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Closed-Loop Identification of Ill-Conditioned Systems Using Rotation

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Abstract: A simple and practical method for closed-loop identification of ill-conditioned systems is presented. The method uses rotation matrices to identify the process in directions important for control. This approach simplifies the identification and improves reliability, because simple first order models are usually adequate in the identification step. An open-loop / closed-loop duality is introduced, which gives a new holistic viewpoint on control-relevant identification of 2×2 ill-conditioned systems. Pitfalls, and some guidelines how to avoid them, in control-relevant identification of ill-conditioned systems are discussed.

Keywords: Identification for control, Process modeling and identification, Ill-conditioned systems, Principal Component Analysis

1. INTRODUCTION

To achieve tight two-point distillation control, a model-based control algorithm, e.g. Model-Predictive Control (MPC) is usually employed. To implement MPC we need an algorithm, which is usually available for today's automation systems, and a model of the controlled process. Naturally, the model must be tailor-made (identified) for each application. An identification experiment must be performed during commissioning, and also later to keep the model up to date and to ensure good control performance.

A process is classified as ill-conditioned when the condition number of the steady-state gain matrix is high. Ill-conditioned processes are characterized by directionality, i.e. some input directions are highly amplified, and some are not. In addition, process dynamics usually go hand in hand with the gain directions. For control purposes, it is important to model and identify both the gains and the dynamics of all gain directions (Jacobsen and Skogestad, 1994).

Hovd et al. (1997) introduced the SVD control concept for a subset of multi-variable processes with frequency-independent rotation matrices of the form

$$\mathbf{G(s)} = \mathbf{W\Sigma(s)V}^H, \mathbf{\Sigma(s)} = diag(\sigma_i(s)) \tag{1}$$

According to Hovd et al. (1997), processes of type (Eq. 1) are quite common in the process industry.

Friman (2020) used Eq. 1 for high-purity distillation models to simplify identification. Eq. 1 provides a structure with fewer states and fewer parameters compared to traditional modeling. Still, this model structure is useful for model-based control of ill-conditioned systems, because it takes into account the dynamics in all gain directions.

Friman (2020) only discussed open-loop identification, but closed-loop identification is sometimes preferred, because identification can often be done during production. In this paper we extend the results to closed-loop identification. In addition, a duality that provides a holistic overview of ill-conditioned system's identification is presented. The duality clarifies the conditions for successful, control-relevant identification of ill-conditioned systems.

Our aim is to introduce a simple and practical closed-loop identification concept that can provide good models for control. The objectives are: 1) to keep the number of parameters and states as low as possible, 2) to use simple ARX identification, 3) to capture the directionality of the process (both gain and dynamics), which is crucial for control, and 4) to provide easy assessment of identified model quality.

It has been estimated that distillation stands for 10-15% of total energy consumption in the world. We believe that tight model-based distillation control can reduce the energy consumption globally, and that the model identification method presented here is so simple and practical that it is well suited and useful for industrial applications.

2. DIFFICULTIES IN IDENTIFICATION OF ILL-CONDITIONED SYSTEMS

Traditionally, models for high-purity distillation control have been obtained by fitting individual transfer function elements, typically first-ordered-plus-dead-time models for each input-output pair (Wood and Berry, 1973), (Waller et al., 1988)

$$\mathbf{G(s)} = \begin{pmatrix} \frac{k_{11}exp(-L_{11}s)}{T_{11}s+1} & \frac{k_{12}exp(-L_{12}s)}{T_{12}s+1} \\ \frac{k_{21}exp(-L_{21}s)}{T_{21}s+1} & \frac{k_{22}exp(-L_{22}s)}{T_{22}s+1} \end{pmatrix} \tag{2}$$

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Jacobsen and Skogestad (1994) used a simple heat exchanger model to demonstrate that, for ill-conditioned systems (like high-purity distillation), first-order models (Eq. 2) are able to fit open-loop data well, but they are badly suited for control purposes because they can not model the fast responses in the low gain direction. To correct this drawback, Jacobsen and Skogestad (1994) suggest the model structure

$$\mathbf{G(s)} = \frac{\begin{pmatrix} k_{11}(z_{11}s+1) & k_{12}(z_{12}s+1) \\ k_{21}(z_{21}s+1) & k_{22}(z_{22}s+1) \end{pmatrix}}{(T_1 s+1)(T_2 s+1)} \tag{3}$$

This model can capture directionality, but compared to the original first principle model of the heat exchanger, which has two states, Eq. 3 has 6 states in the general case. Therefore, it is motivated to use the SVD model structure (Eq. 1) to model the heat exhcanger with the the two-state model

$$\mathbf{G(s)} = \mathbf{W} \begin{pmatrix} \dfrac{k_1}{T_1 s+1} & 0 \\ 0 & \dfrac{k_2}{T_2 s+1} \end{pmatrix} \mathbf{V^T} \tag{4}$$

Here we assume that the rotation matrices are organized such that k_1 is the high gain and k_2 is the low gain, i.e. $k_1 > k_2$. The high gain direction is generally slower than the low gain direction ($T_1 > T_2$). This is a general property of ill-conditioned systems (Häggblom, 2014).

Friman (2020) noticed that open-loop identification of Eq. 4 is very practical, because using uncorrelated input excitation, the rotation matrix $\mathbf{W}$ can be identified from the output data using principal component analysis (PCA) before the actual identification step. As a result, the identification step simplifies from identification of high-order MIMO models in one step to identification of first-order models one output at a time. Simple ARX identification methods can be used.

3. CLOSED-LOOP IDENTIFICATION IN THE ROTATED DOMAIN

In system identification we must disturb the process in some way. Here we consider two methods: 1) setpoint excitation and 2) input excitation. These two methods are discussed below.

Closed-loop identification has two options: the direct and the indirect approach (Gustavsson et al., 1977). In this paper we consider the direct approach, i.e. we identify the process from inputs to outputs without utilizing controller knowledge.

3.1 Closed-Loop Identification Using Setpoint Excitation

We can use the same concept as Friman (2020) also for closed-loop identification of ill-conditioned systems, but with some modifications. The steps needed to identify the parameters of Eq. 4 using setpoint excitation are briefly discussed together with the heat exchanger (Jacobsen and Skogestad, 1994) model

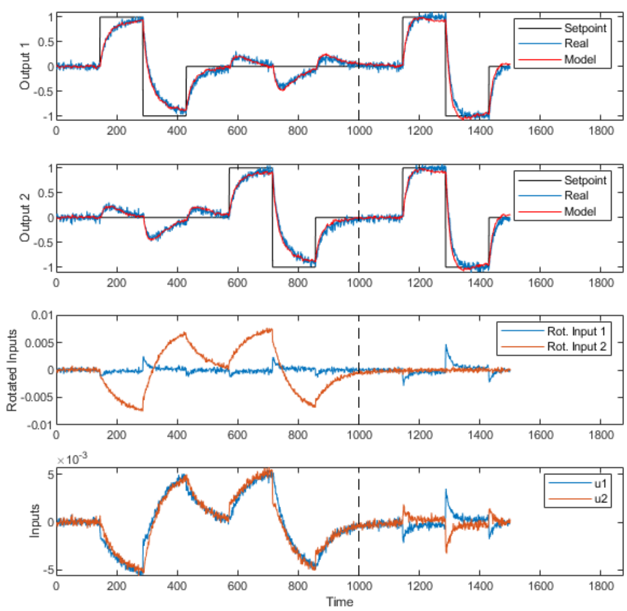

Fig. 1. Closed-loop identification of the heat exchanger example. The first part of the experiment $t < 1000$ is the actual identification experiment. An extension of the experiment, in this case with excitation in the high gain direction was done to improve the identification ($t > 1000$).

$$G(s) = \frac{89.243}{(T_1 s+1)(T_2 s+1)} \begin{pmatrix} -21(T_3 s+1) & 20 \\ -20 & 21(T_3 s+1) \end{pmatrix} \tag{5}$$

where $T_1 = 100$, $T_2 = 2.439$, and $T_3 = 4.762$. We compare each identification step to the corresponding open-loop step (marked "OLID" below). For a more detailed discussion about the identification concept we refer to Friman (2020).

We start by performing an identification experiment that employs a sequence of uncorrelated setpoint changes (OLID: sequence of uncorrelated inputs) and simple decentralized PI control. An example simulation is illustrated in Fig. 1 for $t < 1000$. PCA is scaling dependent, so we scaled the inputs (OLID: outputs) to zero mean and unit variance before applying PCA. PCA gives the input rotation matrix $\mathbf{V}$ (OLID: output rotation matrix $\mathbf{W}$). The upper two subplots show the outputs (blue) with corresponding setpoints (black lines), and the bottom plot shows the input trends. The rotated inputs (i.e. the principal components of the inputs) are plotted in the second plot from below.

A system identification method, e.g. ARX identification, gives a model from rotated inputs to outputs (OLID: from inputs to rotated outputs). We start by identifying the low gain (OLID: high gain) direction. With reference to Fig. 1, we identify the process from the first principal component (red "rotated input" trend) to the outputs. To use simple ARX identification note that $\mathbf{W}$ is an orthogonal rotation matrix with

$$\mathbf{W} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} = \begin{pmatrix} w_{22} & -w_{21} \\ w_{21} & w_{22} \end{pmatrix}, \mathbf{W^{-1}} = \begin{pmatrix} w_{22} & w_{21} \\ -w_{21} & w_{22} \end{pmatrix} \tag{6}$$

and that

$$\mathbf{W}^{-1}\mathbf{y}(s) = \mathbf{y_r}(s) = \begin{pmatrix} \dfrac{k_1 u_{r1}(s)}{T_1 s + 1} \\ \dfrac{k_2 u_{r2}(s)}{T_2 s + 1} \end{pmatrix} \tag{7}$$

where we use notation $\mathbf{y}$ for outputs, $\mathbf{u}$ for inputs, and the subscript r refers to rotated signals. With standard least-squares ARX identification, where we are restricted to multi-input-single-output models, we identify the (inverse) model from outputs to rotated inputs. To identify the low gain direction we have

$$k_2 u_{r2}(s) = (T_2 s + 1)(-w_{21} y_1(s) + w_{22} y_2(s)) \tag{8}$$

and we need to identify parameters a_1, a_2, b_1, b_2 in

$$u_{r2}(s) = a_1 s y_1(s) + a_2 s y_2(s) + b_1 y_1(s) + b_1 y_2(s) \tag{9}$$

to get the model parameters

$$\begin{aligned} k_2 &= \left(b_1^2 + b_2^2\right)^{-1/2} \\ w_{21} &= -b_1 k_2 \\ w_{22} &= b_2 k_2 \end{aligned} \tag{10}$$

At this point both rotation matrices $\mathbf{W}$ and $\mathbf{V}$ are known, so it is straightforward to identify the time constant T_2 and the high gain direction parameters k_1 and T_1 from rotated inputs to rotated outputs.

Model goodness evaluation is done by evaluating the high-gain (OLID: low-gain) fit. If the fit is good, the model has been identified, otherwise we extend the experiment with excitation in the high-gain (OLID: low-gain) direction.

In the example in Fig. 1 the rotated input 1 (blue "rotated input") hardly stands out from noise. Therefore we extended the experiment with setpoint excitation in the high-gain direction ($\mathbf{W}[1, 0]^\top$). At $t = 1500$, we re-identified the high gain parameters (k_1, T_1) by fitting a first-order system from the rotated input (blue line) to rotated output 1 (not shown). Note that, for the extended experiment, the rotation matrices remain unchanged, as they must be determined based on data from uncorrelated outputs ($t < 1000$).

The closed-loop identification method is summarized in Table 1 for a 2×2 system.

3.2 Closed-Loop Identification Using Input Excitation

With closed-loop identification using input excitation we mean adding uncorrelated input disturbances to the inputs, and keeping the setpoints constant. This setup generates highly correlated outputs, and we can use the open-loop identification concept (Friman, 2020) to identify the model.

4. THE ILL-CONDITIONED SYSTEMS IDENTIFICATION DUALITY

In this section we compare open-loop identification and closed-loop identification of ill-conditioned systems. We compare the open-loop concept introduced by Friman (2020) with the closed-loop identification concept suggested in section 3.1 and summarized in Table 1.

In open-loop we use uncorrelated input excitation, which produces highly correlated outputs, and PCA analysis of the outputs gives the output rotation matrix $\mathbf{W}$. Good identification of high gain (k_1, T_1) and difficulties in low gain (k_2, T_2) identification are expected with noise and disturbances present during identification experiment.

For closed-loop identification the situation is the opposite. With uncorrelated outputs, we can employ PCA analysis of the highly correlated inputs to obtain the input rotation matrix $\mathbf{V}$. Good identification of low gain (k_2, T_2) and difficulties in high gain (k_1, T_1) direction are expected in non-ideal circumstances.

The duality for a 2×2 ill-conditioned system is summarized in in Table 2, and the implications for successful identification are discussed below.

5. IMPACT ON IDENTIFICATION RELIABILITY

There are two main pitfalls in control-relevant identification of ill-conditioned systems. First, it is easy to have insufficient perturbation in some gain direction. Secondly, it common to evaluate model goodness in the output domain, and in that case model goodness assessment only considers the gain directions that are present in the data, and the model fit might look good even though it is not.

Table 1. Summary of closed-loop identification using input rotation

Step	Description
1	Perform a standard identification experiment consisting of setpoint excitation. At this point the setpoints must be uncorrelated, preferable steps.
2	A PCA analysis of the inputs gives the rotated inputs $\mathbf{u_r}$ (the first and second principal components). PCA also gives the input rotation matrix $\mathbf{V}$.
3	Identify the output rotation matrix $\mathbf{W}$ (Eq. 6 - 10) and calculate the rotated outputs $\mathbf{y_r}$
4	Using rotated inputs and rotated outputs, identify gains and time constants of all gain directions. Simple SISO ARX identification can be utilized for each rotated input / rotated output pair.
5	Evaluate model goodness, i.e. the high gain model fit.
6	If evaluation result in step 5 is good, model is identified, otherwise continue with setpoint excitation in the high gain direction ($\mathbf{W}[1, 0]^\top$), re-identify the high gain direction (k_1 and T_1) and go to step 5.

Table 2. The Ill-Conditioned Systems Identification Duality

	Open-Loop	Closed-Loop
Assumptions	uncorrelated inputs highly correlated outputs	uncorrelated outputs highly correlated inputs
PCA analysis of	outputs	inputs
... gives	output rotation matrix $\mathbf{W}$	input rotation matrix $\mathbf{V}$
Reliable identification of	high gain direction	low gain direction
... also gives	input rotation matrix $\mathbf{V}$	output rotation matrix $\mathbf{W}$
Model goodness assessment	low gain fit	high gain fit

A typical case where modeling fails, is when we use an open-loop experiment with uncorrelated inputs and identify a model from physical inputs to physical outputs. In that case the process is mainly excited in the high-gain direction, and the model fit looks usually good even though its not. Ignoring the low-gain direction in both identification and model assessment easily results in a model that is useless for control (Jacobsen and Skogestad, 1994).

According to the identification duality in Table 2, the open-loop and closed-loop experiments perfectly complement each other. Hence, one possibility would be to perform both experiments and to pick the best parts from each experiment. This is, however, not very practical. A better way is to choose either method, and to extend the experiment with perturbation in the weakly identified direction when needed (e.g. Fig. 1 for $t > 1000$).

On a general level, we can conclude that identification experiments that uses either uncorrelated inputs or uncorrelated outputs may be incomplete. To perturb all gain directions, we should design the experiment such that we have uncorrelated inputs in one part, and uncorrelated outputs in the rest of the experiment. Input design methods that generate uncorrelated outputs of multi-variable systems have been suggested (e.g. Sadabadi and Poshtan (2009), Kumar and Narasimhan (2016), Häggblom (2018), Häggblom (2019)). These experiments provide similar results as the closed-loop identification concept suggested here, but with the advantage of inputs being unaffected by output noise. However, these methods poorly perturb the process in the high gain direction, so model assessment in the high-gain direction (using e.g. rotation matrices as suggested above) is highly recommended for these methods.

5.1 Integral Controllability

With traditional modeling (models Eq. 2 and 3) it may be difficult to ensure integral controllability (IC). For example, for the heat exchanger (Eq. 5), IC is lost if any gain element deviates more than 10% in an unfavorable direction. On the other hand, with the rotated model (Eq. 4) IC is obtained when the rotation angles deviate less than $90°$ when we select positive gains in all gain directions. Note that Table 2 suggests that the rotation matrices are well identified in both open-loop and closed-loop cases so IC is practically always obtained when identification is done in the rotated domain.

6. SUMMARY AND CONCLUSIONS

We have extended the practical concept of ill-conditioned systems identification in the rotated domain (Friman, 2020) from open-loop to closed-loop. The focus is on practical and simple solutions. No process knowledge is needed prior to the identification experiment, simple ARX identification one output at a time can be used, and minimum number of model parameters and states are used in the models. Still, the suggested models can capture the directionality of ill-conditioned systems, which is important when models are used for control.

The closed-loop option is important, because closed-loop identification experiments can often be performed during production, which is valuable in industrial applications.

By identifying the process in the rotated domain we can avoid many pitfalls associated with ill-conditioned systems. It is easy to ensure integral controllability. The key to successful control-relevant identification is to properly identify each gain direction. With a possibility to evaluate model fit in each gain direction, we know when we need to extend the identification experiment with excitation in the most problematic gain direction. This ensures good models for control.

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