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Influence of the object stiffness on the grasp stability with compliant hand based on energetic approach

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Abstract:
This article presents a stability analysis of object grasping with a compliant multi-fingered robot hand considering the influence of the flexibility of the grasped object on this stability.In this stability analysis we aimed to compute the maximum disturbance that can be withstood by a compliant hand-object system before being destabilized. Here, the case of objects with a compliant behavior that can be synthesized in a stiffness matrix is addressed. The specific example of beam objects and cylindrical grasps is investigated, a computation of the local stiffness matrix of the beam object is proposed using Euler-Bernoulli theory for beam deflection and the influence of the Young modulus of the beam on the stability is evaluated.

1. INTRODUCTION

Grasping and manipulating objects with complex grippers such as multi-fingered hands is an challenging and arduous task in robotics and it is still an active field of research Grossard et al. (2014). The goal of a grasp is to maintain, without damaging it, an object in the gripper under a definite class of external disturbances. Thus the evaluation of the quality of the grasp can help to define the best grasping parameters depending on specific criteria (robustness, task dependence, dexterous behavior, mechanical properties of the grasped object, etc.).

The class of multi-fingered hands encompass numerous types of grippers, one of which is the compliant hand. The compliance of these hands can come either from the fingertips, from the fingers' segments or from the fingers' joints. In the latter case, the compliance can be passive, or active through the joints controller gains. This kind of gripper is conveniently described by an object or contact-level stiffness model.

One of the most important aspects of the grasp quality is the stability of this grasp Bruyninckx et al. (1998) defined for a grasp at an equilibrium. At an equilibrium the sum of all the forces and moments acting on the grasp equals to zero. When all these forces and moments derive from a potential function, this equilibrium is considered stable if the first-order derivative of the potential energy of the grasp is null and the matrix of second-order partial derivatives is positive definite Nguyen (1986).

The study of the stability of a grasp can be done using numerous type of analysis or criteria, however most of them make the assumption that the grasped object is rigid and undeformable Roa and Suárez (2015). This type of hypothesis simplifies greatly the complexity of the models and the computational cost, however it limits the overall amount of class of objects that can be grasped.

One classical way to ensure the stability of a grasp is by inspecting the force-closure property of the grasp Murray et al. (1994), which can be often encountered in the dedicated literature because it implies a stable equilibrium grasp Nguyen (1986). Although it is a useful property when the grasped object is rigid, the same does not applies for the flexible objects Wakamatsu et al. (1996), due to a bounded amount of force from the grasping hand and non-fixed contact points.

Moreover, not all the stable grasps verify the property of force-closure Howard and Kumar (1995), and and other stability criteria exist based on three types of factors, namely the contact points, and the curvature and compliance at these points. We addressed a part of this issue by proposing an energy-based analysis of the grasp with compliant hands Vollhardt et al. (2019), however this analysis only stand for the undeformable objects.

For deformable objects, the complexity of the modeling of the object lead to the study of alternative ways to circumvent the issue by adding vision Mateo et al. (2015) and learning approach Bodenhagen et al. (2014) to track in real time the deformation of the object and thus adapt the grasp. An other approach consists in learning the mechanical properties and the deformation behavior of the grasped object via iterative learning Frank et al. (2010) Langsfeld et al. (2016). Several other studies restricted their applications to some specific type of deformable object or materials, such as clothes Bell and Balkcom (2010) or strings Bell and Balkcom (2008), or limit the study to squeezing a planar object with two fingers Jia et al. (2011).

This paper presents an extension of our energy-based analysis of the grasp stability with a compliant hand Vollhardt et al. (2019) to the flexible objects in order to study the influence of the non-infinite rigidity of the object on the stability of the grasp. This study takes into consideration the elastic deformation of the fingertips as well as the whole geometry of the hand and the compliance brought by the servoing of the joints. Considering the complexity of the modeling of the deformable object, we suppose that the deformation behavior of the object can be describe using a stiffness matrix. The specific case of beam objects and cylindrical grasps is presented. This analysis is task-oriented, and allows to select and tune a grasp guaranteeing the stability towards specified external disturbances.

The paper proposes two contributions. A modeling of the grasping for flexible objects in section 2 and the stability analysis of the grasping towards external disturbances in the section 3. The section 4 presents the computation of the stiffness matrix for the specific case of beam object and cylindrical grasps, a case study follow in section 5, and a conclusion is given in section 6.

2. FLEXIBLE OBJECT MODEL

This section details our contribution over the modeling of the grasp stiffness with a flexible object grasped by a compliant multi-fingered hand.

2.1 Rigid object approach

Considering a rigid object, Vollhardt et al. (2019) presents a stability analysis for the grasp selection with compliant multi-fingered. This stability analysis is based on an energetic approach of the grasp, defines that the stiffness K_o felt by the object due to the grasp is:

$$K_o = G\left(C_{s_h} + J_h(q)^T C_q J_h(q)\right)^{-1} G^T \tag{1}$$

with, C_{s_h} being the structural compliance of the hand, namely the compliance contributed by the passive elements of the multi-fingered hand, such as the compliance of the several fingertips in contact with the object for example, C_q being the active compliance of the hand contributed by the servoing of the joints q, $J_h(q)$ being the hand Jacobian depending on the fingers' configuration and G being the Grasp matrix.

Considering a direction of external disturbance $d_{W_{ext}}$ applied on the object, Vollhardt et al. (2019) shows that the maximal intensity α_{max} of the disturbance $W^w_{ext\to obj} = \alpha d_{W_{ext}}$, so that the grasp was able to withstand before the object drop from it, depends on the several parameters of the grasp. i.e. the position of the contact points X_c, the active compliance of the hand C_q as well as the squeezing force (also named internal forces, f_i) applied by the hand on the object to maintain it in the grasp.

The same analysis is extended to the case of non-infinite object, given the stiffness behavior of the total grasp. This part will be addressed in the next subsection.

2.2 Grasp stiffness of a flexible object

Let us consider a grasp with n fingertips in contact with the object at n contact points. The position and orientation of the object is given by the homogeneous transformation $g_{w\to obj}$ between the world coordinate frame w and the object coordinate frame obj attached to the center of the object. The homogeneous transformations $g_{obj\to C_k}$ define the position and orientation of all the coordinate frames C_k attached to the contact points with respect to the obj frame. In the same way, the homogeneous transformations $g_{S_k\to C_k}$ defines the same position and orientation with respect to the spatial frame S_k of each finger k, namely their base.

Assuming a local stiffness behavior of the object, we consider that it can be modeled by stiffness matrix K_{lk} expressed in the contact point frame C_k, moreover, from (1) the stiffness felt by the object at the contact point K_c is:

$$K_c = \left(C_{s_h} + J_h(q)^T C_q J_h(q)\right)^{-1} \tag{2}$$

Considering these two stiffnesses, and taking into consideration each finger separately, the grasp can be modeled by two springs in series with K_l and K_{c_k} as their spring constant, K_{c_k} being the submatrix of K_c that is responsible for the stiffness at the contact point k (Fig. 1).

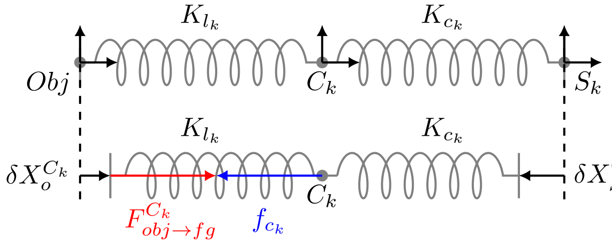

Fig. 1. Simplified representation of the grasp and flexible object for the k^{th} contact point

Let us consider an external disturbance $W^o_{ext\to obj}$ applied on the center of the object in its own frame as the summation of wrenches applied on all the n fingertips by the object:

$$W^o_{ext\to obj} = \sum_{k=0}^{n} W^o_{obj\to fg_k} \tag{3}$$

For a given grasp at equilibrium and for the k^{th} fingertip, the wrench $W^o_{obj\to fg_k}$ applied by the object on the fingertip, the corresponding force $F^{C_k}_{obj\to fg}$ applied on the k^{th} fingertip at the contact point in its own contact frame C_k and the contact forces f_{c_k} applied by the finger on the object at the contact point in C_k are linked by the grasp matrix G (itself depending on the position and orientation $g_{obj\to C_k}$ of C_k with respect to obj frame) as such:

$$W^o_{obj\to fg_k} = G_k(g_{obj\to C_k})F^{C_k}_{obj\to fg_k} = -G_k(g_{obj\to C_k})f_{c_k} \tag{4}$$

Moreover we know that:

$$\begin{cases} F^{C_k}_{obj\to fg} &= K_{l_k}.\delta X^{C_k}_o \\ f_{c_k} &= -K_{c_k}.\delta X^{C_k}_{S_k} \end{cases} \tag{5}$$

$\delta X^{C_k}_o$ and $\delta X^{C_k}_{S_k}$ are the vertical concatenation of the instantaneous displacement and moment of center of the object and of the base of the finger. It is important to take note that both of these displacements result from the combine effect of the external force $W^{C_k}_{ext\to obj}$ as well as the contact forces $f_c = f_r + \lambda.ker(G)$, f_r being the reaction forces, and $\lambda.ker(G)$ the internal forces that can be considered as a pre-load of the springs. Thus, the contribution of the internal force is included in $\delta X^{C_k}_{S_k}$.
So:

$$\begin{cases} W^o_{obj\to fg_k} &= G_k K_{l_k}\delta X^{C_k}_o \\ W^o_{obj\to fg_k} &= -G_k K_{c_k}\delta X^{C_k}_{S_k} \end{cases} \tag{6}$$

By construction of the G_k matrix, it's null space is empty, hence there is no additional term due to the use of the pseudoinverse as such we can say that:

$$\Rightarrow \begin{cases} \delta X^{C_k}_o &= K^{-1}_{l_k}G^+_k W^o_{obj\to fg_k} \\ \delta X^{C_k}_{S_k} &= -K^{-1}_{c_k}G^+_k W^o_{obj\to fg_k} \end{cases} \tag{7}$$

Moreover, we know that the instantaneous displacement and moment of the center of the object with respect to the world frame in its own contact frame $\delta X^o_{o_k}$ is related to $\delta X^{C_k}_o$ and $\delta X^{C_k}_{S_k}$ by:

$$G^T_k \delta X^o_{o_k} = \delta X^{C_k}_o - \delta X^{C_k}_{S_k} \tag{8}$$

So:

$$W^o_{obj\to fg_k} = G_k \left[K^{-1}_{l_k} + K^{-1}_{c_k}\right]^{-1} G^T_k \delta X^o_{o_k} \tag{9}$$

Then if we consider all the contributions of the fingers on the object, we have:

$$W^o_{ext\to obj} = G\left[K^{-1}_l + K^{-1}_c\right]^{-1} G^T \delta X^o_o \tag{10}$$

As such, we have:

$$\delta X^o_o = \left[G\left[K^{-1}_l + K^{-1}_c\right]^{-1} G^T\right]^{-1} W^o_{ext\to obj} \tag{11}$$

Moreover, considering the fact that $K^{-1}_c = C_{s_h} + J^T_h C_q J_h$ (see [8] for more details), C_{s_h} being the structural compliance of the hand and C_q being the active joint compliance of the hand, the equation (11) becomes:

$$\delta X^o_{o_k} = \left[G\left[K^{-1}_l + C_{s_h} + J^T_h C_q J_h\right]^{-1} G^T\right]^{-1} W^o_{ext\to obj} \tag{12}$$

As such both the components K^{-1}_l and C_{s_h} can be put together in a single term of structural compliance C_s:

$$C_s = K^{-1}_l + C_{s_h} \tag{13}$$

We have then the same equation's form as the one in the rigid case, the only difference is that the matrix G should be updated at each step time, because the flexibility of the object produces displacement of contact points changing the grasps matrix G:

$$\delta X^o_{o_k} = \left[G\left[C_s + J^T_h C_q J_h\right]^{-1} G^T\right]^{-1} W^o_{ext\to obj} \tag{14}$$

Based on this result, we are now ready to expose the grasp stability analysis method for a flexible grasped object.

3. STABILITY ANALYSIS

3.1 Plastic deformation energy

The impact of the object's stiffness on the global stiffness of the grasp has been studied in the last section, as such it is possible to compute the displacement of the object under a external perturbation $W^o_{ext\to obj}$, as well as the displacement of the several contact points depending on this external disturbance as well as the contact forces f_c.

However, not all the contact forces are acceptable by a flexible object. Depending on its material and mechanics, the object will be deformed temporarily or definitely or it can even be destroyed depending on the contact forces applied on it. At such it is important to ensure that these forces stay bounded.

This type of constraints can by formalized by a maximal energy E^{max}_{obj} inputted by the contact forces on the object, depending on its deformation.

$$\begin{cases} E_{obj_{max}} &= \dfrac{1}{2}\delta X^{o^T}_{c_{max}} K_l \delta X^o_{c_{max}} \\ E_{obj_{max}} &= \dfrac{1}{2} f^T_{c_{max}} K^{-1}_l f_{c_{max}} \end{cases} \tag{15}$$

Thus, this constraint has to be checked in addition to the friction cone constraints to ensure the stability of the grasp.

Algorithm 1

```
function COMPUTE_XC(λ_fi^final, X_c^o)
    λ_fi = 0
    while λ_fi < λ_fi^final do
        G = Compute_G(X_c^o)
        λ_fi = λ_fi + δλ_fi
        δX_o^c = −K_l^{-1}.δλ_fi.Ker(G)
        X_c^o = X_c^o + δX_c^o
    end while
    return λ_fi, X_c^o
end function
```

3.2 Computation of the internal forces impact on contact points

Without external disturbance on the object, the grasp is supposed to be at an equilibrium state, and the only contribution to the contact forces comes from the internal forces. Thus any variation of the internal forces δf_i will induce a small displacement δX^c_c of the contact point in its own coordinate frame:

$$\delta f_i = \delta\lambda.Ker(G) = -K_l \delta X^c_c \tag{16}$$

As such:

$$\delta X^c_o = K^{-1}_l.\delta\lambda.Ker(G) \tag{17}$$

The fact that the grasp Matrix G is strongly dependent on the position and orientation of the contact frame with respect to the object frame is important here. The computation of δX^c_o is not straightforward as G should be updated each time the contact frame moves with respect to the object frame. Thus, a step by step computation as presented in Algorithm 1 could be used to solve this issue.

3.3 Grasp under external disturbances

For a three-dimensional system, with a given initial set of n contact points located in space by the homogeneous transformations $g_{obj\to C_{i_0}}$ and $g_{S_k\to C_{i_0}}$ $i \in [1, ..., n]$, a joint configuration of the fingers q_i, a joint compliance C_q, an intensity of internal forces λ_{f_i}, an object stiffness K_l, as well as a maximum of deformation energy that can be handled by the object $E_{obj_{max}}$, the maximal intensity of disturbance α_{max} and its corresponding energy $E_{d_{w_o}max}$ is computed by solving the optimization problem presented in Algorithm 2.

Algorithm 2

Set $g_{obj\to C_{i_0}}$, $g_{S_k\to C_{i_0}}$, q_i, C_q, λ_{f_i}, K_l, $E_{obj_{max}}$

$$\alpha_{max}(C_i, k_{p_{ij}}) = \max_{\alpha} \ \alpha$$

$$\begin{aligned} \text{s.t.} \quad & f_{c_{i_z}} \geq 0, \quad i \in [1, ..., n], \\ & \sqrt{f_{c_{i_x}}^2 + f_{c_{i_y}}^2} \leq \mu f_{c_{i_z}}, i \in [1, ..., n], \\ & 0 \leq \alpha, \\ & f_c K_l^{-1} f_c < E_{obj_{max}}, \\ & W_{ext\to obj}^w = \alpha d_{W_{ext}}, \\ & f_{c_i} = New\_State(\delta\alpha) \end{aligned}$$

function NEW_STATE($\delta W_{ext\to obj}^w$)

// Computation of the motions of the object and contact points due to $\delta W_{ext\to obj}$

$\delta W_{ext\to obj_{(k)}}^o \leftarrow Ad_{g_{obj\to w}} \delta W_{ext\to obj}^w$

$K_{o_{(k)}} \leftarrow \left[G_{(k-1)} \left[K_l^{-1} + K_{c_{(k-1)}}^{-1} \right]^{-1} G_{(k-1)}^T \right]^{-1}$

$\delta X_{o_{(k)}}^o \leftarrow K_{o_{(k)}} \delta W_{ext\to obj_{(k)}}^o$

$\delta X_{s_{(k)}}^c \leftarrow K_{c_{(k-1)}}^{-1} G_{(k-1)}^+ W_{ext\to obj_{(k)}}^o$

$\delta X_{o_{(k)}}^c \leftarrow K_l^{-1} G_{(k-1)}^+ W_{ext\to obj_{(k)}}^o$

// Calculation of the reaction forces and the stored energy induced by the motions

$f_{r_{(k)}} \leftarrow f_{r_{(k-1)}} - K_{c_{(k-1)}} \delta X_{s_{(k)}}^c$

$E_{(k)} \leftarrow E_{(k-1)} + X_{o_{(k)}}^{o^T} G_{(k-1)} K_{c_{(k-1)}} G_{(k-1)}^T \delta X_{o_{(k)}}^o$

// Update of the object and contact point pose

$g_{w\to obj_{(k)}} \leftarrow g_{w\to obj_{(k-1)}} \times to\_hgn(\delta X_{o_{(k)}}^o)$

for $i \in [1, ..., n]$ **do**

$g_{obj\to C_{i_{(k)}}} \leftarrow g_{obj\to C_{i_{(k-1)}}} \times to\_hgn(\delta X_{o_{(k)}}^{c_i})$

$g_{S_k\to C_{i_{(k)}}} \leftarrow g_{S_k\to C_{i_{(k-1)}}} \times to\_hgn(\delta X_{s_{(k)}}^{c_i})$

end for

// Update q_i, K_c, G and $W_{ext\to obj}^w$

$q_{i(k)} \leftarrow ikm(g_{S_k\to C_{(k)}})$

$K_{c_{(k)}} = \left(C_s + J_h(q_{i(k)}) C_q J_h(q_{i(k)})^T\right)^{-1}$

$G \leftarrow Compute\_G(g_{obj\to C_{(k)}})$

$W_{ext\to obj_{(k)}}^w \leftarrow W_{ext\to obj_{(k)}}^w + \delta W_{ext\to obj_{(k)}}^w$

return $(f_{r_{(k)}} + \lambda_{f_i} Ker(G_{(k)}))$

end function

The matrix $Ad_{g_{obj\to w}}$ is the adjoint transformation matrix associated with the homogeneous transformation $g_{obj\to w}$, and is used here to map the external disturbance from the world coordinate frame w to the object coordinate frame obj. The vectors $\delta X_{s_{(k)}}^c$ ans $\delta X_{o_{(k)}}^c$ are the concatenation of respectively the $\delta X_o^{C_i}$ and $\delta X_{S_i}^{C_i}$. The functions $to\_hgn()$ and $ikm()$ are respectively in charge of computing the homogeneous transformation corresponding to the instantaneous translation and moment passed as parameter and in charge of computing the inverse kinematic for a given Cartesian position. And Finally μ is the friction coefficient at the contact point, used to check the friction cone constraints.

The function $New\_State()$ is called at each step $\delta\alpha$ of the optimization, to update the complete state of the system depending on the small increase of disturbance intensity $\delta\alpha$. The stability of the system is then checked by using the friction cone constraints and ensuring that the amount of deformation of the object did not went through the maximum bearable.

Hence, in the same way as the rigid case, we proposed a comparable analysis, taking into account the properties of the object and their impact on the stiffness, that is able to compute the maximal amount of perturbation that the object can withstand before falling out of the grasp, and before being permanently damaged due to too high contact forces.

4. SPECIFIC CASE OF BEAM OBJECTS AND CYLINDRICAL GRASPS

It has been shown in the last section that both the rigid and flexible grasp could be analyzed in the same way, given the elastic behavior of the object , that can be computed by a finite element method or in certain case defined by an analytic expression.

In our case, we consider a cylindrical object grasped in a hand using a cylindrical grasp as described in Fig. 3. Given the hypothesis that the surfaces around the punctual contact points between the object and the finger does not change when the finger applies a force on the object, this system can be approximated as a simply supported beam with one load if there is no disturbances (the two fingers on the same side as support and the one on the other as the load), and a simply supported beam with two loads or with one load and three supports depending on the direction of the disturbance. So it is easy to compute beforehand the deformation behavior of the beam, and compute the corresponding stiffness matrix.

4.1 Simply supported beam elastic deflection

The solution of the deflection of a simply support beam under a single load is a well known result in the literature, as well as the two loads case using the superposition principle. We consider that the deflection of the beam follow the Euler-Bernoulli theory, as such only the normal component to the beam before deflection are taken in account. Let us take the following system presented in Fig. 2. f_1, f_2, f_3 are the contact forces applied by the respective fingers on the object, W is the external disturbance applied on the object. a, b, c being the distances between the application points C_1, C_2, C_3, C_w of the several forces, L the sum of these distances, E the Young modulus of the material of the object and J_z the second moment of area of the beam. In this specific case, the equilibrium of the

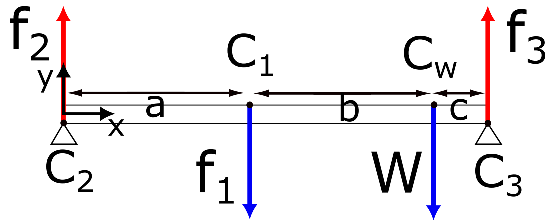

Fig. 2. Simply supported beam with two loads

forces is defined by:

$$\begin{cases} Lf_3 - (a+b)W - af_1 = 0 \\ Lf_2 - cW - (b+c)f_1 = 0 \end{cases} \tag{18}$$

So we have the deflections $v_1(x)$, $v_2(x)$ and $v_3(x)$ along the y-axis, respectively for $x \in [0, a]$, $x \in [a, a+b]$, $x \in [a+b, L]$

$$\begin{cases} v_1(x) = -\dfrac{(b+c)f_1 x}{6LEJ_z}(a^2 + 2a(b+c) - x^2) \\ \quad -\dfrac{cWx}{6LEJ_z}((a+b)^2 + 2c(a+b) - x^2) \\ v_2(x) = -\dfrac{af_1(L-x)}{6LEJ_z}(a^2 - 2xL + x^2) \\ \quad -\dfrac{cWx}{6LEJ_z}((a+b)^2 + 2c(a+b) - x^2) \\ v_3(x) = -\dfrac{af_1(L-x)}{6LEJ_z}(a^2 - 2xL + x^2) \\ \quad -\dfrac{(a+b)W(L-x)}{6LEJ_z}((a+b)^2 - 2x(a+b-2c) + x^2) \end{cases} \tag{19}$$

This deflection can be used to compute the displacement of each contact point as well as the rotation of each contact frame due to the forces f_1, f_2, f_3 and W.

4.2 Local static stiffness

The study of the deflection of a beam under one or several loads suppose that the supports are fixed in space and do not move, as such the deflection $v(x)$ is the absolute displacement of each point of the beam in a world reference frame. However, this displacement is also relative to the straight line between the two support points C_2 and C_3, as such the displacement can be split in two equal parts between both sides of the beam if we consider that the supports are not fixed. Hence these displacements are still with respect to a fixed point that correspond to the center of the object.

Let us define dC_{1y}, dC_{2y} and dC_{3y}, the displacements along the y-axis with respect to the center of the object:

$$\begin{cases} dC_{1y} = \dfrac{v(a)}{2} \\ dC_{2y} = -\dfrac{v(a)}{2} \\ dC_{3y} = -\dfrac{v(a)}{2} \end{cases} \tag{20}$$

Moreover for a small displacement dx and a small variation of force dF, the stiffness K is defined as:

$$K^{-1} = \frac{dx}{dF} \tag{21}$$

As such using (20), (19) and (21), we can define the local static stiffness K_{lyi} at each contact point i with respect to the y-axis as:

$$\begin{cases} K_{ly1}^{-1} = \dfrac{dC_{1y}}{df_1} = \dfrac{a^2(b+c)^2}{6LEJ_z} \\ K_{ly2}^{-1} = -\dfrac{dC_{2y}}{df_2} = \dfrac{a^2(b+c)}{6EJ_z} \\ K_{ly3}^{-1} = -\dfrac{dC_{3y}}{df_3} = \dfrac{a(b+c)^2}{6EJ_z} \end{cases} \tag{22}$$

Besides, it can be seen that:

$$K_{ly1}^{-1} = -\frac{b+c}{L} K_{ly2}^{-1} = -\frac{a}{L} K_{ly3}^{-1} \tag{23}$$

We supposed that the elastic behavior of the system follow the Euler-Bernoulli theory, as such no displacement is allowed in the other directions, thus the local stiffness at each contact point in the x-axis and z-axis are infinite, and:

$$K_{lxi}^{-1} = K_{lzi}^{-1} = 0 \tag{24}$$

Thus, the complete local stiffness for the i-th contact is:

$$K_{li}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{lyi}^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{25}$$

Moreover, using the first derivative of the deflection $v(x)$ at the point $x = 0$, $x = a$ and $x = L$ gives us the rotation angle around the z-axis to compute the new orientation of each contact frame due to the deflection of the beam.

This approach enables us to construct a stiffness matrix corresponding to the elastic behavior of a cylindrical object using the several hypothesis defined previously as such we can use the stability analysis for this type of system. In the next section, we are going to present a study case dealing with the impact of the flexibility on the stability of cylindrical grasp system.

5. CASE STUDY

This section illustrates our analysis extended to the cylindrical grasps based on an example of a pick and place task with a hand. This example will show the impact of the flexibility of the grasped object on the stability of its grasp.

5.1 Considered system

The following ideal 3D example relates to a grasp with three fingers and three contact points and Coulomb friction constraints (Fig. 3). We consider in this study two different objects with the exact same geometry but two different Young modulus, one at $E_1 = 15GPa$ that correspond to some wood, and an other one greater at $E_2 = 69GPa$ that corresponds to aluminium. The section of the object is considered circular with a diameter of $d = 10^{-2}m$, as such its second moment of area is $J_z = \frac{\pi(10^{-2})^4}{64} = 4.91.10^{-10}m^4$. The distances a and c are both equal to $1.10^{-1}m$ and b is null as the contact point C_1 is placed right in front of the center of the object. The intensity of the internal forces applied by the fingers on the object is $f_{i1} = 20N$, $f_{i2} = f_{i3} = 10N$.

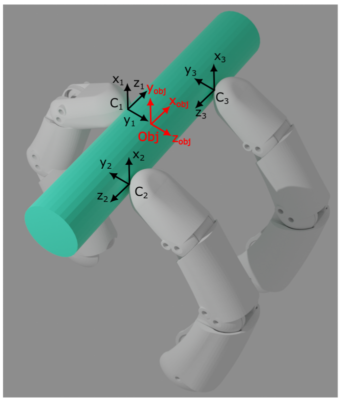

Fig. 3. 3D representation of a cylindrical grasp

5.2 *Pick and Place example task*

Let us consider the following pick and place task, the object is grasped at a position A in space, and moved to a position B along a trajectory that remains in the horizontal plane (x, z) to which belong A and B. Thus all the accelerations withstood by the object also remain inside its plane (x_{obj}, z_{obj}). The pick and place task is designed so that the accelerations along its trajectory induce on the object several directions of disturbances that are listed below:

$$d_{w_{ext}} \in \left\{ \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{-\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix} \right\} \tag{26}$$

5.3 *Analysis for a fixed grasp*

Given the pick and place task, its set of direction of disturbances and the two different grasps, it is possible to analysis the stability of the two systems along the several directions of disturbances. The table I gather de results for the maximal intensity of $W^{w}_{ext \to obj}$ for each type of material and each direction.

Table 1. Maximal intensity of disturbance for the tasks directions and each chosen young modulus

$d^T_{w_{ext}}$	$E = 15GPa$	$E = 69GPa$
$\begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \end{bmatrix}$	13.9	4.4
$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$	7.5	4.4
$\begin{bmatrix} -1 & 0 & 0 \end{bmatrix}$	15.7	14.1
$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$	15.7	14.1
$\begin{bmatrix} \frac{-\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}$	7.5	4.4
$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \end{bmatrix}$	13.9	4.4

It can be seen that a flexible grasped object allows a better stability for the several direction of disturbance chosen for the task. However this comparison is only true for the two chosen young modulus for the objects, that does not prove that a flexible object always allow a better stability than a rigid one.

5.4 *Influence of the flexibility on the stability of the grasp*

Given the same system, with higher internal forces ($f_{i1} = 30N$, $f_{i2} = f_{i3} = 15N$) and considering only one direction of disturbance along the x-axis of the object frame, the figure 4 shows the evolution of the maximal intensity of the disturbance depending on the Young modulus E, $E \in [1.5.10^{10}, 5.10^{10}]$. It can be seen that the stability decrease

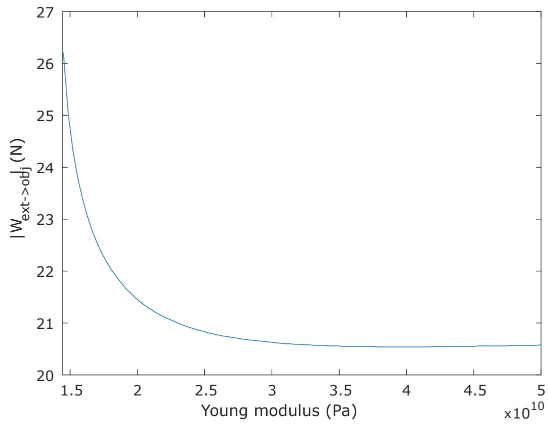

Fig. 4. Influence of the flexibility of the grasped beam on the stability of the grasp

with the increase of the Young modulus for interval and this type of grasp, as such, it can be at least deduced that the stability of the system is better with a flexible object than a rigid one.

6. CONCLUSIONS

This paper extends a stability analysis already existing for rigid objects to flexible objects that can have their flexible behavior defined by a local stiffness matrix. This stability analysis takes explicitly into account in addition to the local stiffness matrix, the same elements as the rigid case, that are: the whole geometry of the hand, the compliance due to mechanical design and the control gains in the joints.

The specific case of beam objects and cylindrical grasp as been addressed to form the local stiffness matrix. The influence of the flexibility on this kind of object has been illustrated with an ideal pick and place task, for several directions of disturbances that it induce and for two different objects. Moreover the influence of the flexibility of the object on the grasp stability has also been studied for only one specific direction of disturbance and an interval of an interval of Young modulus values, which defines the flexibility of the object.

This analysis has shown that including flexibility in the modeling of a grasp allow a finer model, and that the difference between a complete rigid model and a model with some flexibility is not negligible and thus should be considered for object that have flexibility. In future works, the influence of the control gains and the contacts points on the stability of flexible object should be evaluated. Based on these works, a control strategy adapting the control gains, internal forces and contact point locations could be proposed to ensure a stable grasp and cope with external disturbances.

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