Robust H_∞ Estimation of Retarded State-multiplicative Systems

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Abstract:
Linear, discrete-time systems with state-multiplicative noise and delayed states are considered. The problem of robust H_∞ general-type filtering is solved for these systems when the uncertainty in their deterministic parameters is of the polytopic-type. The obtained vertex-dependant solution is based on a modified Finsler lemma which leads to a simple set of LMIs condition. The included numerical example demonstrates the tractability and solvability of the proposed method.

1. INTRODUCTION

In this extended abstract we address the problem of robust H_∞ filtering of uncertain discrete-time retarded systems with stochastic state-multiplicative noise. The solution of this problem is based on the solution of the nominal case [i.e. with no uncertainties] and adaptation of the Finsler lemma (Cvetkovski (2012)).

The control and estimation of retarded state-multiplicative noisy systems have been a central topic within the stochastic control theory by large (see for example Boukas and Liu (2002), Xu et al. (2004), Yue et al. (2009) and Mazenc and Normand-Cyrot (2013)) where many of the techniques that were used for the solution of the deterministic counterpart problems have been adopted (see Fridman (2014) for a comprehensive review). In Gershon and Shaked (2013), a solution of the filtering problem is brought for nominal and for polytopic-type uncertain stochastic systems where in the latter, the solution is obtained for the restrictive case where a single Lyapunov function is assigned to all the polytop vertices [the so called "quadratic solution"]. Here we start with a modified solution for the nominal case and we apply a modified version of the Finsler lemma, resulting in a less conservative solution. A numerical example is brought that demonstrates the theoretical result.

Notation: Throughout the work the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n \times m$ real matrices, $\mathcal{N}$ is the set of natural numbers and the notation $P > 0$, (respectively, $P \geq 0$) for $P \in \mathcal{R}^{n\times n}$ means that P is symmetric and positive definite (respectively, semi-definite). We denote by $L^2(\Omega, \mathcal{R}^n)$ the space of square-integrable $\mathcal{R}^n-$ valued functions on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, $\mathcal{F}$ is a σ algebra of a subset of Ω called events and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. By $(\mathcal{F}_k)_{k\in\mathcal{N}}$ we denote an increasing family of σ-algebras $\mathcal{F}_k \subset \mathcal{F}$. We also denote by $\tilde{l}^2(\mathcal{N}; \mathcal{R}^n)$ the n-dimensional space of nonanticipative stochastic processes $\{f_k\}_{k\in\mathcal{N}}$ with respect to $(\mathcal{F}_k)_{k\in\mathcal{N}}$ where $f_k \in L^2(\Omega, \mathcal{R}^n)$. On the latter space the following l^2-norm is defined:

$$\begin{aligned} ||\{f_k\}||^2_{\tilde{l}_2} = E\{\sum_0^\infty ||f_k||^2\} = \sum_0^\infty E\{||f_k||^2\} < \infty, \\ \{f_k\} \in \tilde{l}_2(\mathcal{N}; \mathcal{R}^n), \end{aligned} \tag{1}$$

where $||\cdot||$ is the standard Euclidean norm. We denote by $\mathrm{Tr}\{\cdot\}$ the trace of a matrix and by δ_{ij} the Kronecker delta function.

2. PROBLEM FORMULATION

We consider the following linear retarded system:

$$\begin{array}{l} x_{k+1} = (A_0 + D\nu_k)x_k + (A_1 + F\mu_k)x_{k-\tau_k} + B_1 w_k, \\ x_l = 0,\ l \leq 0 \\ y_k = C_2 x_k + D_{21} n_k \\ z_k = C_1 x_k \end{array} \tag{2}$$

We seek a filter

$$\begin{array}{l} \hat{x}_{k+1} = A_f \hat{x}_k + B_f y_k,\ \ \hat{x}_0 = 0 \\ \hat{z}_k = C_f \hat{x}_k \end{array} \tag{3}$$

where $x_k \in \mathcal{R}^n$ is the system state vector, $w_k \in \mathcal{R}^q$ is the exogenous disturbance signal, $n_k \in \mathcal{R}^p$ is the measurement noise signal, $y_k \in \mathcal{R}^m$ is the measured output and $z_k \in \mathcal{R}^r$ is the state combination (objective function signal) to be regulated and where the time delay is denoted by the integer τ_k and it is assumed that $0 \leq \tau_k \leq h$, $\forall k$. The

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variables $\{\nu_k\}$, and $\{\mu_k\}$ are zero-mean real scalar white-noise sequences that satisfy:

$$E\{\nu_k\nu_j\} = \delta_{kj},\ E\{\mu_k\mu_j\} = \delta_{kj},\ E\{\mu_k\nu_j\} = 0,\ \ \forall k, j\ \geq 0.$$

The matrices in (2) and (3) are constant matrices of appropriate dimensions.

Denoting: $\xi_k^T = [x_k^T\ \hat{x}_k^T]$ and $\bar{w}_k^T = [w_k^T\ n_k^T]$ we write the following augmented system:

$$\begin{array}{c}\xi_{k+1} = \tilde{A}_0\xi_k + \tilde{A}_1\xi_{k-\tau k} + \tilde{B}\tilde{w}_k + \tilde{D}\xi_k\nu_k + \tilde{F}\xi_{k-\tau k}\mu_k,\\ \tilde{z}_k = \tilde{C}\xi_k,\end{array} \quad (4)$$

where

$$\tilde{A}_0 = \begin{bmatrix} A_0 & 0 \\ B_f C_2 & A_f \end{bmatrix},\ \tilde{A}_1 = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\tilde{B} = \begin{bmatrix} B_1 & 0 \\ 0 & B_f D_{21} \end{bmatrix}, \tilde{D} = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix},\ \tilde{C}^T = \begin{bmatrix} C_1^T \\ -C_f^T \end{bmatrix}, \quad (5)$$

and $\tilde{F} = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix}$. We start by solving the H_∞ filtering problems for the nominal systems of (2) and we then expand our solution to the robust uncertain case. We note that similarly to the H_∞ case, the H_2 counterpart filter for nominal systems can be derived by applying our approach.

3. THE ROBUST OPTIMAL H_∞ FILTER

In this section we start by seeking a filter of the type (3) for the nominal system of (2). Since the solution of the filtering problem in Gershon and Shaked (2013) does not lead to the solution of the robust vertex-defendant case, we apply the Finsler lemma (Cvetkovski (2012)), in the sequel, to nominal systems and we latter show how to extend the nominal solution to the uncertain case. Defining the following performance index:

$$J_F \triangleq ||z_k - \hat{z}_k||_{\tilde{l}_2}^2 - \gamma^2[||w_k||_{\tilde{l}_2}^2 + ||n_{k+1}||_{\tilde{l}_2}^2]. \quad (6)$$

our objective is to find a filter of the type of (3) such that J_F is negative for all nonzero w_k, n_k where $w_k \in \tilde{l}_{\mathcal{F}_k}^2([0,\infty);\mathcal{R}^q),\ n_k \in \tilde{l}_{\mathcal{F}_k}^2([0,\infty];\mathcal{R}^p)$.

Applying the BRL result of Gershon and Shaked (2013) to the nominal system of (2), we obtain that the requirement of $J_F < 0$ is achieved for all nonzero $w \in \tilde{l}_{\mathcal{F}_k}^2([0,\infty);\mathcal{R}^q)$, if there exist $2n\times 2n$ matrices $\tilde{Q} > 0$, $\tilde{R}_1 > 0$ and a $2n\times 2n$ matrix Q_m that satisfy the following:

$$\tilde{\Gamma} \triangleq \begin{bmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & 0 & 0 & \tilde{\Gamma}_{15} & 0 & \tilde{C}^T \\ * & -\tilde{Q} & \tilde{\Gamma}_{23} & Q_m & 0 & \tilde{Q}\tilde{B} & 0 \\ * & * & \tilde{\Gamma}_{33} & 0 & \tilde{\Gamma}_{35} & 0 & 0 \\ * & * & * & -\epsilon_b\tilde{Q} & -h\epsilon_b Q_m^T & 0 & 0 \\ * & * & * & * & -\epsilon_b\tilde{Q} & \epsilon_b h\tilde{Q}\tilde{B} & 0 \\ * & * & * & * & * & -\gamma^2 I_q & 0 \\ * & * & * & * & * & * & -I_r \end{bmatrix} < 0 \quad (7)$$

where

$$\begin{aligned}
\tilde{\Gamma}_{11} &= -\tilde{Q} + \tilde{D}^T\tilde{Q}[1+\epsilon_b h^2]\tilde{D} + \tilde{R}_1,\\
\tilde{\Gamma}_{12} &= \tilde{A}_0^T\tilde{Q} + Q_m^T,\\
\tilde{\Gamma}_{15} &= \epsilon_b h[\tilde{A}_0^T\tilde{Q} + Q_m^T] - \epsilon_b h\tilde{Q},\\
\tilde{\Gamma}_{23} &= \tilde{Q}\tilde{A}_1 - Q_m,\\
\tilde{\Gamma}_{33} &= -\tilde{R}_1 + (1+\epsilon_b h^2)\tilde{F}^T\tilde{Q}\tilde{F},\\
\tilde{\Gamma}_{35} &= \epsilon_b h[\tilde{A}_1^T\tilde{Q} - Q_m^T].
\end{aligned}$$

Applying Schur formula the above condition becomes $\Gamma =$

$$\begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} & 0 & 0 & \Gamma_{1,5} & 0 & \tilde{C}^T & \rho\tilde{D}^T\tilde{Q} \\ * & -\tilde{Q} & \Gamma_{2,3} & Q_m & 0 & \tilde{Q}\tilde{B} & 0 & 0 \\ * & * & \Gamma_{33} & 0 & \Gamma_{3,5} & 0 & 0 & 0 \\ * & * & * & -\epsilon_b\tilde{Q} & -h\epsilon_b Q_m^T & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_b\tilde{Q} & \epsilon_b h\tilde{Q}\tilde{B} & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I_q & 0 & 0 \\ * & * & * & * & * & * & -I_r & 0 \\ * & * & * & * & * & * & * & -\tilde{Q} \end{bmatrix} < 0 \quad (8)$$

where

$$\begin{aligned}
\Gamma_{1,1} &= -\tilde{Q} + \tilde{R}_1,\\
\Gamma_{1,2} &= \tilde{A}_0^T\tilde{Q} + Q_m^T,\\
\Gamma_{1,5} &= \epsilon_b h((\tilde{A}_0^T - I)\tilde{Q} + Q_m^T),\\
\Gamma_{2,3} &= \tilde{Q}\tilde{A}_1 - Q_m,\\
\Gamma_{33} &= -\tilde{R}_1 + (1+\epsilon_b h^2)\tilde{F}^T\tilde{Q}\tilde{F},\\
\Gamma_{3,5} &= \epsilon_b h\tilde{A}_1^T\tilde{Q} - Q_m^T
\end{aligned} \quad (9)$$

and where we denote $\sqrt{1+\epsilon_b h^2}$ by ρ.

Denoting by Γ_0 the matrix that is obtained by substituting $\tilde{Q} = 0$ in Γ and defining the $(12)\times(14n+r+q)$ matrices

$$\bar{E} = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \quad (10)$$

and

$$A_Q = \begin{bmatrix} -\frac{1}{2}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{A}_0 & -\frac{1}{2}I & \tilde{A}_1 & 0 & 0 & \tilde{B} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\epsilon_b}{2}I & 0 & 0 & 0 & 0 & 0 \\ \epsilon_b h(\tilde{A}_0 - I) & 0 & \epsilon_b h\tilde{A}_1 & 0 & -\frac{\epsilon_b}{2}I & \epsilon_b h\tilde{B} & 0 & 0 & 0 \\ \rho\tilde{D} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}I & 0 \\ 0 & 0 & \rho\tilde{F} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}I \end{bmatrix}, \quad (11)$$

we obtain from (8) the following condition:

$$\begin{array}{c}\Gamma = \Gamma_0 + \bar{E}^T diag\{\tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q}\}A_Q + \\ A_Q^T diag\{\tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q}\}\bar{E} < 0.\end{array} \quad (12)$$

Applying the Finsler lemma (Cvetkovski (2012)) to the latter, we obtain that (12) is satisfied iff there exist matrices $\tilde{U}$ and $\tilde{V}$, where $\tilde{U} \in \mathcal{R}^{12n\times(14n+r+q)}$ and $\tilde{V} \in \mathcal{R}^{12n\times 12n}$ that satisfy the following:

$$\bar{\Gamma} = \begin{bmatrix} \Gamma_0 + \tilde{U}^T A_Q + A_Q^T \tilde{U} & \bar{\Gamma}_{1,2} \\ * & -\tilde{V} - \tilde{V}^T \end{bmatrix} < 0, \tag{13}$$

where

$$\bar{\Gamma}_{1,2} = -\bar{E}^T diag\{\tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q},\ \tilde{Q}\} + \tilde{U}^T - A_Q^T \tilde{V}.$$

Next, we define the following $2n \times 2n$ matrix

$$J = \begin{bmatrix} \bar{X} & Y \\ 0 & N \end{bmatrix}$$

where $\bar{X},\ Y$, and $N = \bar{X}^T - Y^T$ are nxn matrices.

Choosing,

$$\tilde{U}^T = \begin{bmatrix} J^T & 0 & 0 & 0 & 0 & 0 \\ 0 & J^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0_{2nx2n} & 0 & 0 & 0 \\ 0 & 0 & J^T & 0 & 0 & 0 \\ 0 & 0 & 0 & J^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0_{(q+r)x2n} & 0 \\ 0 & 0 & 0 & 0 & J^T & 0 \\ 0 & 0 & 0 & 0 & 0 & J^T \end{bmatrix} \quad \text{and}$$

$$\tilde{V} = \varepsilon diag\{J,\ J,\ J,\ J,\ J,\ J\},$$

the condition of (13) is written as:

$$\begin{bmatrix} \hat{\Gamma}_{11} & \hat{\Gamma}_{12} \\ * & \hat{\Gamma}_{22} \end{bmatrix} < 0, \tag{14}$$

where:

$$\hat{\Gamma}_{11} = \left[\begin{matrix} \tilde{R}_1 - \frac{1}{2}(J^T + J) & \tilde{A}_0^T J + Q_m^T & 0 & 0 \\ * & -\frac{1}{2}(J^T + J) & -Q_m + J^T \tilde{A}_1 & Q_m \\ * & * & -\tilde{R}_1 & 0 \\ * & * & * & -\frac{\epsilon_b}{2}(J^T + J) \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{matrix}\right.$$

$$\left.\begin{matrix} \epsilon_b h((\tilde{A}_0^T - I)J + Q_m^T) & 0 & \tilde{C}^T & \rho \tilde{D}^T J & 0 \\ 0 & J^T \tilde{B} & 0 & 0 & 0 \\ \epsilon_b h \tilde{A}_1^T J - Q_m^T & 0 & 0 & 0 & \rho \tilde{F}^T J \\ -\epsilon_b h Q_m^T & 0 & 0 & 0 & 0 \\ -\frac{\epsilon_b}{2}(J^T + J) & \epsilon_b h J^T \tilde{B} & 0 & 0 & 0 \\ * & -\gamma^2 I_q & 0 & 0 & 0 \\ * & * & -I_r & 0 & 0 \\ * & * & * & -\frac{1}{2}(J^T + J) & 0 \\ * & * & * & * & -\frac{1}{2}(J^T + J) \end{matrix}\right],$$

$$\hat{\Gamma}_{12} = \left[\begin{matrix} J^T + \frac{\varepsilon}{2} J - \tilde{Q} & -\varepsilon \tilde{A}_0^T J & 0 \\ 0 & \frac{\varepsilon}{2} J + J^T - \tilde{Q} & 0 \\ 0 & -\varepsilon \tilde{A}_1^T J & 0 \\ 0 & 0 & \frac{\epsilon_b \varepsilon}{2} J + J^T - \tilde{Q} \\ 0 & 0 & 0 \\ 0 & -\varepsilon \tilde{B}^T J & \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right.$$

$$\left.\begin{matrix} -\varepsilon\epsilon_b h(\tilde{A}_0^T - I)J & -\varepsilon\rho\tilde{D}^T J & 0 \\ 0 & 0 & 0 \\ -\varepsilon\epsilon_b h \tilde{A}_1^T J & 0 & -\varepsilon\rho\tilde{F}^T J \\ 0 & 0 & 0 \\ \frac{\varepsilon\epsilon_b}{2} J + J^T - \tilde{Q} & 0 & 0 \\ -\varepsilon\epsilon_b h \tilde{B}^T J & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{\varepsilon}{2} J + J^T - \tilde{Q} & 0 \\ 0 & 0 & \frac{\varepsilon}{2} J + J^T - \tilde{Q} \end{matrix}\right]$$

and where

$$\hat{\Gamma}_{22} = -\varepsilon diag\{J+J^T,\ J+J^T,\ J+J^T,\ J+J^T,\ J+J^T,\ J+J^T\}.$$

In the latter, denoting $F_A = A_f^T N$ and $F_B = B_f^T N$, the following products appear:

$$\tilde{A}_0^T J = \begin{bmatrix} A_0^T & C_2^T B_f^T \\ 0 & A_f^T \end{bmatrix} \begin{bmatrix} \bar{X} & Y \\ 0 & N \end{bmatrix} = \begin{bmatrix} A_0^T \bar{X} & A_0^T Y + C_2^T F_B \\ 0 & F_A \end{bmatrix},$$

$$\tilde{A}_1^T J = \begin{bmatrix} A_1^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{X} & Y \\ 0 & N \end{bmatrix} = \begin{bmatrix} A_1^T \bar{X} & A_1^T Y \\ 0 & 0 \end{bmatrix},$$

$$\tilde{B}^T J = \begin{bmatrix} B_1^T & 0 \\ 0 & D_{21}^T B_f^T \end{bmatrix} \begin{bmatrix} \bar{X} & Y \\ 0 & N \end{bmatrix} = \begin{bmatrix} B_1^T \bar{X} & B_1^T Y \\ 0 & D_{21}^T F_B \end{bmatrix}, \tag{15}$$

$$\tilde{D}^T J = \begin{bmatrix} D^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{X} & Y \\ 0 & N \end{bmatrix} = \begin{bmatrix} D^T \bar{X} & D^T Y \\ 0 & 0 \end{bmatrix},$$

$$\text{and} \quad J + J^T = \begin{bmatrix} \bar{X} + \bar{X}^T & Y + Y^T \\ Y^T + Y & \bar{X} + \bar{X}^T - Y - Y^T \end{bmatrix}.$$

The decision variables in the resulting LMI are: $\bar{X},\ Y,\ F_A,\ F_B, C_f,\ \tilde{R}_1,\ Q_m,\ \tilde{Q}$ and γ^2. Two scalar parameters ε and ϵ_b have to be tuned. Now, application of the Finsler lemma (Cvetkovski (2012)) has no bearing on the nominal system (i.e with no uncertainty). The merit of the latter application is that now we can assign a unique Lyapunov function $\tilde{Q}$, in the polytopic case, to each vertex of the polytope (noting that $\tilde{Q}$ does not multiply any of the system matrices). We thus turn to the uncertain case and we assume that the system parameters lie within the following polytope:

$$\bar{\Omega} \triangleq [\, A_0\, A_1\, B_1\, C_1\, C_2\, D_{21}\ D\ F\,], \tag{16}$$

which is described by the vertices:

$$\bar{\Omega} = \mathcal{C}o\{\bar{\Omega}_1, \bar{\Omega}_2, ..., \bar{\Omega}_N\}, \tag{17}$$

where $\bar{\Omega}_i \triangleq$

$$\left[A_0^{(i)} A_1^{(i)}\, B_1^{(i)}\, C_1^{(i)}\ C_2^{(i)}\ D_{21}^{(i)}\, D^{(i)}\, F^{(i)}\right] \tag{18}$$

and where N is the number of vertices. In other words:

$$\bar{\Omega} = \sum_{i=1}^{N} \bar{\Omega}_i f_i, \quad \sum_{i=1}^{N} f_i = 1 \quad , f_i \geq 0. \tag{19}$$

We thus arrive to the following result in the uncertain case:

Theorem 1 Consider the system of (2) where the system matrices lie within the polytope $\bar{\Omega}$ of (16). For a prescribed scalar $\gamma > 0$ and two positive tuning scalars

ϵ_b and ϵ, there exists a filter of the structure (3) that achieves $J_F < 0$, where J_F is given in (6), for all nonzero $w \in \tilde{l}^2([0,\infty);\mathcal{R}^q)$, $n \in \tilde{l}^2([0,\infty);\mathcal{R}^p)$, if there exist $n \times n$ matrices $\bar{X} > 0$, $Y > 0$, $2n \times 2n$ matrices $\tilde{R}_1 > 0$ and Q_m, $n \times n$ matrix F_A, $m \times n$ matrix F_B, $r \times n$ matrix C_f and matrices $\tilde{Q}^i$, $\quad i = 1, 2, ..N$, where N is the number of the vertices, that satisfy (14) and where A_0, A_1, B_1, C_1, C_2, D_{21}, D and F are replaced by $A_0^{(i)}$ $A_1^{(i)}$ $B_1^{(i)}$ $C_1^{(i)}$ $C_2^{(i)}$ $D_{21}^{(i)}$ $D^{(i)}$ and $F^{(i)}$, respectively.

The parameters of the filter should then be given by: $A_f = N^{-T}F_A^T$, $B_F = N^{-T}F_B^T$ and C_f.

Since the resulting transfer function matrix of the optimal filter is given by:

$$H(z) = C_f(zI - N^{-T}F_A^T)^{-1}N^{-T}F_B^T = C_f(zN^T - F_A^T)^{-1}F_B^T$$

we use the fact that $N^T = \bar{X} - Y$ and find that the filter's transfer function matrix is:

$$H(z) = C_f(z(\bar{X} - Y) - F_A^T)^{-1}F_B^T \tag{20}$$

4. EXAMPLE

We consider the system of (2) with the following system matrices:

$$A = \begin{bmatrix} 0.1 & 0.6 \pm a \\ -1 & -0.5 \end{bmatrix},\ D = \begin{bmatrix} 0 & 0.189 \\ 0 & 0 \end{bmatrix},$$

$$F = \begin{bmatrix} 0 & 0.01 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} -0.225 \\ 0.45 \end{bmatrix}, D_{21} = 0.01,$$

$$C_1 = \begin{bmatrix} -0.5 & 0.4 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix},$$

and $C_2 = [\,0\ 1\,]$. Taking $a = 0$ for the nominal case and applying the result of (Gershon and Shaked (2013), see Chapter 5), we obtain for a delay bound of $h = 12$, a near minimum attenuation level of $\gamma = 2.14$ for $\epsilon_b = 0.001$. Taking $a \in [-0.3 \quad 0.3]$, and applying the result of the robust quadratic solution (by assigning the same Lyapunov function over all the uncertainty polytope), we obtain for the latter delay bound a near minimum attenuation level of $\gamma = 23.56$ for $\epsilon_b = 1e - 7$. Applying the result of the less conservative solution of Theory 1 we obtain a near minimum attenuation level of $\gamma = 20.1$.

5. CONCLUSIONS

In this work, the solution of the H_∞ filtering problem for nominal retarded stochastic systems has been extended to the robust vertex-dependant case resulting in a less conservative solution compared to the quadratic solution (where a single Lyapunov function is assigned overall the uncertainty polytop). The improved vertex-dependent solution is achieved by applying the Finsler lemma to a modified solution of the nominal case, resulting in a simple and tractable set of LMIs. We note that solution of the nominal case and therefore the solution of the robust uncertain case is based on the BRL solution, which in turn is based on the application of the input-output approach. The latter approach entails an over-design in the solution method however, the vertex-dependant approach results in a less conservative condition. The numerical example demonstrates the less conservative nature of our solution method compared to the quadratic solution.

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