Presented as late breaking results contribution
21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Decoupling Problems for Switching Linear Systems without Knowledge of the Switching Signal

Giuseppe Conte * **Anna Maria Perdon** * **Elena Zattoni** **

* *Dipartimento di Ingegneria dell'Informazione, Università Politecnica delle Marche, 60131 Ancona, Italy.*
(e-mail: gconte@univpm.it, perdon@univpm.it)
** *Department of Electrical, Electronic, and Information Engineering "G. Marconi", Alma Mater Studiorum Università di Bologna, 40136 Bologna, Italy*
(e-mail: elena.zattoni@unibo.it).

Abstract: The disturbance decoupling problem is considered in the framework of switching linear systems assuming that no information on the actual value of the switching signal or on the current mode of the system is available. By introducing the novel notion of strong conditioned invariant subspace, the solvability of the problem is completely characterized by means of necessary and sufficient conditions. Constructive procedures to check the solvability conditions and to construct solutions, if any exists, are given.

Keywords: disturbance decoupling; switching systems; geometric methods.

1. INTRODUCTION

In recent years, a large research effort has been devoted to the study of switching systems in reason of their effectiveness in modeling complex dynamical behaviors. Stability and stabilization problems, as well as control and regulation problems, have been investigated using different methods and tools. Examples and results can be found in several books and collections of papers, like e.g. Antsaklis and Nerode (1998), Antsaklis (2000), Van der Schaft and Schumacher (2000), Savkin and Evans (2002), Liberzon (2003), Sun and Ge (2005), Haddad et al. (2006), Goebel et al. (2012), Zattoni et al. (2020).
In considering control and regulation problems that involve switching systems, a quite standard assumption is that the value of the switching signal at each time, and hence the current mode of the system, is known. This allows the implementation of switching controllers that switch synchronously with the system to be controlled or regulated. Actually, unless the switching is controlled by the designer or by the system manager himself, the validity of such assumption may be questionable and, in some situations, for instance when the switching is due to unpredictable failures, it is likely that the assumption is violated. For this reason, it is interesting to explore the possibility of solving specific control and regulation problems without any knowledge about the switching signal, except the fact that it belongs to a given class, although obviously this leads to tight solvability conditions.
In this paper, we consider the disturbance decoupling problem for linear switching systems and we look for solutions that work without any information on the actual value of the switching signal or of the current mode of the system. We provide a complete characterization of solvability of the problem in terms of structural necessary and sufficient conditions and we give also an algorithmic procedure to test the conditions and to construct solutions, if any exist.
In our approach, we adopt a structural point of view that exploits the geometric approach for linear systems (Basile and Marro, 1987; Wonham, 1985) and we introduce a suitable, novel notion of controlled invariance, called strong controlled invariance. Our results can be seen as an extension to the case in which the switching signal is not known of the results obtained on the same problem in Otsuka (2010), Zattoni et al. (2013), Conte et al. (2014), Zattoni et al. (2014), Zattoni et al. (2016), Perdon et al. (2017).
The symbols $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{Z}^+$ are used to denote the sets of real numbers, nonnegative real numbers and nonnegative integer numbers, respectively. Real vector spaces and subspaces are denoted by calligraphic letters, like $\mathcal{V}$. Linear maps between vector spaces and the associated matrices are denoted by the same slanted capital letters, like A. Therefore, the statements $A \in \mathbb{R}^{p\times q}$ and $A : \mathbb{R}^q \to \mathbb{R}^p$ are consistent. The image and the kernel of A are denoted by $\operatorname{Im} A$ and $\operatorname{Ker} A$, respectively.

2. PRELIMINARIES AND STATEMENT OF THE PROBLEM

A continuous-time switching linear system Σ_σ is a dynamical system described by the equations

$$\Sigma_\sigma \equiv \begin{cases} \dot{x}(t) \,=\, A_{\sigma(t)}\, x(t) + B_{\sigma(t)}\, u(t), \\ y(t) \,=\, C_{\sigma(t)}\, x(t), \end{cases} \tag{1}$$

where $t \in \mathbb{R}^+$ is the time, $x \in \mathcal{X} = \mathbb{R}^n$ the state, $u \in \mathcal{U} = \mathbb{R}^m$ the input, $y \in \mathcal{Y} = \mathbb{R}^p$ the output. Letting $\mathcal{I} = \{1, \ldots, N\}$ denote a finite index set, the function $\sigma : \mathbb{R}^+ \to \mathcal{I}$ is a piece-wise constant function that represents the switching time signal. For any value $i \in \mathcal{I}$, A_i, B_i, C_i are real matrices of suitable dimensions. The time-invariant linear systems

$$\Sigma_i \equiv \begin{cases} \dot{x}(t) \,=\, A_i\, x(t) + B_i\, u(t), \\ y(t) \,=\, C_i\, x(t), \end{cases} \qquad \text{with } i \in \mathcal{I}, \tag{2}$$

are called the *modes* of Σ_σ. The indexed family $\Sigma = \{\Sigma_i\}_{i\in\mathcal{I}}$ is said to be the family of the modes of Σ_σ and the active mode at the time t is specified by the value of $\sigma(t) \in \mathcal{I}$.

We say that the switching signal σ is **measurable** if its value $\sigma(t)$ is known at each time instant t and its knowledge can be used for control purposes. Otherwise, we say that $\sigma(t)$ is **unmeasurable**. Letting $\{t_\ell,\ \ell \in \mathbb{Z}^+,\ t_0 = 0\}_\sigma$ be the set of the discontinuity points of σ, we denote by τ_σ the dwell time of σ, defined by $\tau_\sigma = \inf\{t_{\ell+1} - t_\ell,\ \ell \in \mathbb{Z}^+\}$. In order to avoid Zeno phenomena, only switching signals with dwell time greater than 0 are admissible. The set of all switching signals satisfying this condition is denoted by $\mathscr{S}_0$.
Now, assume that Σ_σ is subject to an additional disturbance input, so that its equations take the form

$$\Sigma_{D\sigma} \equiv \begin{cases} \dot{x}(t) = A_{\sigma(t)}\, x(t) + B_{\sigma(t)}\, u(t) + D_{\sigma(t)}\, d(t), \\ y(t) = C_{\sigma(t)}\, x(t), \end{cases} \tag{3}$$

where $d \in \mathbb{R}^q$ is a disturbance input. We want to study the problem of decoupling the output $y(t)$ of $\Sigma_{D\sigma}$ from the disturbance $d(t)$ by means of a state feedback in the case in which the switching signal σ is unmeasurable, and hence the current mode of the system is not known. Clearly, in such case we need to employ a non-switching feedback. The resulting problem can then be stated as follows.

Problem 1. Given a switching linear system $\Sigma_{D\sigma}$ of the form (3) for which the switching signal σ is unmeasurable, the Strong Disturbance Decoupling Problem (SDDP) consists in finding a state feedback $u(t) = F\, x(t)$, such that the compensated system $\Sigma^F_{D\sigma}$ described by the equations

$$\Sigma^F_{D\sigma} \equiv \begin{cases} \dot{x}(t) = (A_{\sigma(t)} + B_{\sigma(t)}\, F)\, x(t) + D_{\sigma(t)}\, d(t), \\ y(t) = C_{\sigma(t)}\, x(t), \end{cases} \tag{4}$$

satisfies the following structural condition:

$\mathcal{R}\,1.$ the output $y(t)$ of $\Sigma^F_{D\sigma}$ is independent from the disturbance $d(t)$ for any $\sigma \in \mathscr{S}_0$, that is $y(t) = 0$ for all $t \in \mathbb{R}^+$, for all disturbance input $d(t)$ and for any $\sigma \in \mathscr{S}_0$, provided that $x(0) = 0$.

3. PROBLEM SOLUTION

To deal with the problem stated in the previous section, we introduce a notion that is stronger than the notion of hybrid controlled invariance by which the disturbance decoupling problem was solved in the case of measurable switching signals (Otsuka, 2010; Conte and Perdon, 2011; Conte et al., 2014; Zattoni et al., 2014, 2016).

Definition 1. Given a switching linear system Σ_σ of the form (1), a subspace $\mathcal{V} \subseteq \mathcal{X}$ is said to be **strong hybrid controlled invariant**, or **robust strong (A_i, B_i)-invariant**, if there exists a feedback map $F : \mathbb{R}^n \to \mathbb{R}^m$ such that

$$(A_i + B_i\, F)\mathcal{V} \subseteq \mathcal{V} \quad \text{for every } i \in \mathcal{I}. \tag{5}$$

Any map $F : \mathbb{R}^n \to \mathbb{R}^m$ satisfying (5) is called a **strong friend** of $\mathcal{V}$.

Denoting by $\mathcal{V}^N \subseteq (\mathbb{R}^n)^N$ the subspace

$$\mathcal{V}^N = \begin{pmatrix} \mathcal{V} \\ 0 \\ 0 \\ \\ \vdots \\ 0 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ \mathcal{V} \\ 0 \\ \\ \vdots \\ 0 \end{pmatrix} \oplus \ldots \oplus \begin{pmatrix} 0 \\ 0 \\ 0 \\ \\ \vdots \\ \mathcal{V} \end{pmatrix},$$

(5) is equivalent to

$$\begin{pmatrix} A_1 \\ \vdots \\ A_N \end{pmatrix} \mathcal{V} \subseteq \mathcal{V}^N + Im \begin{pmatrix} B_1 \\ \vdots \\ B_N \end{pmatrix}. \tag{6}$$

Proposition 2. Given a switching linear system Σ_σ of the form (1) and a subspace $\mathcal{W} \subseteq \mathbb{R}^n$, the set of all strong hybrid controlled invariant subspaces contained in $\mathcal{W}$ forms a semi-lattice with respect to inclusion and sum of subspaces, therefore there exists a maximum element of that set, denoted by $\mathcal{V}^*_S(\mathcal{W})$, or simply by $\mathcal{V}^*_S$ if no confusion arises.

Proposition 3. The sequence of subspaces defined by

$$\begin{aligned} \mathcal{V}_0 &= \mathcal{W} \\ \mathcal{V}_{k+1} &= \mathcal{V}_k \bigcap \begin{pmatrix} A_1 \\ \vdots \\ A_N \end{pmatrix}^{-1} \left(\mathcal{V}^N_k + Im \begin{pmatrix} B_1 \\ \vdots \\ B_N \end{pmatrix} \right) \end{aligned} \tag{7}$$

converges in a finite number of steps. Its limit is the maximum strong hybrid controlled invariant subspace $\mathcal{V}^*_S(\mathcal{W})$ contained in $\mathcal{W}$.

Now, we can give a complete characterization of solvability of the SDDP using the notions introduced above.

Theorem 1. Let $\Sigma_{D\sigma}$ with modes $\{\Sigma_{Di},\ i \in \mathcal{I}\}$, be a continuous-time switched linear system of the form (3) for which the switching signal σ is unmeasurable and let $\mathcal{V}^*_S$ denote the maximum robust strong (A_i, B_i)-controlled invariant subspace contained in $\bigcap_{i \in \mathcal{I}} \operatorname{Ker} C_i$. Then, the SDDP for $\Sigma_{D\sigma}$ is solvable if and only if for all $i \in \mathcal{I}$ one has

$$\operatorname{Im} D_i \subseteq \mathcal{V}^*_S \tag{8}$$

Remark 1. Clearly, the necessary and sufficient solvability condition of Theorem 1 is tight, but this has to be expected, since the fact that σ is unmeasurable makes the information on the system quite poor. It has to be remarked, however, that condition (8) can be checked by constructing $\mathcal{V}^*_S$ by means of the procedure given in Proposition 3 and that a friend F can be synthesized without any information on σ. A solution to the SDDP, if any exists, can therefore be practically implemented.

4. CONCLUSION

New structural notions have been introduced and shown to be useful for characterizing structural solvability of the disturbance decoupling problem for switching linear systems without information on the actual value of the switching signal, i.e. on the current mode, and for synthesizing solutions. The same problem with the additional requirement of global asymptotic stability for $\Sigma^F_{D\sigma}$ is much more complex to deal with, since it involves a simultaneous stabilization problem, whose solvability, in general, cannot be expressed by rational conditions, as shown in Blondel (1994), Blondel (1999), Patel (1999). Further investigations on this subject will be carried on in the future.

REFERENCES

Antsaklis, P.H. (2000). Special issue on hybrid systems. *Proceedings of IEEE*, 88(7).

Antsaklis, P.H. and Nerode, A. (1998). Special issue on hybrid control systems. *IEEE Trans. Automatic Control*, 43(4).

Basile, G. and Marro, G. (1987). On the robust controlled invariant. *Systems & Control Letters*, 9(3), 191–195.

Blondel, V.D. (1994). *Simultaneous stabilization of linear systems*. Springer.

Blondel, V.D. (1999). Simultaneous stabilization of linear systems and interpolation with rational functions. In V.D. Blondel, E. Sontag, M. Vidyasagar, and J.C. Willems (eds.), *Open Problems in Mathematical Systems and Control Theory*, 53–60.

Conte, G. and Perdon, A.M. (2011). A necessary condition for disturbance decoupling with quadratic stability in switched linear systems. In *19th Mediterranean Conference on Control and Automation*, 1349–1354. Corfu, Greece.

Conte, G., Perdon, A.M., and Otsuka, N. (2014). The disturbance decoupling problem with stability for switching dynamical systems. *Systems & Control Letters*, 70, 1–7.

Goebel, R., Sanfelice, R.G., and Teel, A.R. (2012). *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*. Princeton University Press, Princeton, New Jersey.

Haddad, W.M., Chellaboina, V.S., and Nersesov, S.G. (2006). *Impulsive and Hybrid Dynamical Systems*. Princeton Series in Applied Mathematics. Princeton University Press, Princeton.

Liberzon, D. (2003). *Switching in Systems and Control*. Birkhauser, Boston.

Otsuka, N. (2010). Disturbance decoupling with quadratic stability for switched linear systems. *Systems & Control Letters*, 59(6), 349–352.

Patel, V.V. (1999). Solution to the “champagne problem” on the simultaneous stabilization of three plants. *Systems & Control Letters*, 37, 173–175.

Perdon, A.M., Zattoni, E., and Conte, G. (2017). Disturbance decoupling in hybrid linear systems with state jumps. *IEEE Transactions on Automatic Control*, 62(12), 6552–6559.

Savkin, A.V. and Evans, R.J. (2002). *Hybrid dynamical systems: controller and sensor switching problems*. Birkhauser, Boston.

Sun, Z. and Ge, S. (2005). *Switched Linear Systems: Control and Design*. Springer-Verlag, London.

Van der Schaft, A.J. and Schumacher, J.M. (2000). *An Introduction to Hybrid Dynamical Systems*. Springer-Verlag, New York.

Wonham, W.M. (1985). *Linear Multivariable Control: A Geometric Approach*. Springer-Verlag, New York, 3 edition.

Zattoni, E., Perdon, A.M., and Conte, G. (2013). The output regulation problem with stability for linear switching systems: a geometric approach. *Automatica*, 49(10), 2953–2962.

Zattoni, E., Perdon, A.M., and Conte, G. (2014). Disturbance decoupling with stability in continuous-time switched linear systems under dwell-time switching. *IFAC Proceedings Volumes*, 47(3), 164–169.

Zattoni, E., Perdon, A.M., and Conte, G. (2016). Disturbance decoupling with closed-loop modes stability in switched linear systems. *IEEE Transactions on Automatic Control*, 61(10), 3115 – 3121.

Zattoni, E., Perdon, A.M., and Conte, G. (eds.) (2020). *Structural Methods in the Study of Complex Systems*, volume 482 of *Lecture Notes in Control and Information Sciences*. Springer, Cham, Switzerland.