear model predictive control (NMPC) scheme for a benchmark plant-wide control problem. The focus of our research is on the performance of direct optimizing control for a complex large-scale process which is subject to plant-model mismatch and external disturbances. As a benchmark case for control and monitoring applications, the Tennessee Eastman Challenge (TEC) process has been widely employed in many publications. We present a first NMPC implementation for this where only economics criteria are used for the control of the process. The results obtained demonstrate the viability of plant-wide economics optimizing NMPC. We also address the issue of robustness against model uncertainties and employ multi-stage NMPC to tackle these. Different possible multi-stage NMPC implementations are discussed and the trade-offs between economic performance and robustness are highlighted.

Keywords: robust NMPC, economics optimizing control, multi-stage optimization, plant-wide control, Tennessee Eastman Challenge Problem.

1. INTRODUCTION

The optimal control of large scale chemical processes that consist of many interconnected units poses a number of challenges, due to the complexity of the models, the effects of recycles, and the presence of a large number of uncertain parameters and disturbances. Reviews of plant-wide control (PWC) as e.g those found in Downs and Skogestad (2011) or Vasudevan and Rangaiah (2012), emphasize that the approaches can be grouped according to the scale on which they address the control task. Stewart et al. (2010) have shown that distributed approaches can ensure stability and may obtain convergence to the plant-wide optimum. The classical approach to PWC involves treating the plant-wide optimization of the process as one big task, while the implementation of the solution is typically done in a top-down multi-layer approach. Some of the main challenges of PWC are the selection of the control structure and the control hierarchy, as highlighted by Skogestad (2000). These choices have an impact on both the stability and the economic profitability of the process. In this approach, the top layer optimizes the plant profitability, usually based on a rigorous steady-state model, while one or two more lower layers are employed for tracking the set-points provided by the upper layer and ensuring that process constraints are not violated (see Skogestad (2000), Engell (2007), Ochoa et al. (2010), Skogestad (2012)). However, from an economics point of view, the two-layer approach does not always lead to the best results. A single layer, direct optimizing control (DOC), or economic-oriented MPC (eMPC), structure can bridge the gap between plant economics and low-level control in a more systematic manner and lead to better results (e.g Engell (2007), Engell (2009), Ellis et al. (2014)).
The rise in popularity of DOC was triggered by the evolution of the optimization algorithms and tools, which now make it possible to solve large scale optimization problems much more efficiently and reliably than before, as demonstrated by e.g Biegler and Zavala (2009) for the dynamic plant-wide control of a polyethylene plant. Furthermore, it is now possible to implement PWC approaches where the control task is formulated solely by means of economic performance criteria and the satisfaction of process and quality constraints is directly implemented in the form of constraints in the optimization problem. The application of direct optimizing or economic MPC approach to real plants has been described e.g. in Toumi and Engell (2005), Kuepper and Engell (2007) and Hasskerl et al. (2018b). Plant-wide control of several units with recycles by DOC has been studied in a simulation study by Ochoa et al. (2010), but there still are relatively few contributions in the literature on this topic.
Since the introduction of the Tennessee Eastman Challenge process in 1992, numerous studies of plant-wide control of this process have proposed low-level stabilizing control or multi-layer structures, which we will review in Section 2 of this article. This paper focuses on the investigation of the potential of DOC when applied to the TEC process, more precisely on the use of non-linear MPC methods to solve an economically motivated control problem. Our aim is to realize a PWC structure which maximizes the plant profitability and handles all relevant constraints. Since the profitability of the plant is affected by disturbances and plant-model mismatch, we extend our investigation to study the performance of a robust multi-stage NMPC controller under the influence of uncertainty. The results presented in this work aim at validating the use of multi-stage NMPC in a plant-wide application and to discuss the selection of the NMPC parameters in order to enhance plant profitability. Furthermore, we contribute to the discussion of the TEC and provide a point of reference for further NMPC applications to this benchmark problem.

2. THE TENNESSEE EASTMAN CHALLENGE PROCESS

2.1 TEC process description and dynamic model

The Tennessee Eastman Challenge (TEC) was proposed by (Downs and Vogel, 1992) as a benchmark case for plant-wide control strategies and process monitoring. The challenge is based on a real industrial process, but details as the names of the components, physical properties or kinetics have been modified by the authors to protect the intellectual property of the company. The model of the process has already been discussed in several papers, e.g. Yan and Ricker (1995), Vallerio et al. (2014) or Jockenhövel et al. (2003), the last one being the formulation that was selected for the implementation of the dynamic model in this paper. Due to space limitations, only the reactor model will be re-iterated here. This is because the reactor is the unit which is most sensitive to disturbances and control inputs, as will be discussed in the next section. The energy and composition balances shown in (2) are illustrative and offer an impression of the model complexity. For an overview of the TEC and its full model and a description of the parameters, please consult Downs and Vogel (1992) and Jockenhövel et al. (2003). A very detailed analysis and a re-fitting of the model parameters was performed by Jockenhövel (2004).

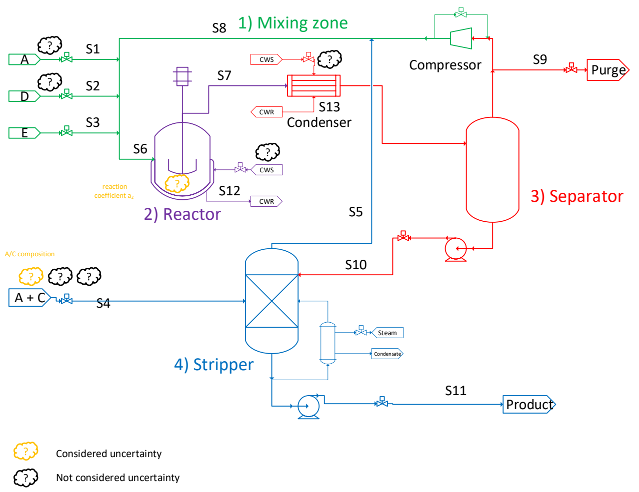

Fig. 1. The schematic representation of the TEC process. The main units are depicted in colour and the most relevant process disturbances are marked with cloud signs.

Three exothermic, irreversible gas-phase reactions occur inside the reactor. The products G and H are the result of the first two reactions, while the third reaction yields one by-product F, as shown in Eq. (1).

$$\begin{aligned} &A_g + C_g + D_g && \rightarrow G_l \\ &A_g + C_g + E_g && \rightarrow H_l \\ &\frac{1}{3}A_g + \frac{1}{3}E_g + D_g && \rightarrow F_l \end{aligned} \tag{1}$$

The reactor model is given in (2). Here $\dot{R}_i$ represent the reaction rates, which depend on the reaction rate coefficients a_i and on the gas volume $V_{g,R}$ as well as on the partial pressure $p_{i,R}$. The variables x_i give the molar concentrations. $V_{L,R}$ designates the liquid volume in the reactor, $\Delta H_{R,k}$ the heat of reaction. T_{Rc} denotes the temperature of the cooling water and $\dot{Q}_R$ the transferred energy via the surface with the heat transfer coefficient kA and the temperature of incoming cooling water $T_{cw,in,R}$. m_{CWR} is here the mass of the cooling water.

$$\begin{aligned}
\frac{dn_{i,R}}{dt} &= \dot{n}_6 y_{i,6} - \dot{n}_7 y_{i,7} + \sum_{k=1}^{3} \nu_{i,k}\dot{R}_k \\
\dot{R}_1 &= a_1\, V_{g,R}\, exp(44.06 - \frac{42600}{RT_R})\; p_{A,R}^{1.08}\; p_{C,R}^{0.311}\; p_{D,R}^{0.874} \\
\dot{R}_2 &= a_2\, V_{g,R}\, exp(10.27 - \frac{19500}{RT_R})\; p_{A,R}^{1.15}\; p_{C,R}^{0.370}\; p_{E,R} \\
\dot{R}_3 &= a_3\, V_{g,R}\, exp(59.50 - \frac{59500}{RT_R})\; p_{A,R}\; (0.77 p_{D,R} + p_{E,R}) \\
p_{i,R} &= \gamma_{i,R}\, x_{i,R}\, p_{sat,i}(T_R) \\
V_{L,R} &= \sum_{i=D}^{H} n_i \frac{M_i}{\rho_i}, \quad P_R = \sum_{i=A}^{F} p_{i,R} \\
\dot{S}_6 &= 0.8334[\frac{kmol}{sMPa^{0.5}}]\sqrt{p_M - p_R} \\
\dot{S}_7 &= 1.5355[\frac{kmol}{sMPa^{0.5}}]\sqrt{p_R - p_{Se}} \\
\Delta H_{R,k} &= \dot{R}_k \sum_{i=A}^{H} \nu c_{p,vap} T_r + H_{k,form} \\
\frac{dT_R}{dt} &= \frac{\dot{n}_6 (\sum_{i=A}^{H} y_{i,6} c_{p,vap,i})(T_6 - T_R) - \dot{Q}_r - \sum_{k=1}^{3} \Delta H_{R,k}}{\sum_{i=A}^{C} n_{i,R} c_{p,vap,i} + \sum_{i=D}^{H} n_{i,R} c_{p,liq,i}} \\
\frac{dT_{Rc}}{dt} &= \frac{\dot{m}_{CWR} c_{p,cw} (T_{cw,in,R} - T_{Rc}) + \dot{Q}_R}{c_{p,cw} m_{CWR}} \\
\dot{Q}_R &= \frac{\Delta T_{1,R} - \Delta T_{2,R}}{ln(\frac{\Delta T_{1,R}}{\Delta T_{2,R}})} kA_R
\end{aligned} \tag{2}$$

The full model of the TEC consists of 30 ODEs. Similarly to the reactor model, it is based on energy and component balances for the remaining units. Algebraic expressions, like the total pressures in the mixing unit (P_m), the separator (P_{se}), and the stripper (P_{st}), as well as the liquid hold-ups in the units, are used to impose safety and operational constraints on the production. The units are coupled by vapour streams S_5, S_6, S_7, S_8, S_9 and by one liquid stream S_{10}. In total the model defines 14 algebraic expressions. In Fig. 1 also the 11 control inputs of the TEC are depicted. Downs and Vogel (1992) note that only the pressures and temperatures in the units, the flowrates, stream compositions and stream temperatures can be measured. However, in the present work we consider full state information about the process units, focusing explicitly on the feasibility of state-feedback NMPC schemes.

3. ECONOMIC NMPC AND MULTI-STAGE NMPC

Quite a few control approaches have so far been reported for the Tennessee Eastman Challenge, amongst which we discuss here only some of the optimization-based approaches. A first NMPC application to the TEC with six manipulated control variables was presented by Ricker and Lee (1994), which included some additional PID loops to stabilize the process. The same group also presented an NMPC approach which considered soft constraints (Yan and Ricker, 1995). A self-optimizing control approach was discussed by Larsson et al. (2001), while newer NMPC approaches were proposed by Jockenhövel (2004), who introduced a dynamic model of the process and performed dynamic optimization for reference tracking based on the original prescriptions by Downs and Vogel (1992). Karra et al. (2008) introduced an adaptive MPC structure and, in later work, (Zakharov et al., 2013) employed a two-phase NMPC. Vallerio et al. (2014) have shown an application of multi-objective optimisation (Pareto optimality) for a reduced case of the TEC.

In the present work we discuss the implementation of direct optimizing control and multi-stage NMPC (MS-NMPC) with the platform *do-mpc* (Lucia et al. (2017)). The multi-stage MPC approach is one of the least conservative robust approaches for handling plant model mismatch in the form of uncertain model parameters. It was proposed for applications where the uncertainty cannot be eliminated by model enhancement methods, as e.g. model adaptation or parameter estimation. The robustness to parametric uncertainty is realized with the help of a branching prediction tree (see Fig. 2), where the nodes represent the future predicted states and the arcs represent the control actions and evolutions of the parameters. This formulation results in a

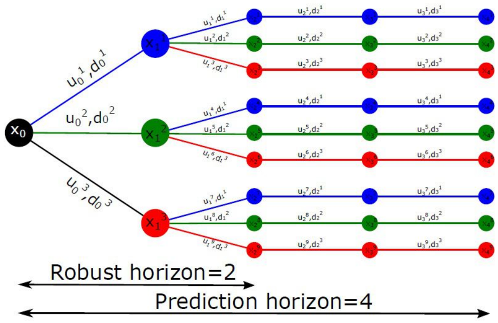

Fig. 2. Plant-model mismatch represented as a scenario tree

large scale optimization problem of the form:

$$\min_{x_k^j, u_k^j \forall (j,k) \in I} \sum_{i=1}^{N} \omega_i J_i(X_i, U_i), \tag{3}$$

subject to:

$$\begin{aligned}
x_{k+1}^j &= f(x_k^{p(j)}, u_k^j, d_k^{r(j)}), && \forall (j, k+1) \in I, \\
g(x_{k+1}^j, u_k^j) &\leq 0\,, && \forall (j, k+1) \in I, \\
u_k^j &\in \mathbb{U}\,, x_k^j \in \mathbb{X}\,, && \forall (j,k) \in I, \\
u_k^j &= u_k^l \text{ if } x_k^{p(j)} = x_k^{p(l)} && \forall (j,k),(l,k) \in I,
\end{aligned}$$

where $g : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_g}$ represents general and possibly nonlinear constraints on the states and the inputs of the control problem evaluated at each node of the tree. The superscript j denotes the position in the prediction horizon, while i denotes the scenario number. k is used to address individual values in the state and input vectors. The cost of each scenario S_i with weight ω_i is denoted by $J_i : \mathbb{R}^{n_x \times N_P+1} \times \mathbb{R}^{n_u \times N_P} \to \mathbb{R}$. The non-anticipativity constraints ensure that the decisions u_k^j with the same parent node $x_k^{p(j)}$ are the same. Additionally the states and inputs are restricted to feasible sets $\mathbb{X}$ and $\mathbb{U}$, respectively.

The complexity of the problem increases with the number of scenarios N, which grows exponentially with the number of stages. Since recourse is explicitly modelled in our problem by embedding the measurement feedback at each sampling time, it is typically enough to consider only one or two stages where the tree branches. This is called the robust horizon. After the robust horizon the uncertain parameters are assumed to be constant. This assumption reduces the size of the problem and enables real-time implementation. For further details on the assumptions of the multi-stage approach and a discussion about the robustness of the method, the reader is referred to the papers by Lucia and Engell (2012), Lucia et al. (2014) and Lucia and Engell (2015). The *do-mpc* platform has been successfully used over the past years in a number of NMPC studies. Lucia et al. (2017) and Tatulea-Codrean et al. (2019) discussed different details of NMPC implementation with *do-mpc*. Hasskerl et al. (2018a) and Hasskerl et al. (2018b) showed real application results of *do-mpc* at a reactive distillation column. In this work, we present the first implementation of a plant-wide optimizing control scheme using *do-mpc*. The efficiency of the solution of NMPC problems for large-scale nonlinear models is greatly enhanced by the use of efficient symbolic representations of the process model provided by the framework CasADi (Andersson et al., 2019) and the solution of the optimization problems by the nonlinear solver IPOPT (Wächter and Biegler, 2006). The platform *do-mpc* is built on top of these tools.

3.1 Model uncertainty and sensitivity analysis

A sensitivity analysis of the economics-oriented NMPC implementation for the TEC process was carried out based on the original information on the uncertainties and the recommendations provided in Downs and Vogel (1992). The model uncertainty and the possible disturbances are considered as parametric disturbances and were implemented in this way in *do-mpc*. A list of the uncertain parameters is given in Tab. 1 along with their nominal values. The range of uncertainty considered here is $\pm 20\%$ of the nominal values except for the temperatures, where it is $\pm 20K$. The sensitivity analysis was conducted for a simulation time of 2 hours, which is sufficient for reaching a steady state. The uncertainties mostly affect the initial dynamic phase, therefore extending the simulation time did not result in qualitatively different results. The different parameter values were applied as constant value throughout the entire simulation time, and the results are listed in Tab. 2. The economic NMPC controller solves the problem given in (4) and has prediction and control horizons equal to 2000 seconds, realized in 20 steps (i.e. $N_p = 20$ and sampling time $= 100s$). The stage cost function $J_k(x,u)$ comprises different parts that

Table 1. Uncertain parameters and their nominal values according to Downs and Vogel (1992)

No.	Parameter		Nominal	Unit
1	A/C feed ratio, B constant (Stream 4)	x_A	0.485	mol/mol
		x_C	0.51	mol/mol
2	B composition, A/C constant (Stream 4)	x_A	0.485	mol/mol
		x_B	0.005	mol/mol
		x_C	0.51	mol/mol
3	D feed temperature (Stream 2)		318.15	K
4	CWR temperature		308	K
5	CWC temperature		313	K
6	A feed loss (Stream 1)		20	kmol/h
7	C header pressure loss (Stream 4)		700	kmol/h
8	Reactor kinetics	a_1	1.039916	-
		a_2	1.0113731	-
		a_3	1.0	-

account for the costs and sources of revenue of the process. The costs are labelled c_i and the revenue for the products are labelled p_i. A list of the costs for the components $i = A,..F$, as well as the cost of steam and cooling water are given in the original paper of Downs and Vogel (1992). As for the revenue from the products, an educated guess is to set them as $p_{G,H} = 100\ \$/kmol$. In addition to the economic terms in J_k, the soft constraint penalty terms $\varepsilon_c, c = 1..9$ are included in order to account for the process safety and operational constraints. The cost function and the constraints are given below:

$$
\begin{aligned}
&\min_{x,u} \sum_{k=1}^{N_P} J_k(x,u) \\
&\text{subject to:} \\
&\varepsilon_{1..9} && \geq 0 \\
&T_R - 423K - \varepsilon_1 && \leq 0 \\
&p_R - 2895kPa - \varepsilon_2 && \leq 0 \\
&V_{L,R} - 21.3m^3 - \varepsilon_3 && \leq 0 \\
&-V_{L,R} + 11.8m^3 - \varepsilon_4 && \leq 0 \\
&-V_{L,Se} + 3.3m^3 - \varepsilon_5 && \leq 0 \\
&V_{L,Se} - 9.0m^3 - \varepsilon_6 && \leq 0 \\
&p_{Se} - p_R + 5kPa - \varepsilon_7 && \leq 0 \\
&-V_{L,Str} + 3.5m^3 - \varepsilon_8 && \leq 0 \\
&V_{L,Str} - 6.6m^3 - \varepsilon_9 && \leq 0 \\
&\text{where } J_k(x,u) = \Bigg[\dot{n}_{11} \sum_{D}^{F} (x_{i,str} c_i) + \dot{n}_9 \sum_{A}^{F} (y_{i,se} c_i) + \\
&W_{compr} c_{compr} + m_{steam} c_{Steam} - \dot{n}_{11} \sum_{G}^{H} (x_{i,str} p_i) \Bigg] 10^{-6} + \\
&\sum_{c=1}^{9} \varepsilon_c + 0.01 \sum_{j=1}^{11} (\Delta u_j)^2 .
\end{aligned}
\tag{4}
$$

The effect of the uncertainties on the overall eNMPC performance is shown in Tab. 2. It can be seen that uncertainty No. 1, which is the composition of feed stream 4, has a big influence on the overall profit, which can become much lower than in the nominal case. Also this is the only case which leads to constraint violations. A second parameter with a big effect on the total profit is coefficient No. 8.2, the reaction rate coefficient a_2. The minimal value of these parameters results in a significantly reduced profit, while the maximum value has a positive impact, with both cases exhibiting above-average changes in the profit. The other uncertainties have a relatively small impact on the total profit, therefore it was decided to only select these two uncertainties for the multi-stage NMPC implementation.

3.2 Scenario tree selection

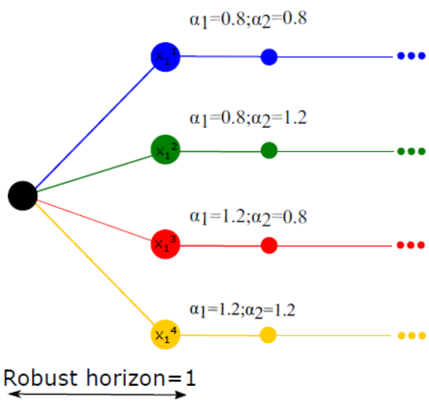

Fig. 3. The result of the scenario tree structure selection: A tree with 4 branches and a robust horizon of 1 step was selected for the economic NMPC implementation.

Table 2. Uncertain parameters and their impact on the total profit

No.	Configuration	Computation time [s]	Profit [$]	Constraint violation
0	default	1349	65938	0
1	min	1192	41683	0
1	max	1197	46917	0.000511
2	min	1327	66099	0
2	max	1339	65606	0
3	min	1319	65938	0
3	max	1322	65938	0
4	min	1338	65877	0
4	max	1352	66002	0
5	min	1340	65974	0
5	max	1510	65894	0
6	min	1439	65278	0
6	max	-	-	-
7	min	1408	63274	0
7	max	-	-	-
8.1	min	1388	65457	0
8.1	max	1415	66218	0
8.2	min	1385	61085	0
8.2	max	1420	69956	0
8.3	min	1385	66045	0
8.3	max	1399	65831	0

With the selection of two parameters for the multi-stage NMPC implementation, the next choice is to decide on the number of realizations of the uncertainty for each parameter. Implementations with two and with three values for each parameter were compared based on three decision criteria (see Fig. 4). For the first case, the candidate values are given by the minimum and the maximum of the respective parameter: $\alpha_1, \alpha_2 \in \{0.8, 1.2\}$. For the second case, the nominal value is included in the scenario tree, so that the candidate values are now $\alpha_1, \alpha_2 \in \{0.8, 1.0, 1.2\}$. The length of the robust horizon was varied between $RH = 0$ (nominal case) and $RH = 1, 2$, while keeping the length of the prediction horizon constant. A set of

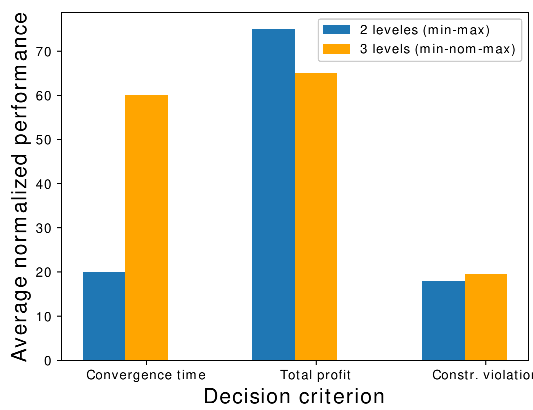

Fig. 4. Performance comparison of two economic MS-NMPC schemes. Scenario trees with 4 branches (blue) were compared to trees with 9 branches (orange).

15 simulations was performed for each of the two MS-NMPC implementations, each one for a different uncertainty realization sampled from the available parameter set. In each case the controlled plant was simulated for 6 hours, with a sampling rate of 100 seconds and a prediction horizon of 2 hours. The results are summarized in the diagram in Fig. 4. The MS-NMPC implementations with three levels of uncertainty for each parameter

achieved an average total profit of 151,615 \$ and required an average computation time of 58 sec/iteration. In comparison, the implementation with only 2 levels of uncertainty for each parameter achieved an average profit of 160,025 \$ and an average computation time of 21 sec/iteration. The results depicted in the figure are normalized with respect to their highest and lowest values. It was found that values of the robust horizon larger than one ($RH = 1$) do not produce an improvement in the solution. This can be attributed to the embedding of feedback in the MS-NMPC structure, which makes it unnecessary to consider larger values of the robust horizon (as discussed also in Lucia et al. (2014)). For this test case it was found that considering only two levels of the uncertainties produces better economics results, while also guaranteeing constraint satisfaction and demanding a lower computational effort. Since the constraint violation is comparable, the second criterion was the computation time. The implementation with only two levels of the uncertainties is much faster and was thus selected for the discussion of the results presented in the next section.

4. ECONOMIC NMPC PERFORMANCE UNDER THE INFLUENCE OF UNCERTAINTY

We begin the investigation of the performance of the eNMPC controller by selecting three control modes from the ones proposed by Downs and Vogel (1992), which are combined in the cost function given in Eq. (5). To this end, we formulated an optimizing controller which can manage dynamic switches between the three modes of operation. The first term in the objective function contains the economics part, which is computed based on the reactant and energy costs from the original TEC publication. The next two terms in the function represent the tracking of a fixed production-rate, as well as the tracking of a fixed product composition in the stream S_{11}. They can be switched on or off during the simulation, by setting the parameter $p_{tv,(1,2)}$ to 1, for activation, and to 0 in order to deactivate the term. The constraints structure is identical to the one in Eqs. (4). The soft constraints, as well as the additional term for input movement penalization, are added in order to implement a smooth operation of the plant. The initial values correspond to those reported by Jockenhövel (2004), which are the base case values. For each case the NMPC was run with a prediction horizon of 2 hours and a sample time of 200 seconds.

$$\begin{aligned}
J_{eNMPC} = {} & 0.5\left[\sum_{j=1}^{4}\dot{n}_j\sum_{A}^{E}(x_{i,j}c_i) - \dot{n}_{11}\sum_{G}^{H}(x_{i,str}p_i) + \right.\\
& \left. + W_{compr}c_{compr} + m_{steam}c_{Steam}\right]10^{-6}\\
& + 0.25p_{tv,0}\left[\sum_{G}^{H}(w_{i,str}-s_i)^2\right] + 0.25p_{tv,1}\left[\sum_{G}^{H}(\dot{n}_{11}M_i x_{i,str}-s_i)^2\right]\\
& + \sum_{c=1}^{9}\varepsilon_c + 0.01\sum_{j=1}^{11}(\Delta u_j)^2.
\end{aligned} \tag{5}$$

The results shown in Fig. 5 are for a switch from purely economics NMPC to economics combined with a fixed product composition and fixed production rate. One can see that the optimal control strategy leads to producing more of the component H, because the reactants for this product are cheaper than the reactants required for G. The profits during the first 3 hours of operation reach a maximum of 24,970 $\$/h$, while all the reactor feed streams and the liquid level are at the constraints. Therefore, the production cannot be further optimized. After 3 hours the two tracking terms are activated. This corresponds to fixed relative demands, which in this case are a mass-ratio of 40/60 between products G and H and a combined production rate of 14,000 kg/h. The new operating point is slightly less profitable and the profit stabilizes at 23,460 $\$/h$ after 4 further hours. The average computation time of the eNMPC was 10.20 sec/iteration, while the maximum was 23 seconds and the minimum was 7.5 seconds. For this test case the results confirm that this implementation is real-time feasible.

Next we show the results of multi-stage NMPC under the influence of plant-model mismatch. The optimization problem implements a scenario tree with robust horizon $RH = 1$ and four branches, as discussed in the previous section and depicted in Fig. 3. The results in Fig. 6 were obtained for the values of the uncertain parameters $[\alpha_1, \alpha_2] = [0.9, 1.1]$, which correspond to a reduced purity of the stream S_4 and an increase in the reaction rate. This means that the simulator is running with a set of values for the two parameters which do not belong to any of the branches that are considered in the scenario tree, but are within the assumed min-max range.

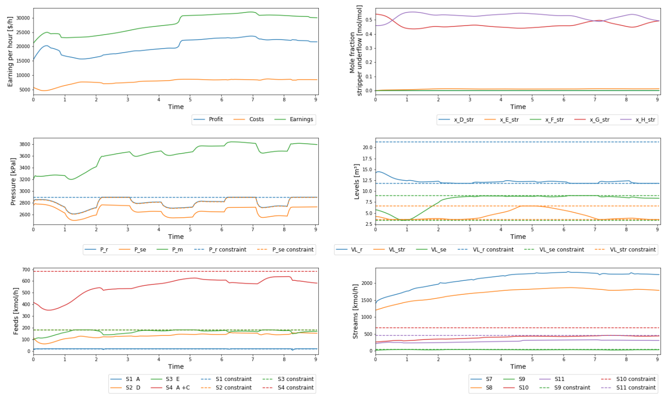

Fig. 7. The influence of random parametric uncertainty on the performance of economic MS-NMPC with added tracking terms. The uncertainties $[\alpha_1, \alpha_2]$ were varied each hour.

The same two different modes of operation are shown in Fig. 6 for the multi-stage case. In the first 3 hours, a purely economic cost function is employed. The average profit is slightly lower than in the nominal case, which can be explained by the necessary back-off of the robust controller and the negative impact of the reactant impurity. At 3 hours, an additional tracking term for the product concentrations is added to the cost function. This results in a temporary increase on the profit that cannot be maintained over a longer operation time. The profit decreases even more, before eventually converging to the value of 21,950 $\$/h$, as the product stream stabilizes around the desired values. The computation times for the MS-NMPC simulation are higher due to the complexity of the NLP and the effect of the uncertainties. An average time of approx. 35 sec/iteration was achieved for this test case. All iterations were in the range of 28 to 50 seconds, with the iterations around t=3 h being slightly more intensive. It is known that the convergence time depends greatly on the choice of the linear solvers. Here the HSL routine $MA86$ was used, which is well suited for large scale problems and provides parallelization features. Another viable alternative that we tested was the solver $MA57$.

In the simulation results depicted in Fig. 7, the combined influence of the uncertain parameters on the performance of the economic MS-NMPC is shown. It is assumed that the purity

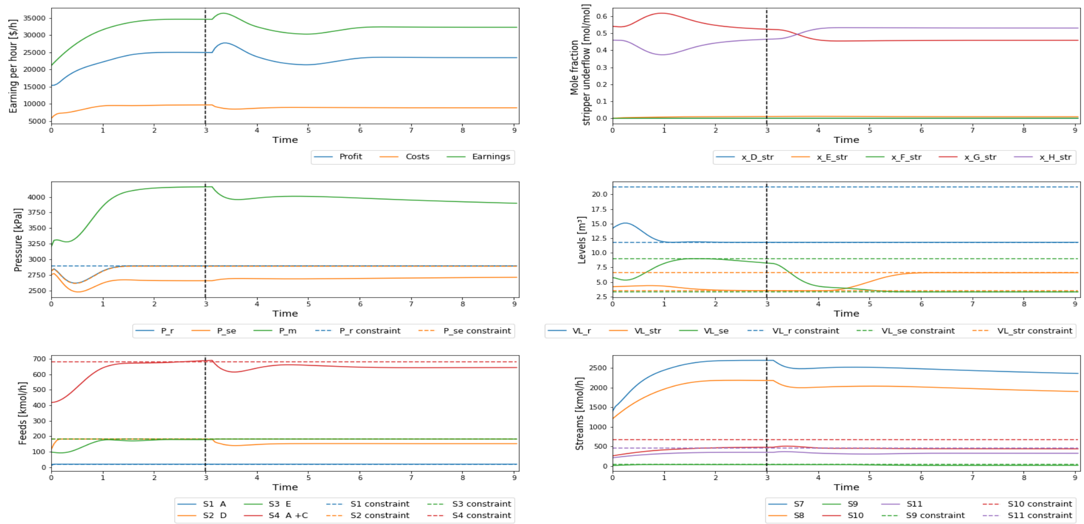

Fig. 5. Plant-wide optimizing NMPC with one switch of control objective. The additional tracking terms are activated at t=3 h.

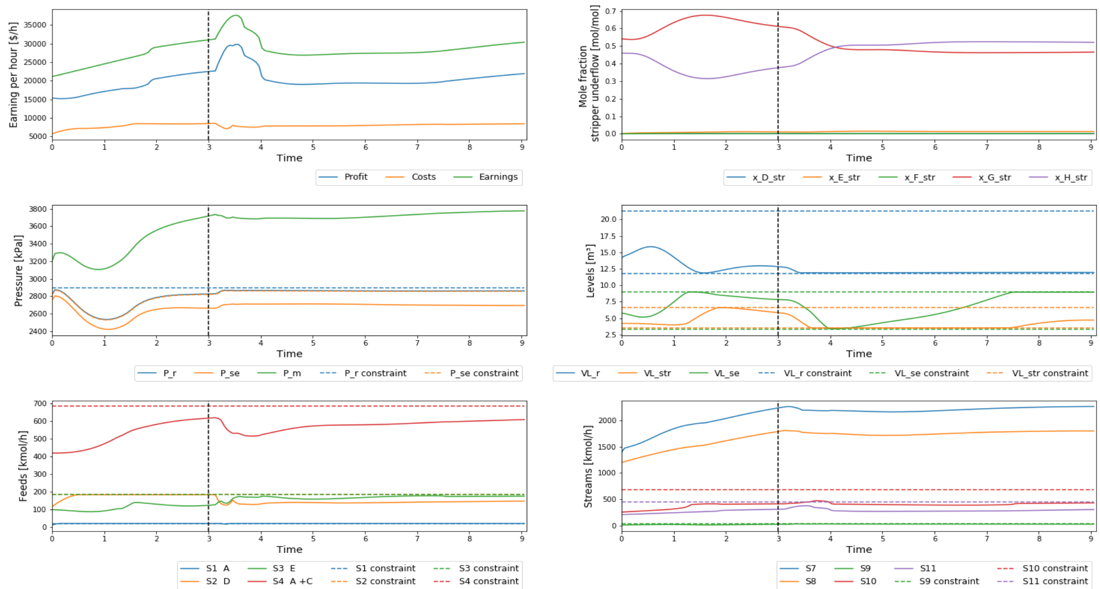

Fig. 6. Multi-stage NMPC results under the influence of parametric plant-model mismatch. A control goal switch is performed, such that the first 3 h are run with the purely economic criterion in Eq. (5) and the last 6 h are run with tracking of concentrations in stream S_{11} and tracking of the production rate.

of stream S_4 changes randomly every hour within the assumed range of $\pm 20\%$ around the nominal value. The reaction rate coefficient was also changed every 3 hours to cover the set [0.8, 1.0, 1.2]. The variations of the purity of reactant stream S_4 cause fluctuations in the pressure responses of the process units that can be handled by the optimizer. The computational effort is slightly higher for this test case, with the average computation time being around 38 sec/iteration and the maximum time being around 70 sec/iteration. The profitability fluctuates more widely because some of the combinations of the uncertainties are more profitable, while others are detrimental to the process performance.

5. CONCLUSIONS

We have presented a new formulation of the complex plant-wide control problem for the Tennessee Eastman benchmark as a direct optimizing control problem which was solved using the *do-mpc* platform. To our knowledge the results represent the first reported instance of the application of eNMPC to the TEC and demonstrate the applicability of the approach to large scale plant-wide control problems through the careful design of the NMPC strategy. We provided results with an economics optimizing NMPC controlling the nominal plant model and of a robust multi-stage NMPC controller for the TEC with parameter variations. We analysed which MS-NMPC formulation is most efficient to handle errors in the parameters that have the largest influence on the plant profit and achieved a robust behaviour using only 4 different scenarios. Using the *do-mpc* platform, which integrates state of the art algorithms and software tools, the solution for the economics NMPC problem is feasible in real time.

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